Potential for a new muon g-2 experiment

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A new high-precision experiment to measure the muon g-2 factor is proposed. The developed experiment can be performed on an ordinary storage ring with a noncontinuous and nonuniform field. When the total length of straight sections of the ring is appropriate, the spin rotation frequency becomes almost independent of the particle momentum. In this case, a high-precision measurement of an average magnetic field can be carried out with polarized proton beams. A muon beam energy can be arbitrary. Possibilities to avoid a betatron resonance are analyzed and corrections to the g-2 frequency are considered.

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I. INTRODUCTION

Measurement of the anomalous magnetic moment of the muon is very important because it can in principle bring a discovery of new physics. Experimental data dominated by the BNL E821 experiment, $a_{\mu\pm}^{exp} = 116592080(63) \times 10^{-11}$ (0.54 ppm), are not consistent with the theoretical result, $a_{\mu\pm}^{the} = 116591790(65)\times 10^{-11}$, where $a = (g-2)/2$. The discrepancy is 3.2σ: $a_{\mu\pm}^{exp} - a_{\mu\pm}^{the} = +290(90) \times 10^{-11}$ [1]. In this situation, the existence of the inconsistency should be confirmed by new experiments. The past BNL E821 experiment [2] was based on the use of electrostatic focusing at the "magic" beam momentum $p_m = mc/\sqrt{a}$ ($\gamma_m = \sqrt{1 + 1/a} \approx 29.3$). An upgraded (but not started up) experiment, E969 [3], with goals of $\sigma_{syst} = 0.14$ ppm and $\sigma_{stat} = 0.20$ ppm is based on the same principle.

Since the muon g-2 experiment is very important, a search for new methods of its performing is necessary. One of new methods has been proposed by Farley [4]. Its main distinctions from the usual g-2 experiments are *i*) noncontinuous magnetic field which is uniform into circular sectors, *ii*) edge focusing, and *iii*) measurement of an average magnetic field with polarized proton beams instead of protons at rest. A chosen energy of muons can be different from the "magic" energy. Its increasing prolongs the lab lifetime of muons. As a result, a measurement of muon g-2 at the level of 0.03 ppm appears feasible [4].

In the present work, we develop the ideas by Farley. We adopt his propositions to measure the average magnetic field with polarized proton beams and to use a ring with a noncontinuous field for keeping the independence of the spin rotation frequency from the particle momentum. We also investigate the most interesting case when the beam energy can be arbitrary. However, we propose to perform the high-precision muon g-2 experiment on an ordinary storage ring with a nonuniform field created by superconducting magnets. We prove that the independence of the spin rotation frequency from the particle momentum can be reached not only in a continuous uniform magnetic field [2, 3] and a noncontinuous and locally uniform one [4] but also in a usual storage ring with a noncontinuous and nonuniform magnetic field. In the last case, the total length of straight sections of the ring should be appropriate. We also analyze possibilities to avoid the betatron resonance $\nu_x = 1$ (ν_x is the horizontal tune) and consider corrections to the g-2 frequency.

The system of units $\hbar = c = 1$ is used.

II. g-2 RING WITH A NONCONTINUOUS MAGNETIC FIELD AND MAGNETIC FOCUSING

Let us consider spin dynamics in a usual storage ring with a noncontinuous magnetic field and magnetic focusing. The general equation for the angular velocity of spin precession in the cylindrical coordinates is given by (see Ref. [5])

$$\begin{aligned}\boldsymbol{\omega}^{(a)} = -\frac{e}{m}\Biggl\{ a\boldsymbol{B} - \frac{a\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{B}) \\ +\frac{1}{\gamma}\left[\boldsymbol{B}_{\parallel} - \frac{1}{\beta^2}\left(\boldsymbol{\beta}\times\boldsymbol{E}\right)_{\parallel}\right] \\ +\frac{\eta}{2}\left(\boldsymbol{E} - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{E}) + \boldsymbol{\beta}\times\boldsymbol{B}\right)\Biggr\}, \quad \boldsymbol{\beta} = \frac{\boldsymbol{v}}{c}. \qquad (1)\end{aligned}$$

Eq. (1) is useful for analytical calculations of spin dynamics with allowance for field misalignments and beam oscillations. This equation does not contain small terms which can be neglected. $\eta = 4dm/e$ is an analogue of the g factor for the electric dipole moment, d. The sign $\parallel$ denotes a horizontal projection for any vector. Thereinafter, the electric dipole moment will be disregarded. The vertical magnetic field, B_z, is the main field in the muon g-2 experiment. The spin precession caused by this

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field is defined by

$$\omega_z^{(a)} = -\frac{e}{m} a B_z. \qquad (2)$$

Let $\mathbf{\Omega}^{(a)}$ denotes the average value of $\boldsymbol{\omega}^{(a)}$. The spin coherence is kept when

$$\frac{d\Omega_z^{(a)}}{dp} = 0. \qquad (3)$$

For a storage ring with a noncontinuous field, the quantity B_z should be averaged.

This condition defines a spin-isochronous ring, i.e., the spin precession frequency is independent of the momentum at the first order.

Condition (3) can be satisfied for ordinary storage rings with magnets creating nonuniform field (Fig. 1). Beam direction is normal to the magnet faces and there is not edge focusing. The number of bending sections can be different. If the field created by the magnets is given by $B_z(\rho) = const \cdot \rho^{-n}$, the field index and betatron tunes into bending sections are equal to

$$n = -\frac{R_0}{B_0}\left(\frac{\partial B_z}{\partial \rho}\right)_{\rho=R_0}, \quad \nu_x^{(b)} = \sqrt{1-n}, \quad \nu_z^{(b)} = \sqrt{n},$$

where $B_0 \equiv B_z(R_0)$, $x = \rho - R_0$, and R_0 is the ring radius. Average angular frequency of spin precession is given by

$$\Omega^{(a)} = \frac{\omega_z^{(a)}\pi\rho}{\pi\rho + L} = -\frac{\pi e a \rho B_z(\rho)}{m(\pi\rho + L)}, \qquad (4)$$

where L is a half of the total length of the straight sections (Fig. 1). The muon anomaly is equal to

$$a_\mu = \frac{g_p - 2}{2}\frac{m_\mu}{m_p}\frac{\Omega_\mu^{(a)}}{\Omega_p^{(a)}}, \qquad (5)$$

where the fundamental constants g_p and m_μ/m_p are measured with a high precision. The magnetic field is the same for muons and protons when they move on the same trajectory. In this case, their momenta coincide.

When the momentum increases ($p > p_0$), the magnetic field becomes weaker, but the time of flight in the magnetic field becomes longer. The spin precession is defined by

$$\frac{d\Omega_z^{(a)}}{dp} = \frac{d\Omega_z^{(a)}}{d\rho}\left(\frac{dp}{d\rho}\right)^{-1}, \quad \frac{dp}{d\rho} = (1-n)eB_z(\rho). \quad (6)$$

Condition (3) leads to $d\Omega_z^{(a)}/d\rho = 0$ and is satisfied when

$$L = L_0 = \frac{n}{1-n}\pi R_0, \qquad (7)$$

where R_0 corresponds to p_0 and B_0. In this case

$$R_0 = \frac{p_0}{|e|B_0}, \quad \Omega^{(a)} = \Omega_0^{(a)} = -(1-n)\frac{eaB_0}{m}, \qquad (8)$$

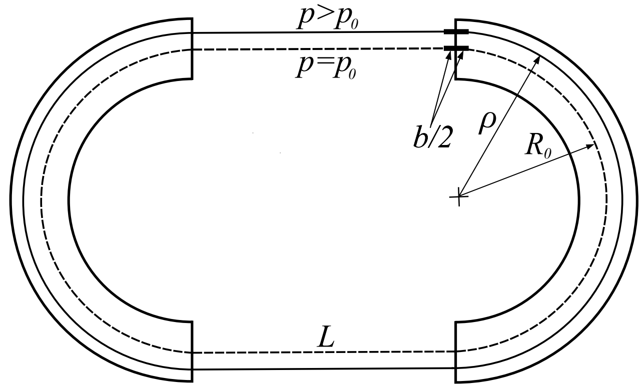

FIG. 1: The storage ring.

and the following relation takes place:

$$\frac{\Delta C}{C_0} = \frac{\Delta p}{p_0} = (1-n)\frac{x}{R_0},$$

where C is the orbit circumference. As a result, the momentum compaction factor is

$$\alpha = \frac{\Delta C/C_0}{\Delta p/p_0} = 1. \qquad (9)$$

Since

$$\frac{\Delta C}{C_0} = \frac{1}{\gamma_0^2}\frac{\Delta p}{p_0} + \frac{\Delta T}{T_0},$$

where T is the revolution period, the definition of α can be brought to the usual form:

$$\frac{\Delta T}{T_0} = \left(\alpha - \frac{1}{\gamma_0^2}\right)\frac{\Delta p}{p_0}.$$

Evidently, the spin-isochronous ring ($\alpha = 1$) is not isochronous in the usual sense, i.e., the beam revolution frequency depends on the momentum.

Eq. (7) is not exact because it does not include a correction for the fringe field. This field also contributes to the average field, but it is independent of ρ. The fringe field is important only near the magnet edges and causes the correction to L_0 of order of the ratio of the magnet gap to the ring radius ($\sim 10^{-2}$). This correction depends on the number of the straight sections and can be analytically and numerically calculated because the magnet field is known with a needed accuracy.

Evidently, the correction to the local value of $\omega_z^{(a)}$ is given by

$$\delta\omega_z^{(a)}/\omega_z^{(a)} = \delta B_z/B_z.$$

The corrected values of L_0 also coincide for muons and protons because particles with equal momenta move in the same field.

Two other corrections to the angular velocity of spin precession caused by the longitudinal magnetic field and the vertical betatron oscillations are considered in Section IV. While these corrections are different for the muons and protons, they are rather small (~ 1 ppm).

The real value of the length of the straight section, L, can slightly differ from L_0. In the general case,

$$\alpha = \frac{\pi R_0}{\pi R_0 + L - L_0}. \quad (10)$$

The difference between the real and nominal values of the average angular frequency of spin rotation is given by

$$\frac{\Omega^{(a)} - \Omega_0^{(a)}}{\Omega_0^{(a)}} = n \cdot \frac{L - L_0}{L_0} \cdot \frac{p - p_0}{p_0} - \frac{n}{2(1-n)} \cdot \frac{(p - p_0)^2}{p_0^2}. \quad (11)$$

It is important that Eq. (11) does not depend explicitly on B. The first term in the r.h.s. of this equation disappears if we *define* $L_0 = L$. In this case, p_0 is the vertex of a parabola in the momentum space. To find p_0 and adjust the ring lattice, one can make measurements with proton beams. Three measurements with different values of p are sufficient. The average proton momentum can be kept with radio frequency (RF) cavities put into straight sections of the ring. The longitudinal electric field in the cavities does not influence the spin dynamics.

III. AVOIDING A BETATRON RESONANCE

Condition (3) leading to Eq. (9) should not be exactly satisfied. It can be shown that the relation $\alpha = 1$ leads to the betatron resonance $\nu_x = 1$ which results in zeroth frequency of horizontal coherent betatron oscillation (CBO) of the beam as a whole and a loss of the beam [6]. Therefore, the total length of the straight sections should slightly differ from L_0 so that the CBO tune would be small but nonzero:

$$\nu_{CBO} \equiv |1 - \nu_x| = \left|1 - \sqrt{1+\lambda}\right| \ll 1, \;\; \lambda = \frac{L - L_0}{L_0} n. \quad (12)$$

Typically, in a weak focusing ring $\alpha > 1$. Eq. (10) results in $L < L_0$. We expect that the CBO tune about 0.01 is sufficient to keep the beam. In this case, the appropriate choice of the total length of straight sections $\lambda \sim 0.01$ reduces the dependence of the spin rotation frequency on the beam momentum by two orders of magnitude. As a result, the use of proton beams for measuring the average magnetic field becomes quite possible.

Experimental details depend on the beam momentum. If it is higher than in the completed experiment (see Ref. [4]), the muon lifetime in the laboratory frame increases and the RF cavities may be helpful not only for protons but also for muons to keep the spin coherence. Otherwise, the use of low muon momentum (~ 0.3 GeV/c) and much higher statistics (see Ref. [7]) may even be more preferable. In this case, the RF cavities are unnecessary for muons.

IV. CORRECTIONS TO THE g-2 FREQUENCY

The problem of taking into account corrections to the g-2 frequency is very important. One of the main problems is an influence of the radial and vertical betatron oscillations on the average vertical magnetic field. We can consider the case when the velocity of unperturbed motion, v_0, coincides with the absolute value of the velocity of perturbed motion. For the latter motion, the average longitudinal component of the velocity is approximately equal to

$$v_\phi = v_0 \left(1 - \frac{< v_\rho^2 + v_z^2 >}{2v_0^2}\right) = v_0 \left(1 - \frac{v_{0\rho}^2 + v_{0z}^2}{4v_0^2}\right). \quad (13)$$

It can be shown that the average magnetic field for the perturbed motion, $\overline{B_p}$, slightly differs from that for the unperturbed motion, $\overline{B_u}$:

$$\overline{B_p} = \frac{1-n}{1 + \lambda\left(1 + \frac{v_{0\rho}^2 + v_{0z}^2}{4v_0^2}\right)} B_0, \quad \overline{B_u} = \frac{1-n}{1+\lambda} B_0, \quad (14)$$

where λ is given by Eq. (12). Approximately,

$$\overline{B_p} = \left(1 - \frac{\lambda}{1+\lambda} \cdot \frac{v_{0\rho}^2 + v_{0z}^2}{4v_0^2}\right) \overline{B_u}. \quad (15)$$

When $v_{0\rho}/v_0 \sim v_{0z}/v_0 \sim 0.001$, $\lambda \sim 0.01$, the correction to the average vertical magnetic field for the betatron oscillations is rather small and may be even negligible.

A noncontinuous vertical magnetic field leads to a longitudinal magnetic field on the edges of the magnets. Possibly, the latter field is a reason of the main correction to the g-2 frequency. It was asserted in Ref. [8] that this field causes "the need to know $\int \boldsymbol{B} \cdot d\boldsymbol{l}$ for the muons to a precision of 10 ppb". However, we should take into account that the longitudinal magnetic field cannot be neglected only on small segments of the beam trajectory near edges of magnets. As a result, the above estimate of precision should be decreased by several orders of magnitude.

The correction for the longitudinal magnetic field can be carefully examined. As $\mathrm{curl}\,\boldsymbol{B} = 0$ and $B_\phi = (z/\rho)(\partial B_z/\partial\phi)$, the longitudinal magnetic field acting on a particle oscillates. When the vertical velocity oscillation (pitch) is given by $v_z/v_0 = \psi_0 \cos(\omega_v t + \delta)$,

$$B_\phi = z_0 \frac{\partial B_z}{\partial l} \sin(\omega_v t + \delta), \;\; z_0 = \frac{\psi_0 \omega_c^{(b)} R_0}{\omega_v} = \frac{\psi_0 R_0}{\sqrt{n}}, \quad (16)$$

where l is the trajectory length and $\omega_c^{(b)} = v_0/R_0$ is the cyclotron frequency into bending sections. Evidently, $\int B_\phi dl = z_0 B_0 \sin(\omega_v t + \delta)$.

To estimate the correction, we can suppose that $(\partial B_z)/(\partial l) \approx B_0/b$. The length of the considered trajectory segment at the magnet edge is b. Calculations can be simplified if we present the angular velocity of the spin precession in the form

$$\boldsymbol{\omega}_a = a_0 \boldsymbol{e}_z + a_2 \sin(\omega_v t + \delta)\boldsymbol{e}_\phi,$$
$$a_2 = -\frac{eB_0}{m} \cdot \frac{(a+1)\psi_0 R_0}{\sqrt{n}\gamma b} \quad (17)$$

and suppose that $a_0 \approx -eB_0 a/(2m) = const.$ This is nothing but an estimate because the vertical magnetic field strongly varies within the considered trajectory segment.

To calculate the correction, we can use the results presented in Ref. [5]. The corrected local angular frequency is given by

$$\omega_z^{(l)} = a_0 \left[1 - \frac{a_2^2}{4(\omega_v^2 - a_0^2)}\right]. \quad (18)$$

The correction to the average angular velocity of the spin precession caused by the longitudinal magnetic field is equal to

$$\begin{aligned} \delta\Omega_l^{(a)} &= \frac{\omega_z^{(a)}(2\pi\rho - b/2) + \omega_z^{(l)} b}{(2\pi\rho - b/2) + b + (2L - b/2)} - \Omega^{(a)} \\ &= -\Omega^{(a)} \frac{a_2^2}{2\left[4\omega_v^2 - (\omega_z^{(a)})^2\right]} \cdot \frac{b}{2\pi\rho} \\ &\approx -\Omega^{(a)} \frac{(a+1)^2 \psi_0^2 R_0}{4\pi n(4n - a^2\gamma^2) b}, \end{aligned} \quad (19)$$

where $\omega_z^{(a)}$ and $\Omega^{(a)}$ are given by Eqs. (2) and (4), respectively.

Eq. (19) defines only the correction for one segment of the beam trajectory. To obtain the total correction, we should take into account the Maxwell equation $\oint \boldsymbol{B} \cdot d\boldsymbol{l} = 0$. If the g-2 precession did not take place, the total effect of the longitudinal magnetic field would vanish. However, the total correction is nonzero owing to a non-commutativity of rotations and is provided by the spin component orthogonal to the beam polarization at the beginning of a beam turn. Therefore, the total correction, $\Delta\Omega_l^{(a)}$, can be obtained with the multiplication of $\delta\Omega_l^{(a)}$ by the additional factor:

$$\Delta\Omega_l^{(a)} \sim F \cdot \delta\Omega_l^{(a)}, \; F = \begin{cases} \omega_z^{(a)}/\omega_c^{(b)} = a\gamma \text{ when } a\gamma < 1 \\ 1 \text{ when } a\gamma \geq 1 \end{cases} \quad (20)$$

The quantity b is usually of the order of the magnet gap. If we substitute the parameters of the BNL E821 experiment into Eqs. (19),(20), we obtain $|\Delta\Omega_l^{(a)}/\Omega^{(a)}| \sim 1$ ppm for both muons and protons.

To measure the total correction with an absolute accuracy of 0.01 ppm, one should determine the magnetic field parameters with a relative accuracy of $10^{-3} \div 10^{-2}$. Since the field of magnets is well known, extra measurements may be unnecessary. When the muon beam momentum is significantly decreased as compared with the BNL E821 experiment (see Ref. [7]), the correction for the muons becomes an order of magnitude less. For low-momentum beams, one can suppress the vertical betatron oscillations and additionally reduce the corrections for both the muons and protons.

In the proposed experiment, the correction for the vertical betatron oscillations (pitch correction) [9] (see also Ref. [5]) should also be taken into account. Known formulas [5, 9] give the order of magnitude of this correction ($\sim 0.1 \div 1$ ppm). The pitch correction can also be reduced with a suppression of the vertical betatron oscillations for low-momentum beams. Specific calculations should allow for a noncontinuity and a nonuniformity of the magnetic field.

In any case, all the corrections can be determined with an accuracy of 0.01 ppm or even better.

V. DISCUSSION AND SUMMARY

The stabilization and monitoring the magnetic field is an important and rather difficult problem. To stabilize the magnetic field in a few minutes needed for measuring the proton spin precession frequency, superconducting magnets can be used. It is more difficult to avoid a change of the magnetic field when switching from muon to proton storage. However, such a change can be properly determined. The average magnetic field can be calculated if the beam momentum and the average radius or frequency of the beam orbit are known. A change of the average magnetic field brings a corresponding change of the average radius and frequency of the beam orbit. Therefore, measuring the frequencies [10] or positions of the muon and proton beam orbits allows to determine the shift of the average magnetic field. The average proton momentum is defined by the RF cavities. In addition to the muon measurements, proton beams before and/or after muon runs can be used. The use of these methods should provide a determination of the shift of the average magnetic field with a relative accuracy of 0.1 ppm or even better. As a result, the muon and proton measurements can be related with a high precision.

The methods of measurement of the g-2 precession in the proposed experiment and the Farley's one are very similar. The important advantage of a noncontinuous nonuniform ring versus a noncontinuous uniform one is a possibility to avoid much shimming needed for creating the uniform magnetic field. Shimming is even more difficult for the noncontinuous uniform ring than for a continuous uniform one because of the fringe field. We expect that the proposed experiment can be carried out with one of existing rings.

The systematical errors considered above do not prevent to measure the muon g-2 factor with a high precision. The sum of all systematical errors considered in the manuscript causes less systematic uncertainty than that in the planned E969 experiment [3]. While there are many other systematical errors, we expect that the precision of the proposed experiment may be approximately the same or better than that of the planned E969 experiment.

A more detailed theoretical analysis should be based on the matrix method. The use of the matrix method is necessary for further theoretical investigations. However, any theoretical analysis is not sufficient to calculate the spin dynamics in specific g-2 rings with a needed accuracy. Nevertheless, necessary calculations can be carried out with spin tracking.

Since the theoretical predictions and the experimental data do not agree, performing new experiments based on different ring lattices is necessary. Such experiments will be very important even if they will not provide better precision as compared with the usual g-2 experiments [2, 3].

In this work, we propose the new experiment to measure the muon g-2 factor. The developed experiment does not require much shimming. This experiment could provide an independent experimental result with different systematics and the advantages mentioned in the Farley's paper [4].

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[1] F. Jegerlehner and A. Nyffeler, Phys. Rep. 477 (2009) 1.
[2] G.W. Bennett *et al.* (Muon $g-2$ Collaboration), Phys. Rev. D 73 (2006) 072003.
[3] R.M. Carey *et al.* (New $g-2$ Collaboration), "The New $(g-2)$ Experiment: A Proposal to Measure the Muon Anomalous Magnetic Moment to ± 0.14 ppm Precision," http://www.fnal.gov/directorate/program_planning/Mar2009PACPublic/Proposal_g-2-3.0Feb2009.pdf
[4] F.J.M. Farley, Nucl. Instr. Meth. A 523 (2004) 251.
[5] A.J. Silenko, Phys. Rev. ST Accel. Beams 9 (2006) 034003.
[6] E.D. Courant and H.S. Snyder, Ann. Phys. (N.Y.) 281 (2000) 360 [reprinted from 3 (1958) 1].
[7] T. Mibe, "New g-2 experiment at J-PARC," http://indico.ihep.ac.cn/getFile.py/access?contribId=50&sessionId=13&resId=0&materialId=slides&confId=619
[8] J. P. Miller, E. de Rafael and B. L. Roberts, Rept. Prog. Phys. 70 (2007) 795.
[9] S. Granger and G.W. Ford, Phys. Rev. Lett. 28 (1972) 1479; F.J.M. Farley, Phys. Lett. B 42 (1972) 66; F.J.M. Farley and E. Picasso, in *Quantum Electrodynamics*, edited by T. Kinoshita (World Scientific, Singapore, 1990); J.H. Field and G. Fiorentini, Nuovo Cimento Soc. Ital. Fis. A 21 (1974) 297.
[10] F.J.M. Farley, private communication (2008).