AN APPROACH TO DECISION MAKING UNDER UNCERTAINTY

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Abstract. Decision making in real environment is under consideration. An approach based on modeling by precedence and intended for controlling systems with an indeterminate change of outer and inner conditions is proposed. A possibility of realizing the approach in a class of discrete events systems is considered.

Introduction

A decision-making problem (DMP) always arises when it is necessary to solve a problem of an effective control of the behavior or functioning of systems, processes, etc. In a broad sense it is investigated in the field of cybernetics and is typical for operations research problems. Numerous theoretical and practical results have been obtained in the framework of these disciplines. At the same time a decision-making problem still remains a topical one.

The development of telecommunication, information and computer technologies creates the necessary prerequisites for the accumulation and processing of a great amount of data in an electronic form. There arises a question concerning an effective use of the data that calls for automation of many processes including decision making.

Each particular DMP has peculiar features that are naturally reflected in the information and thus influence the approach to decision making. The necessity of decision making in new, sometimes weakly formalized (with relation to mathematics) fields calls for the need of the advancement of the existing approaches and the development of new ones. The solution of DMPs under uncertainty holds a particular scientific and practical interest. An approach to decision making in changing environment based on modeling by precedence [1] is proposed in the paper.

Decision-making problems under uncertainty

Mathematical methods, in the absence of the necessary technical base, are usually oriented to the solution of problems under laboratory conditions and on numerous occasions are of theoretical nature. The disregard of interference and dynamics of the outer environment does not allow to reach an effective solution of many important practical problems. A possibility of extraction, processing and use of a large amount of information (including real-time mode) created the necessary prerequisites for the solution of problems in real environment. Simultaneously this called for the development of distinctly new investigation methods oriented for practical implementation.

The degree of adequacy of the made decisions in many respects depends on the successful choice of the formalization language (deterministic, probabilistic or fuzzy) and a decision-making model (analytic, simulation or situation, etc.). They in their turn depend on the completeness of information and the attitude of the researcher to its nature. The central problem of modeling is an overcoming of uncertainty concerned with additional structuring, addition and elimination of contradictions in the source information.

In the course of real systems functioning decision making, as a rule, is carried out repeatedly. DMPs in this case are solved on the basis of the analysis of the current state depending on functioning conditions (decision-making situations). Following the operation of the system a sequence of situations is determined and, consequently, a natural description in terms of situation control is allowed.

Under serial solution of one-type problems it is topical to use the accumulated experience: known and justified regularities, expert properties of the object or samples of problems that have been considered. Under real (weakly formalized) environment examples are often the only possible information. As a result a decision making can be carried out on the basis of modeling by precedence: comparison of the current situation with the known examples and choice of an alternative effective in a "similar" situation. Modeling by precedence is realized in the framework of the mathematical theory of pattern recognition.

Decision making as a problem of a choice of control action

In the course of operation of real systems a non-determined change of outer and inner conditions of their functioning takes place. The changes are conditioned by numerous unknown factors (*deforming action Γ*), the formalization of which can't be done. As a result of such deformation a system transition to a new state takes place (observed as *a deformation trace)*. The change of state requires a real-time intervention and realization of *a redeforming control action* Γ^{-1} (see Fig.1).

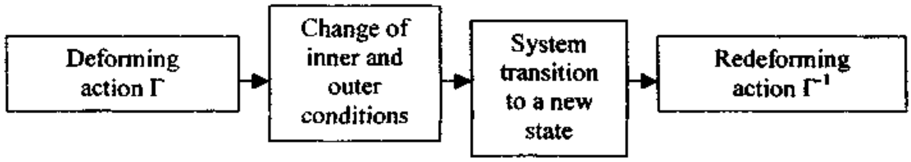

Fig.1

Under such conditions a real-time control of the system is carried out on the basis of tracking according to the following scheme.

Let d_o is an initial state of the system. Let $d_k = d_o$.

1. We choose a control action (control) $a^*(d_k)$, effective in the current state d_k.
2. If at a certain stage $a^*(d_k)$ looses its effectiveness (i.e. it ceases to correspond to the state of the system), then we set k= k +1 and go to item 1.

Realization of the described scheme of control with feedback comes to:
- building of a set of allowable types of control (alternatives);
- determination of the extent of local effectiveness;
- detection of the decision-taking moment (detection of the discord situation);
- choice of control on the basis of the system state evaluation (decision making);
- realization of the solution.

As a result a problem of control can be described by the quadruple <A, Q, R, P>, where A is a set of allowable alternatives; Q is the extent of local effectiveness; R and P are respectively a rule for determining the discord situation and the choice of an effective alternative (decision-making algorithm).

Thus, the above scheme is naturally realized in the framework of the known problem of situation control. In this case a change of control reduces to a DMP which, in its turn, is solved when analyzing the current state of the system by observations. A link between observations and the state of the system is weakly formalized as a rule, hence a DMP is considered under uncertainty and requires untraditional methods of investigation.

Let M is a system and D is a set of its states. Any $d \in D$ is determined by outer and inner conditions of functioning. Let's call a set of these conditions *a situation*.

It's easy to see that D can be represented as:

$$D = D^{+} \cup D^{-}, \quad (1)$$

where D^{+} and D^{-} are subsets of normal states (corresponding to the aim of control) and conditions of discord. It's evident that only the states of d that belong to the subset D^{-} should be analyzed.

Each $d \in D$ is associated with a set of allowable alternatives $A(d)$, ($A = \bigcup_{d \in D} A(d)$ are allowable alternatives of the system M). Then the extent of local effectiveness can be specified by the representation Q: $S \times A \rightarrow [0,\infty]$.

The state of discord is determined by tracking the system dynamics on software and hardware levels. One possible solution of this problem, for example, is associated with the construction of splitting (1) and subsequent classification of the current state.

Let's consider a problem of choosing alternatives (DMP).

Let S^{-} is a set of situations typical for D^{-}. For an arbitrary situation $s \in S^{-}$ it is necessary to build locally optimal control satisfying the condition

$$Q(s, a^{*}(s)) = \max_{a \in A}(Q(s,a)). \quad (2)$$

In the course of system operation there arises numerous one-type problems Z_C represented in (2). Each $z \in Z_C$ is a labour-consuming problem of a searching type demanding too much time. But when problems have a serial nature it is possible to use the knowledge of the considered DMPs and their solution. In this case a decision making can be modeled on the basis of precedence.

The suggested approach is well realized in the framework of the mathematical theory of pattern recognition [1]. "Close" conditions of functioning (situations) are associated with "close" states of the system (the number of which is finite). As a result S^{-} is naturally divided into subsets of one-type situations

$$S^{-} = \bigcup_{i=1}^{N} S_i, \ N<\infty, \quad (3)$$

having a nonempty subset of common solutions $A_i^{*} \subseteq A$ for all $s \in S_i$, $1 \le i \le N$. In this case the solution of the problem <s,A,Q>, $s \in S$ can be reduced to the classification of the situation s with respect to splitting (3) with subsequent choice of any control from $A_{i(s)}^{*}$, $1 \le i(s) \le N$. It is evident that the classification should be made with the accuracy up to the subset of classes $\{A_{i_1}^{*},\dots,A_{i_k}^{*}\}$, $1 \le i_1,\dots,i_k \le N$ such that $\bigcap_{j=1}^{k} A_{i_j}^{*} \ne \varnothing$

As splitting (3) is unknown, recognition with training can be used for the realization of the approach. In this case it is necessary to fulfill the following stages:

- construction of a representative sampling;
- modeling and formation of data (training information);
- data preprocessing;
- modeling of the environment (determination of splitting (3) and construction of subsets A_i^{*}; $1 \le i \le N$);
- construction of a deciding classification rule.
- the training sample $Z_M^0 \subset Z_M$ should satisfy the following conditions:
- the set $S_M^0 = \{s \in : \langle s,A,Q \rangle \in Z_M^0\}$ contains all types of situations allowable for S^{-};

- $\| Z_M^o \| << \| Z_M \|$.

The training information is formed by the solution (complete searching of A) of problems from Z_M^o and can be represented as $I(Z_M^o)=\{(m(s), A_s^*): s \in S_M^o\}$, where m(s) is a description of situation s, A_s^* is a set of "effective" for s control actions.

Preprocessing of the training information can be reduced to the solution of the following problems.

1. Construction of a subset of identical situations

$$S_M^o = \bigcup_{j=1}^{q} I_j . \tag{4}$$

When the parameter q is unknown splitting (4) can be obtained by clusterization with the outer aim without a tutor. In this case for each I_j, $1 \le j \le q$ the general information $m(I_j)$ is selected and subsets $A^*(I_j)=\bigcap_{s \in I_j} A_s^*$ are constructed. $I(Z_C^o)$ is replaced by the pair $\{(m(I_j), A^*(I_j)), 1 \le j \le q$.

2. Formation of space for describing situations $s \in S$.
3. Determination of local optimum threshold of δ.

Environment modeling (training) reduces to the solution of three major problems.

1. Splitting into classes of close situations (classes of equivalence)

$$S^- = \bigcup_{j=1}^{L} K_j , \tag{5}$$

$$K_i=\{ I_{i_1},\ldots,I_{i_{t(i)}} \} \Rightarrow A^*(K_i)= \bigcap_{j=1}^{t(i)} A^*(I_{i_j}) \neq \emptyset,\ i=1,\ldots,L. \tag{6}$$

(When the parameter L is unknown splitting (5) can be also obtained by clusterization with the outer aim without a tutor (6)).

2. Construction of a complete and irredundant set of solutions $A^*(S) \subseteq \bigcup_{i=1}^{N} A_i^*$.

(It is possible to show that the given problem is equal to a minimal cover problem).

3. Determination of orders possible on the set (5).

The analysis of the structure allows to determine properties contributing to simplification of choice control procedure.

Conclusion

A possibility of applying the described approach to systems with discrete events (with an unpredictable spasmodic change of state at random discrete time moments) is discussed [2].

References

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