AN AUTOMATED PROCEDURE OF INDENTIFYING *SARIMA* MODELS FOR MACROECONOMIC TIME SERIES

Kravtsov M.K., Luka Y.Z., Podkopaev D.P.

Economic Research Institute of the Ministry of Economy of the Republic of Belarus, Department of economico-mathematical modeling, Slavinskogo str., 1/1, 105, e-mail: The_philosopher@mail.ru

Abstract: An automated procedure of building the best *SARIMA* model for a given time series is developed and implemented. Adjusted mean squared error of one-step-ahead forecast, mean absolute percentage value and mean relative range of confidence intervals are chosen as selection criteria. Short-term and mid-term forecasts of some indicators of the commodity-producing sector of Belorussian economy are built.

Mathematical models and software tools are used in forecasting the social and economic evolution of the Republic of Belarus. They provide the opportunity of a more profound study of economic mechanisms, producing well-grounded conclusions and states, finding of optimal economic solutions, estimating the consequences of implementing particular administrative decisions or complex measures of economic policy within the bounds of the state economic regulation. However, up to now there are no functioning models of the transitional economy as a single whole, which could describe all the economic processes adequately and fully enough. In practice, only models of separate economic sectors are created, operating with indices of one or several profiles [6, 4, 9, 10].

There are two quite different approaches in the forecasting of different indicators, macroeconomic particularly, in the theory of mathematical statistics [1]. Each of them has many variants. The first approach is based on discovering of causal-investigatory mechanism of the explored process and requires that factors affecting the behavior of indicator under investigation are taken into account. Consecutive derivation of this approach results in econometric modeling of macroeconomic processes by means of systems of simultaneous equations [5, 11]. This approach allows to come from kinematic description of analyzed indicator to its dynamics, because it reveals driving force, influencing on explored object. However, using complicated econometric models for forecasting of macroeconomic indices of the Republic of Belarus is difficult because of the volatility of economic conjuncture, insufficiency and unreliability of statistical data and problems with adaptation this data to National Accounts System. A second approach consists of isolated analysis of only past observed values of the macroeconomic indicator under investigation. It is based on the methods of time series forecasting. One of such methods involves building up *SARIMA* models.

Wide class of random processes may be described by means of stochastic seasonal autoregressive and integrated moving average (*SARIMA*) models. Henceforth we use a standard notation, introduced in [2, 13]. In this notation, a mentioned model is denoted by $SARIMA(p, d, q)(P, D, Q)_s$, were p is the number of auto-regressive parameters; d is the number of differencing passes; q is the number of moving average parameters, P, D, Q – are the corresponding seasonal parameters; s is the seasonal lag, i. e. the number of observations in one seasonal cycle. Each model has $p + d + q + P + D + Q+1$ unknown parameters, if a constant was included into it and $p + d + q + P + D + Q$ unknown parameters otherwise. Building up *SARIMA* model is reduced to finding such a combination of these parameters that given this combination the model will be the best in terms of particular criteria. Box and Jenkins [2] proposed a procedure for identifying p, d, q, P, D and Q, that includes analysis of estimated auto-correlation and partial auto-correlation functions' values and shapes of correlogram and partial correlogram. But that procedure is complicated and not easily conducted [3].

We implemented an automated procedure for selecting the best *SARIMA* model of a given time series. This procedure is used to obtain short-term and mid-term forecasts of macroeconomic indicators of the commodity-producing sector of Belorussian economy. The list of considered indicators includes 55 entries, namely, growth rates of GDP and its basic components (6 indicators); gross output in industry, agriculture, construction and trade (6 indices); particular types of industry output (27 indices) and agriculture output (9 indices); output of transport and communications (6 indices), et al. The data sources are monthly bulletin of the Ministry of statistics and analysis of the Republic of Belarus from January 1994 to March 2002 [14] and quarterly data of Annual statistical collection "Quarterly calculations of GDP" from 1992 to 2001 [12].

The following characteristics were chosen as criteria for selecting the best models: adjusted mean squared error of one-step-ahead forecast (*MS*), mean absolute percentage value (*MAPE*) and mean relative range of confidence intervals (*Up-Low*).

Adjusted mean squared error is given by formula [7]:

$$MS = \frac{1}{N}\sum_{i=1}^{N}(x_t - \tilde{x}_t)^2 ,$$

where $N=n-p-q-P-D^*s-Q$, x_t and $\tilde{x}_t$ are the observed value and forecast at time t correspondingly, n is the length of the time series

Mean absolute percentage value is calculated as:

$$MAPE = \frac{100}{\tau}\sum_{t=1}^{\tau}\frac{|x_t - \tilde{x}_t|}{x_t},$$

where x_t and $\tilde{x}_t$ are observed value and forecast at a time t correspondingly, and $\tau = n_2 - n_1 + 1$ – is the forecasting span ($0 < n_1 \le n_2 \le n$). This value is the relative error of a retrospective forecast. It has two important properties. First, it is very illustrative and clear for economists. Secondly, this characteristic is not dependent on measured units. That is why the opportunity to compare forecasts of different indices appears. It should be noted, that *MS* does not possess of such properties and is not so easily interpretable.

Mean relative range of confidence intervals is computed as:

$$Up\text{-}Low = \frac{100}{2\tau}\sum_{t=1}^{\tau}\frac{|I_{up} - I_{low}|}{x_t},$$

where I_{up} and I_{low} - are respectively upper and lower confidence limit; τ - is the time span of forecasting.

There are other criteria for selecting models, for example Akaike information criterion (AIC), F-adjusted AIC, Hannan-Quinn criterion and Bayesian information criterion. It would be reasonable to develop an automated procedure, which chooses the model meeting all the above-mentioned criteria. But this problem in general form is rather complicated because of its multi-criterion nature. In addition, we don't know a statistical package, where all mentioned criteria are implemented. For example, in *Statistica 6.0* both AIC and BIC criteria for time series are not realized. That is why the problem of choosing the best model is considered in a single-criterion definition.

The automated procedure consists of the following operations:

- building up *SARIMA* models, obtained as a result of running over all combinations of its parameters, belonging to a particular set of values;
- calculation of various characteristics (*MS, MAPE, Up-Low*), that determine quality and adequacy of the models obtained;
- Choosing the best model in terms of minimization of one of three criteria: *MS, MAPE, Up-Low*.

SARIMA models with $p, q, P, Q \in \{0;1;2;3\}$ and $d, D \in \{0;1;2\}$ were examined. A total of 2295 models were considered. ($2295=4\cdot3\cdot4\cdot4\cdot3\cdot4 - 9$. Here 9 are the combinations with $p=q=P=Q=0$ and $d, D \in \{0;1;2\}$, because for launching the procedure at least one positive value among p, q, P and Q is needed).

Three year's time span of retrospective forecast was chosen.

The exact maximum likelihood method was used to estimate coefficients. This method implemented by means of Melard's algorithm [8].

A model was included in the consideration if its coefficients met two conditions: they all were significant and their absolute values were less than one (to ensure stationarity).

If $d=D=0$ then a constant was included into the model, for the rest it hadn't, because differenced series always of zero mean.

Calculations were made on Pentium III-750, 128 RAM. Full running over, building up of 2295 models and choosing of the best of them took approximately one hour per one series.

The best forecasts are obtained for the indicators of transport and communications and agriculture (15 indicators in all). For the 12 the minimal *MAPE* was less than 7%. For the indicators of growth rates of GDP and its basic components and for particular types of output in industry (40 indicators in total) the results are less accurate. For 33 of 40 indicators *MAPE* was no more than 15%. As a whole, 8 of 55 indicators have models with *MAPE* less than 5%; 26 – less than 10%; 49 – less than 15%. Among the models with minimal *MS*, 5 models have *MAPE* less than 5%; 21 – less than 10%; 29 – less than 15%. Absolutely minimal *MAPE* was 0.22% and absolutely minimal *Up-Low* – 2.3%.

Conclusions

On the basis of carried out analysis of *SARIMA* models built up for 55 mentioned time series of macroeconomic indicators, the following conclusions are made.

- *SARIMA* model with minimal *MS* does not necessarily have minimal *MAPE*. Only three of 55 indicators have models with both minimal *MS* and minimal *MAPE*;
- there are several models for any indicator under analysis with all significant coefficients, by means of which one can obtain relatively accurate retrospective forecast. For example, $SARIMA(1,1,1)(0,0,2)_{12}$ model for the number of passengers carried by all types of general–purpose transport has *MAPE*=4.46%, *MS*=45.81, *Up-Low*=31.47%. All of these characteristics are worse than that of the model $SARIMA(1,1,3)(0,0,2)_{12}$, but *MAPE* is not large (less than 5%) so this model may be used for forecasting along with the model $SARIMA(1,1,3)(0,0,2)_{12}$;
- For any indicator there are models with at least one non-significant coefficient, which have better values of *MS*, *MAPE* and *Up-Low* in comparison with the models with all significant coefficients;

- Some indicators may be forecasted accurately by means of even non-stationary models. For example, $SARIMA(1,1,1)(3,0,3)_{12}$ model for the number of livestock has $MAPE$=0,696%, at the same time this model has no significant coefficients;
- there are only 100-150 models with all significant coefficients from the total of 2295 analyzed. Significance of all of the coefficients does not imply the model is accurate in forecasting. For example, $SARIMA(3,2,2)(0,1,1)_{12}$ model for the volume of primary oil refining has all significant coefficients but very unstable in forecast. There are contrary examples. $SARIMA(3,0,3)(1,1,3)_{12}$ model for the volume of beer output has non-significant coefficients, but produce relatively accurate retrospective forecast. On the whole, removal of significance restriction from the model coefficients often allows improvement in model characteristics;
- It is considered [2], that the numbers of the p, q, P or Q parameters very rarely need to be greater than 2. In practice, 15 models with at least one of the p, q, P or Q parameters greater then 2 were found among the models, which have minimal *MS*, 15 – among the models, which have minimal *MAPE*, 18 – among the models, which have minimal *Up-Low*.
- Residuals of the models with minimal *MAPE* and minimal *Up-Low* deviate much more from normal distribution than those of the models with minimal MS as a rule. That is why we recommend preference of models with minimal MS when MAPE of that models is sufficiently small;
- Reducing the time span for retrospective forecast usually allows for the improvement of minimal *MAPE* and *Up-Low*. However, generally speaking, the parameters of a model may vary at the same time. Let us consider the time series of beer production volume. As the forecasting time span was reduced to 12 month, minimal MAPE for $SARIMA(1,0,1)(3,2,0)_{12}$ became 9.12%, that is 2.96% less than minimal *MAPE* for three-year forecasting time span (attained by $SARIMA(0,1,0)(1,1,0)_{12}$). It should be noted, that for above-mentioned $SARIMA(0,1,0)(1,1,0)_{12}$ *MAPE* increased from 12% to 14%, as the time span was reduced to 12 months. Thus, *MAPE* is not a characteristic of a specific model, but a class of models. Minimal *MAPE* value means how accurate retrospective forecast given time span one can obtain considering a particular subset of *SARIMA* models. For the models with minimal *MS* the value of *MAPE* can be improved greatly. Let us consider $SARIMA(3,0,0)(3,1,0)_{12}$ model of the above mentioned indicator of beer production. Its *MAPE* decreased from 29.62% to 12.07% and *Up-Low* – from 54.3% to 36.11% when reducing forecasting time span to one year. It should be pointed out, that in this case *MAPE* became almost as low as minimal: it increased only by 2.95%. This fact allows us to suppose, that *MS* is more preferred as a criterion than *MAPE*.

Now let us summarize all the above-mentioned facts and give some practical recommendations for choosing *SARIMA* models. First, one must select models, that meet several criteria at the same time. We found only three time series from 55, for which such models exist. For the rest we recommend that a certain threshold *MAPE* value is specified and find models with minimal *MS*, which meet this restriction. Only when no such models' exist should one take models with minimal *MAPE*. Suppose, for example, that threshold *MAPE* value is equal to 5%. Then for the time series of passengers' number, carried by all types of transport $SARIMA(1,1,3)(0,0,2)_{12}$ model with minimal *MS* and *MAPE* equal to 3.81% should be selected rather than $SARIMA(0,0,0)(1,1,0)_{12}$ model with minimal *MAPE* equal to 3.08%. The threshold value suggested should not be higher than 10-15%. With larger threshold values forecasting error greatly increases. In this case using such a forecast may become inadmissible. If the threshold *MAPE* value is supposed to be equal to 10%, then we can find satisfactory models for 26 indicators from 55, which is almost a half of a total. However, as we said earlier, *MAPE* usually improves when reducing a retrospective forecast's time span. Thereby maximum threshold *MAPE* value for short-term forecasting can be increased by 2-3%. For example, having increased threshold value by 2% we expand the set of indicators, for which satisfactory models exist, up to 42.

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