TIME-REVERSAL-VIOLATING OPTICAL GYROTROPY

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Time-reversal-violating interactions of the electrons and nucleus cause the appearance of new optical phenomena. These phenomena are not only very interesting from fundamental point of view, but give us a new key for studying the time-reversal-violating interactions of the elementary particles.

Violation of time reversal symmetry has been observed only in K_0-decay many years ago [1], and remains one of the great unsolved problems in elementary particle physics. Since the discovery of the CP-violation in decay of K_0-mesons, a few attempts have been undertaken to observe this phenomenon experimentally in different processes. However, those experiments have not been successful. At the present time novel more precise experimental schemes are actively discussed: observation of the atom [2] and neutron [3] electric dipole moment (EDM) through either spin precession or light polarization plane rotation caused by pseudo-Zeeman splitting of atom (molecule) levels by external electric field $\overrightarrow{E}$ due to interaction of atom level EDM $\overrightarrow{d_a}$ with electric field $W = -\overrightarrow{d_a} \cdot \overrightarrow{E}$ [4, 5, 6, 7, 8](this effect is similar to magneto-optic effect Macaluso-Corbino [9]). It should be noted that the mentioned experiments use the possible existence of such intrinsic quantum characteristic of atom (molecule) as static EDM. According to [10, 11, 12] together with the EDM there is one more characteristic of atom (molecule) describing its response to the external field effect - the T- and P-odd polarizability of atom (molecule) β^T. This polarizability differs from zero even if EDM of electron is equal to zero and pseudo-Zeeman splitting of atom (molecule) levels is absent. According to [12, 13, 17] the T-odd polarizability β^T yields to the appearance of new optical phenomena - photon polarization plane rotation and circular dichroism in an optically homogeneous isotropic medium exposed to an electric field caused by the Stark mixing of atom (molecule) levels. This T-odd phenomenon is a kinematic analog of the well known T-even phenomenon of Faraday effect of the photon polarization plane rotation in the medium exposed to a magnetic field due to Van-Vleck mechanism. Similarly to the well known P-odd T-even effect of light polarization plane rotation for which the intrinsic spin spiral of atom is responsible [15], this effect is caused by the atom magnetization appearing under external electric field action (see section 3 below). Moreover, according to [16] and section 3 below, the magnetization of atom appearing under action of static electric field causes the appearance of induced magnetic field $\overrightarrow{H_{ind}}$. The energy of interaction of atom magnetic moment $\overrightarrow{\mu_a}$ with this field is $W_H = -\overrightarrow{\mu_a} \cdot \overrightarrow{H_{ind}}(\overrightarrow{E})$. Therefore, the total splitting of atom levels is determined by energy $W_T = -\overrightarrow{d_a} \cdot \overrightarrow{E} - \overrightarrow{\mu_a} \cdot \overrightarrow{H_{ind}}(\overrightarrow{E})$. As a result, the effect of polarization plane rotation deal with the energy levels splitting is caused not only by $\overrightarrow{d_a}$ interaction with electric field but by $\overrightarrow{H_{ind}}(\overrightarrow{E})$ interaction with $\overrightarrow{\mu_a}$, too. It is easy to see, that even for $\overrightarrow{d_a} = 0$ the energy of splitting differs from zero and the T-odd effect of polarization plane rotation exists. One more interesting T-odd phenomenon appears when the photon beam is incident orthogonally to the external electric field $\vec{E}$ (or magnetic field $\vec{H}$ or both electric and magnetic fields). This is birefringence effect [13] (i.e. the effect when plane polarized photons are converted to circular polarized ones and vice versa).

Also the T-odd phenomenon of photon polarization plane rotation and circular dichroism appears at photon passing through non-center-symmetrical diffraction grating [12].

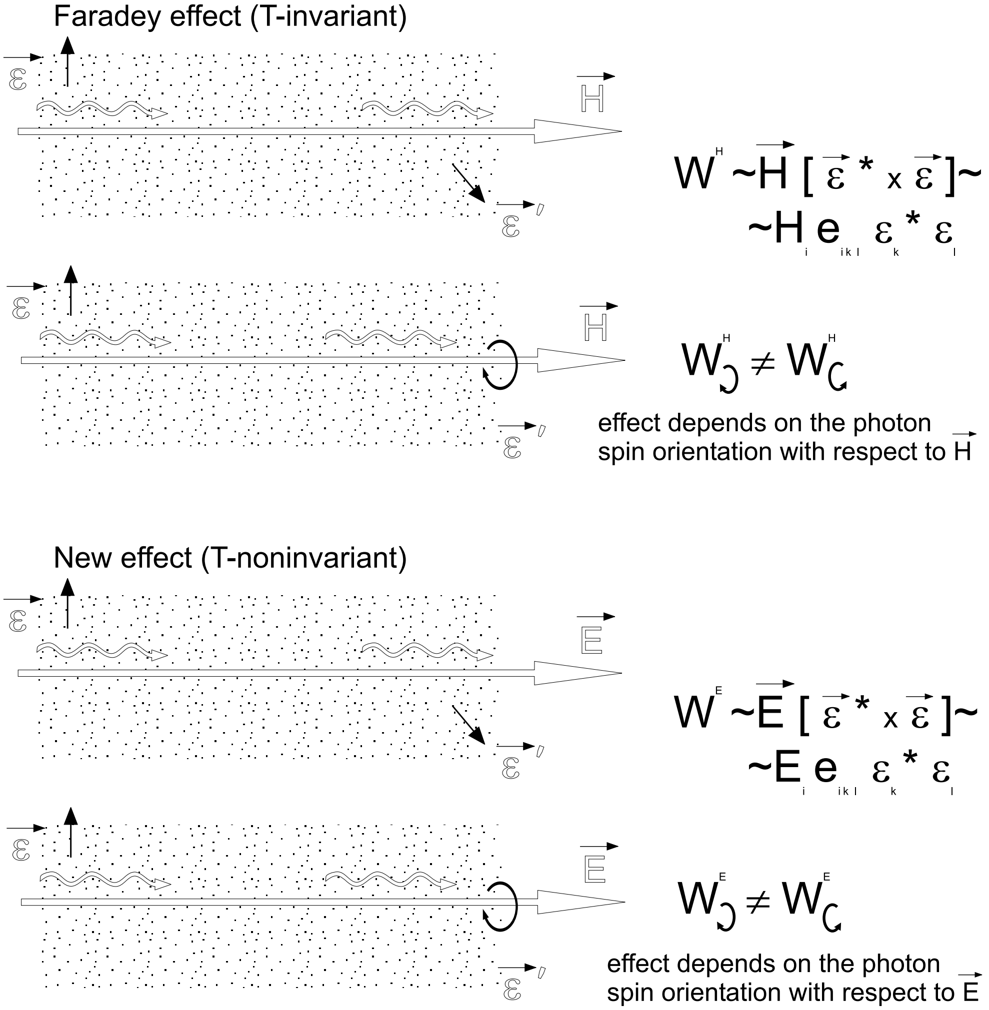


Figure 1: Phenomenon of the time-reversal-violating photon polarization plane rotation and circular dichroism by an electric field.

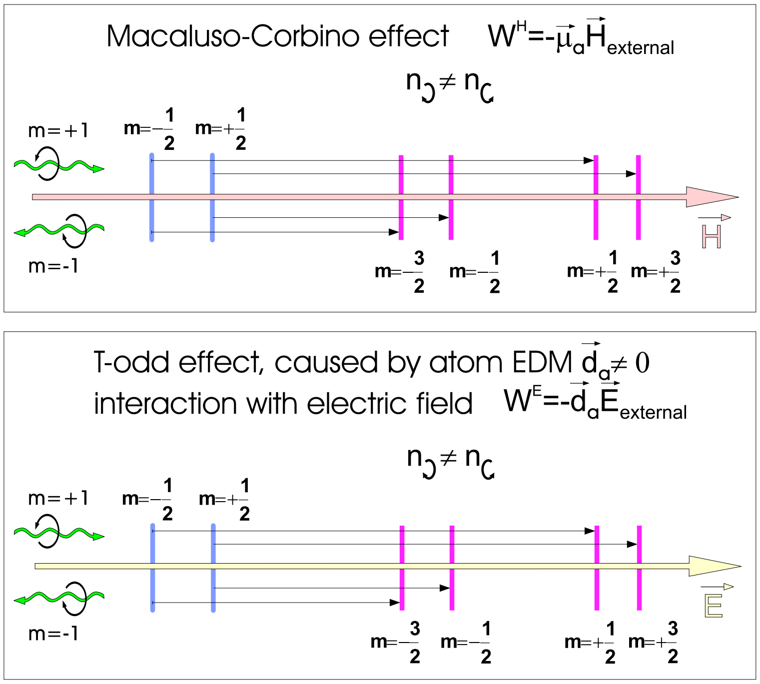
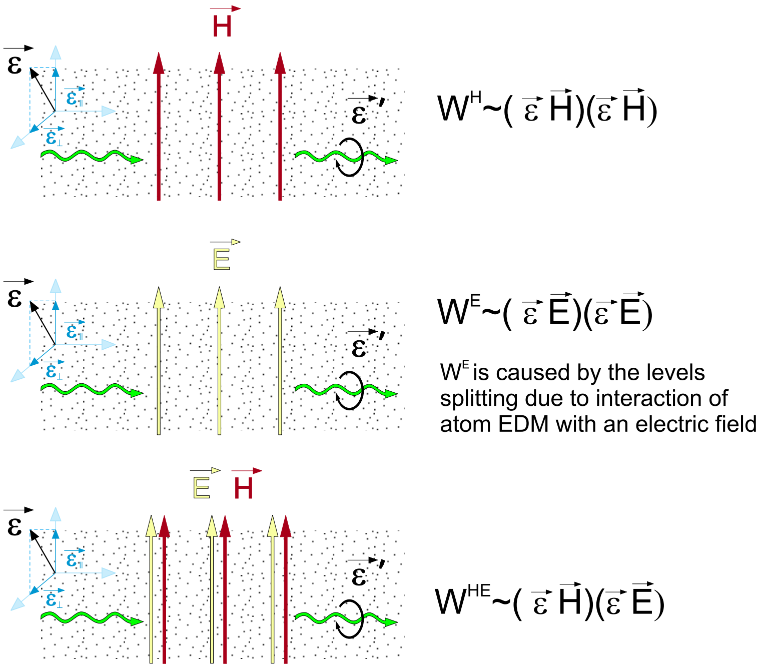


Figure 2: Polarization plane rotation phenomena.



W^E is caused by the levels splitting due to interaction of atom EDM with an electric field

Figure 3: T-noninvariant birefringence effect.

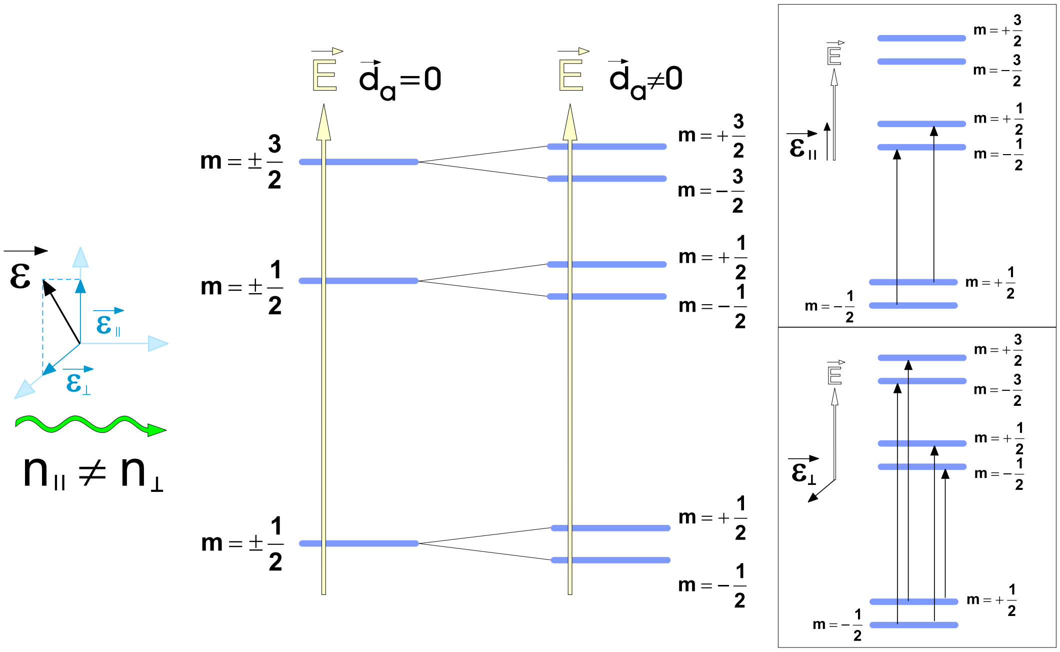


Figure 4: Levels splitting.

1. Phenomenon of the time-reversal-violating photon polarization plane rotation by an electric field.

In this section the T-odd phenomenon of the photon polarization plane rotation (circular dichroism) in an atomic (molecular) gas exposed to an electric field is considered. The expression for the T-non-invariant polarizability of an atom (molecule) in an electric field is obtained. It is shown that the T-odd rotation angle ϑ^T grows up when the energy of interaction of an atom (molecule) with an electric field is of the order of the opposite parity levels spacing.

In accordance with [10, 11, 12] the T-reversal violating dielectric permittivity tensor ε_{ik} for an optically diluted medium ($\varepsilon_{ik} - \delta_{ik} \ll 1$, δ_{ik} is the Kronecker symbol) is given by

$$\varepsilon_{ik} = \delta_{ik} + \chi_{ik} = \delta_{ik} + \frac{4\pi\rho}{k^2} f_{ik}(0), \tag{1}$$

where χ_{ik} is the polarizability tensor of a medium, ρ is the number of atoms (molecules) per cm^3, k is the photon wave number; $f_{ik}(0)$ is the tensor part of the zero angle amplitude of elastic coherent scattering of a photon by an atom (molecule) $f(0) = f_{ik}(0) e_i'^* e_k$. Here $\vec{e}$ and $\vec{e}\,'$ are the polarization vectors of initial and scattered photons. Indices $i = 1, 2, 3$ are referred to coordinates x, y, z, respectively, repeated indices imply summation.

Let photon be scattered by nonpolarized atoms (molecules) interacting with an electric field $\vec{E}$. When photon propagates along the electric field the amplitude $f_{ik}(0)$ can be written as [12]:

$$f_{ik}(0) = f_{ik}^{ev} + \frac{\omega^2}{c^2}[i\beta_s^P \varepsilon_{ikl} n_{\gamma l} + i\beta_E^T \varepsilon_{ikl} n_{El}], \tag{2}$$

where f_{ik}^{ev} is the T-, P-even (invariant) part of $f_{ik}(0)$, β_s^P is the scalar P-violating polarizability of an atom (molecule), β_E^T is the scalar T- and P-violating polarizability of an atom (molecule), ε_{ikl} is the totally antisymmetric unit tensor of the rank three, $\vec{n}_\gamma = \frac{\vec{k}}{k}$, $\vec{k}$ is the photon wave vector, $\vec{n}_E = \frac{\vec{E}}{E}$.

The term proportional to β_s^P describes the T-invariant P-violating photon polarization plane rotation (and circular dichroism) in a gas [15]. The corresponding refractive index N is as follows:

$$N = 1 + \frac{2\pi\rho}{k^2}\left\{ f_{ik}^{ev}(0) e_i^* e_k + i\frac{\omega^2}{c^2}(\beta_s^P \vec{n}_\gamma + \beta_E^T \vec{n}_E)[\vec{e}^{\,*} \times \vec{e}\,] \right\}. \tag{3}$$

The unit vectors describing the circular polarization of photons are: $\vec{e}_+ = -\frac{\vec{e}_1 + i\vec{e}_2}{\sqrt{2}}$ for the right and, $\vec{e}_- = \frac{\vec{e}_1 - i\vec{e}_2}{\sqrt{2}}$ for the left circular polarization, where $\vec{e}_1 \perp \vec{e}_2$, $\vec{e}_2 = [\vec{n}_\gamma \times \vec{e}_1]$ are the unit polarization vectors of a linearly polarized photon, $[\vec{e}_1 \times \vec{e}_2] = \vec{n}_\gamma$, $\vec{e}_1 = -\frac{\vec{e}_+ - \vec{e}_-}{\sqrt{2}}$, $\vec{e}_2 = -\frac{\vec{e}_+ + \vec{e}_-}{i\sqrt{2}}$.

Let an electromagnetic wave propagates through a gas along the electric field $\vec{E}$ direction. The refractive indices for the right N_+ and for the left N_- circular polarized photons can be written as:

$$N_{\pm} = 1 + \frac{2\pi\rho}{k^2} f_{\pm}(0) = 1 + \frac{2\pi\rho}{k^2}\left\{ f^{ev}(0) \mp \frac{\omega^2}{c^2}\left[\beta_s^P + \beta_E^T(\vec{n}_E\vec{n}_\gamma)\right]\right\}, \qquad (4)$$

where $f_+(0)(f_-(0))$ is the zero angle amplitude of the elastic coherent scattering of the right (left) circular polarized photon by an atom (molecule).

Let photons with the linear polarization $\vec{e}_1 = -\frac{\vec{e}_+ - \vec{e}_-}{\sqrt{2}}$ fall in a gas. The polarization vector of a photon in a gas $\vec{e}'_1$ can be written as:

$$\begin{aligned} \vec{e}'_1 &= -\frac{\vec{e}_+}{\sqrt{2}} e^{ikN_+L} + \frac{\vec{e}_-}{\sqrt{2}} e^{ikN_-L} = \qquad (5)\\ &= e^{\frac{1}{2}ik(N_+ + N_-)L}\left\{ \vec{e}_1 \cos\frac{1}{2}k(N_+ - N_-)L - \vec{e}_2 \sin\frac{1}{2}k(N_+ - N_-)L \right\}, \end{aligned}$$

where L is the photon propagation length in a medium.

As one can see, the photon polarization plane rotates in a gas. The angle of rotation ϑ is

$$\begin{aligned} \vartheta &= \frac{1}{2}kRe(N_+ - N_-)L = \frac{\pi\rho}{k}Re\left[f_+(0) - f_-(0)\right]L = \qquad (6)\\ &= -\frac{2\pi\rho\omega}{c}\left[\beta_s^P + \beta_E^T(\vec{n}_E\vec{n}_\gamma)\right]L, \end{aligned}$$

where $ReN_{\pm}$ is the real part of $N_{\pm}$. It should be noted that $\vartheta > 0$ corresponds to the right rotation of the light polarization plane and $\vartheta < 0$ corresponds to the left one, where the right (positive) rotation is recording by the light observer as the clockwise one.

In accordance with (6) the T-odd interaction results in the photon polarization plane rotation around the electric field $\vec{E}$ direction. The angle of rotation is proportional to the polarizability β_E^T and the $(\vec{n}_E\vec{n}_\gamma)$ correlation. Together with the T-odd effect there is the T-even P-odd polarization plane rotation phenomenon determining by the polarizability β_s^P and being independent on the $(\vec{n}_E\vec{n}_\gamma)$ correlation. The T-odd rotation dependence on the electric field $\vec{E}$ orientation with respect to the $\vec{n}_\gamma$ direction allows one to distinguish T-odd and T-even P-odd phenomena experimentally.

The refractive index $N_+(N_-)$ has both real and imaginary parts. The imaginary part of the refractive index $(ImN_{\pm} \sim Im\beta_E^T(\vec{n}_E\vec{n}_\gamma))$ is responsible for the T-reversal violating circular dichroism. Due to this process the linearly polarized photon takes circular polarization. The sign of the circular polarization depends on the sign of the scalar production $(\vec{n}_E\vec{n}_\gamma)$ that allows us to separate T-odd circular dichroism from P-odd T-even circular dichroism. The last one is proportional to $Im\beta_s^P$.

Before the detailed description of the effect let us consider simple observations to demonstrate clearly how the effect appears (see also [18]). Let us expose atom with a single valence electron being in a ground state $S_{1/2}$ to an electric field. P and T odd interactions and interaction with electric field yield to the admixture of opposite parity states to the ground state. Considering only mixing with the nearest $nP_{1/2}$ state one can write atom wave function $|\widetilde{s}_{1/2}>$

$$|\widetilde{s}_{1/2}> = \frac{1}{\sqrt{4\pi}}(R_0(r) - R_1(r)(\vec{\sigma}\vec{n})\eta - R_1(r)(\vec{\sigma}\vec{n})(\vec{\sigma}\vec{E})\delta)|\chi_{1/2}\rangle, \qquad (7)$$

here $\vec{\sigma}$ - are the Pauli matrices, $\vec{n} = \vec{r}/r$ is the unit vector along $\vec{r}$ direction, R_0 and R_1 are the radial parts of $nS_{1/2}$ and $nP_{1/2}$ wave functions, respectively, $|\chi_{1/2}\rangle$ is the spin part of wave function, η is the mixing coeffiecient for $P_{1/2}$ state due to P and T noninvariant interactions and δ is that caused by electric field.

Let us consider orientation of electron spin in atom. In order to find the spatial distribution of spin direction one should average spin operator over spin part of atom wave function. Only terms containing both δ and η contribute to PT-odd rotation of polarization plane of light. The change of spin direction due to these terms is

$$\begin{aligned}\Delta\vec{s}(\vec{r}) &= \frac{\eta\delta}{8\pi}R_1^2\left\langle\chi_{1/2}|(\vec{\sigma}\vec{n})\vec{\sigma}(\vec{\sigma}\vec{n})(\vec{\sigma}\vec{E}) + (\vec{\sigma}\vec{E})(\vec{\sigma}\vec{n})\vec{\sigma}(\vec{\sigma}\vec{n})|\chi_{1/2}\right\rangle \\ &= \frac{\eta\delta R_1^2}{8\pi}\left(4\vec{n}(\vec{n}\vec{E}) - 2\vec{E}\right) \qquad (8)\end{aligned}$$

The vector field $(4\vec{n}(\vec{n}\vec{E}) - 2\vec{E})$ is shown in Fig.5. Since $\Delta\vec{s}$ does not depend on initial direction of atom spin, this spin structure appears even in non-polarized atom. It should be noted that spin vector averaged over spatial variables differs from zero and is directed along the electric field $\vec{E}$. Photons with angular moment parallel and antiparallel to the electric field can interact with such spin structure in a different ways that causes rotation of polarization plane of light.

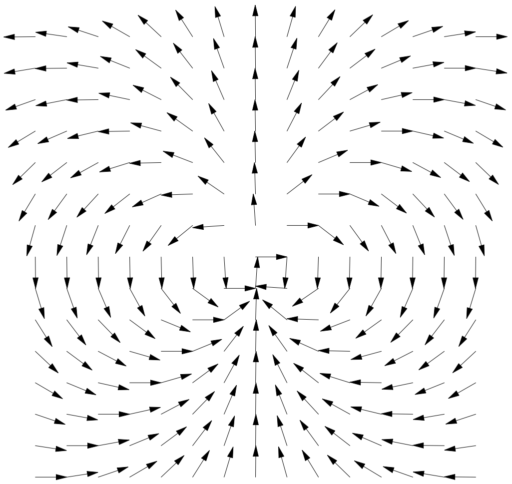


Figure 5: Vector field $4\vec{n}(\vec{n}\vec{E}) - 2\vec{E}$. Vectors in figure show direction of atom spin in $S_{1/2}$ state with the admixture of $P_{1/2}$ state due to PT-noninvariant interactions and external electric field.

In order to estimate the magnitude of the effect one should obtain the T-odd polarizability β_E^T or (that is actually the same, see (48,6)) the amplitude $f_\pm(0)$ of elastic coherent scattering of a photon by an atom (molecule). According to quantum electrodynamics the elastic coherent scattering at zero angle can be considered as the succession of two processes: the first one is the absorption of the initial photon with the momentum $\overrightarrow{k}$ and the transition of the atom (molecule) from the initial state $|N_0\rangle$

with the energy E_{N_0} to an intermediate state $|F\rangle$ with an energy E_F; the second one is the transition of the atom (molecule) from the state $|F\rangle$ to the final state $|F'\rangle = |N_0\rangle$ and irradiation of the photon with the momentum $\vec{k}\,' = \vec{k}$.

Let H_A be the atom (molecule) Hamiltonian considering the weak interaction between electrons and nucleus and the electromagnetic interaction of an atom (molecule) with the external electric $\vec{E}$ and magnetic $\vec{H}$ fields. It defines the system of eigenfunctions $|F\rangle$ and eigenvalues $E_F = E_F(\vec{E}, \vec{H})$:

$$H_A\,|F\rangle = E_F\,|F\rangle\,, \tag{9}$$

$F-$set of quantum numbers describing the state $|F\rangle$:

The matrix element of the process determining the scattering amplitude in the forward direction in the dipole approximation is given by [20]:

$$\mathfrak{M}_{N_0} = \sum_F \left\{ \frac{\langle N_0|\,\vec{d}\,\vec{e}^{\,*}\,|F\rangle\,\langle F|\,\vec{d}\,\vec{e}\,|N_0\rangle}{E_F - E_{N_0} - \hbar\omega} + \frac{\langle N_0|\,\vec{d}\,\vec{e}\,|F\rangle\,\langle F|\,\vec{d}\,\vec{e}^{\,*}\,|N_0\rangle}{E_F - E_{N_0} + \hbar\omega} \right\}, \tag{10}$$

where $\vec{d}$ is the electric dipole transition operator, ω is the photon frequency.

It should be reminded that the atom (molecule) exited levels are quasistationary: $E_F = E_F^{(0)} - \frac{i}{2}\Gamma_F$, $E_F^{(0)}$ is the atom (molecule) level energy, Γ_F is the level width. The matrix element (10) can be written as:

$$\mathfrak{M}_{N_0} = \alpha_{ik}^{N_0} e_i^* e_k, \tag{11}$$

where $\alpha_{ik}^{N_0}$ is the tensor of dynamical polarizability of an atom (molecule)

$$\alpha_{ik}^{N_0} = \sum_F \left\{ \frac{\langle N_0|\,d_i\,|F\rangle\,\langle F|\,d_k\,|N_0\rangle}{E_F - E_{N_0} - \hbar\omega} + \frac{\langle N_0|\,d_k\,|F\rangle\,\langle F|\,d_i\,|N_0\rangle}{E_F - E_{N_0} + \hbar\omega} \right\} \tag{12}$$

In general case atoms are distributed to the levels of ground state N_0 with the probability $P(N_0)$. Therefore, $\alpha_{ik}^{N_0}$ should be averaged over all states N_0. As a result, the polarizability can be written

$$\alpha_{ik} = \sum_{N_0} P(N_0)\alpha_{ik}^{N_0} \tag{13}$$

The tensor α_{ik} can be expanded in the irreducible parts as

$$\alpha_{ik} = \alpha_0\delta_{ik} + \alpha_{ik}^s + \alpha_{ik}^a, \tag{14}$$

where $\alpha_0 = \frac{1}{3}\sum_i \alpha_{ii}$ is the scalar, $\alpha_{ik}^s = \frac{1}{2}(\alpha_{ik} + \alpha_{ki}) - \alpha_0\delta_{ik}$ is the symmetric tensor of rank two, $\alpha_{ik}^a = \frac{1}{2}(\alpha_{ik} - \alpha_{ki})$ is the antisymmetric tensor of rank two,

$$\alpha_{ik}^a = \frac{\omega}{\hbar}\sum_{N_0} P(N_0) \sum_F \left\{ \frac{\langle N_0|\,d_i\,|F\rangle\,\langle F|\,d_k\,|N_0\rangle - \langle N_0|\,d_k\,|F\rangle\,\langle F|\,d_i\,|N_0\rangle}{\omega_{FN_0}^2 - \omega^2} \right\}, \tag{15}$$

where $\omega_{FN_0} = \frac{E_F - E_{N_0}}{\hbar}$.

Let atoms (molecules) be nonpolarized. The antisymmetric part of polarizability (15) is equal to zero in the absence of TP-odd interactions and external magnetic field. It should be reminded that for the P-odd and T-even interactions the antisymmetric part of polarizability differs from zero only for both the electric and magnetic dipole transitions [15].

For further analysis suppose the external magnetic field be equal to zero (electric field differs from zero). We can evaluate the antisymmetric part α_{ik}^{a} of the tensor α_{ik} of dynamical polarizability of atom (molecule), and, as a result, obtain the expression for β_E^T by the following way. According to (4,6) the magnitude of the T-odd effect is determined by the polarizability β_E^T or (that is actually the same, see (48,6)) by the amplitude $f_{\pm}(0)$ of elastic coherent scattering of a photon by an atom (molecule). If $\vec{n}_E \parallel \vec{n}_\gamma$ the amplitude $f_{\pm}(0)$ in the dipole approximation can be written as $f_{\pm} = \mp\frac{\omega^2}{c^2}\beta_E^T$. As a result, in order to obtain the amplitude $f_{\pm}$, the matrix element (10,11) for photon polarization states $\vec{e} = \vec{e}_{\pm}$ should be found.

The electric dipole transition operator $\vec{d}$ can be written in the form:

$$\vec{d} = d_+ \vec{e}_+ + d_- \vec{e}_- + d_z \vec{n}_\gamma, \tag{16}$$

with $\vec{d}_+ = -\frac{d_x - i d_y}{\sqrt{2}}$, $\vec{d}_- = \frac{d_x + i d_y}{\sqrt{2}}$. Let photon polarization state $\vec{e} = \vec{e}_+$. Using (10,11) we can present the polarizability β_E^T as follows:

$$\beta_E^T = \frac{\omega}{\hbar}\sum_{N_0} P(N_0) \sum_F \left\{ \frac{\langle N_0|\, d_- \,|F\rangle \langle F|\, d_+ \,|N_0\rangle - \langle N_0|\, d_+ \,|F\rangle \langle F|\, d_- \,|N_0\rangle}{\omega_{FN_0}^2 - \omega^2} \right\}. \tag{17}$$

For further analysis the more detailed expressions for atom (molecule) wave functions are necessary. The weak interaction constant is very small. Therefore, we can use the perturbation theory. Let $|f, E\rangle$ be the wave function of an atom (molecule) interacting with an electric field $\vec{E}$ in the absence of weak interaction . Switch on weak interaction ($V_w \neq 0$). According to the perturbation theory [20] the wave function $|F\rangle$ can be written in this case as:

$$|F\rangle = \left|f, \vec{E}\right\rangle + \sum_n \frac{\left\langle n, \vec{E}\right| V_w \left|f, \vec{E}\right\rangle}{E_f - E_n} \left|n, \vec{E}\right\rangle \tag{18}$$

It should be mentioned that both numerator and denominator of (17) contain V_w. Suppose V_w to be small one can represent the total polarizability β_E^T as the sum of two terms

$$\beta_E^T = \beta_{mix}^T + \beta_{split}^T, \tag{19}$$

where

$$\beta_{mix}^T = \frac{\omega}{\hbar}\sum_{N_0} P(N_0) \sum_f \frac{1}{\omega_{fn_0}^2 - \omega^2} \sum_l \tag{20}$$

$$\left\{ \frac{2Re\left[\left\{\left\langle n_0 \vec{E}\right| d_- \left|f \vec{E}\right\rangle \left\langle f \vec{E}\right| d_+ \left|l \vec{E}\right\rangle - \left\langle n_0 \vec{E}\right| d_+ \left|f \vec{E}\right\rangle \left\langle f \vec{E}\right| d_- \left|l \vec{E}\right\rangle\right\} \left\langle l \vec{E}\right| V_w \left|n_0 \vec{E}\right\rangle\right]}{E_{n_0} - E_l} + \right.$$

$$
+\frac{2Re\left[\left\langle n_0\vec{E}\right|d_-\left|l\vec{E}\right\rangle\left\langle l\vec{E}\right|V_w\left|f\vec{E}\right\rangle\left\langle f\vec{E}\right|d_+\left|n_0\vec{E}\right\rangle-\left\langle n_0\vec{E}\right|d_+\left|l\vec{E}\right\rangle\left\langle l\vec{E}\right|V_w\left|f\vec{E}\right\rangle\left\langle f\vec{E}\right|d_-\left|n_0\vec{E}\right\rangle\right]}{E_f-E_l}\Bigg\}
$$

and

$$
\begin{aligned}
\beta^T_{split} &= \frac{\omega}{\hbar}\sum_{N_0}P(N_0)\sum_F\left\{\frac{\langle n_0|\,d_-\,|f\rangle\,\langle f|\,d_+\,|n_0\rangle-\langle n_0|\,d_+\,|f\rangle\,\langle f|\,d_-\,|n_0\rangle}{\omega^2_{FN_0}-\omega^2}\right\}= \\
&= \frac{\omega}{\hbar}\sum_{N_0}P(N_0)\sum_F\left\{\frac{\langle n_0|\,d_-\,|f\rangle\,\langle f|\,d_+\,|n_0\rangle-\langle n_0|\,d_+\,|f\rangle\,\langle f|\,d_-\,|n_0\rangle}{(\omega_{FN_0}-\omega)(\omega_{FN_0}+\omega)}\right\}= \\
&= \frac{1}{2\hbar}\sum_{N_0}P(N_0)\sum_F\left\{\frac{\langle n_0|\,d_-\,|f\rangle\,\langle f|\,d_+\,|n_0\rangle-\langle n_0|\,d_+\,|f\rangle\,\langle f|\,d_-\,|n_0\rangle}{(\omega_{FN_0}-\omega)}\right\} \qquad (21)
\end{aligned}
$$

$$
\omega_{FN_0}=\frac{E_F(\vec{E})-E_{N_0}(\vec{E})}{\hbar},
$$

It should be reminded that according to all the above (see also section 3) energy levels E_F and E_{N_0} contain shifts caused by interaction of electric dipole moment of the level with electric field $\vec{E}$ and magnetic moment of the level with T-odd induced magnetic field $\vec{H}_{ind}(\vec{E})$.

It should be noted that radial parts of the atom wave functions are real [21], therefore the matrix elements of operators $d_\pm$ are real too. As a result, the P-odd T-even part of the interaction V_w does not contribute to β^T_{mix} because the P-odd T-even matrix elements of V_w are imaginary [15]. At the same time, the T- and P-odd matrix elements of V_w are real [15], therefore, the polarizability $\beta^T_{mix}\neq 0$. It should be mentioned that in the absence of electric field ($\vec{E}=0$) the polarizability $\beta^T_E=0$ and, therefore, the phenomenon of the photon polarization plane rotation is absent.

Really, the electric field $\vec{E}$ mixes the opposite parity levels of the atom . The atom levels have the fixed parity at $\vec{E}=0$. The operators $d_\pm$ and V_w change the parity of the atom states. As a result, the parity of the final state $|N_0'\rangle = d_+\ d_-\ V_w\ |N_0\rangle$ appears to be opposite to the parity of the initial state $|N_0\rangle$. But the initial and final states in the expression for β^T_E are the same. Therefore β^T_E can not differ from zero at $\vec{E}=0$.

It should be emphasized once again that polarizability β^T_E differs from zero even if EDM of electron is equal to zero. The interaction of electron EDM with electric field gives only part of contribution to the total polarizability of atom (molecule). The new effect we discuss is caused by the Stark mixing of atom (molecule) levels and weak T- and P-odd interaction of electrons with nucleus (and with each other).

Therefore, according to (19) the total angle of polarization plane rotation includes two terms $\vartheta=\vartheta_{mix}+\vartheta_{split}$, where $\vartheta_{mix}\sim\beta^T_{mix}$ is caused by the considered above effect similar to Van Vleck that and $\vartheta_{split}\sim\beta^T_{split}$ is caused by the atom levels splitting both in electric field $\vec{E}$ and magnetic field $\vec{H}_{ind}(\vec{E})$. The contributions given by β^T_{mix} and β^T_{split} can be distinguished by studying the frequency dependence of $\vartheta=\vartheta(\omega)$. According to (20,21) $\vartheta_{mix}\sim\frac{1}{\omega_{fn_0}-\omega}$ whereas $\vartheta_{split}\sim\frac{1}{(\omega_{fn_0}-\omega)^2}$. So, ϑ_{split} decreases faster then ϑ_{mix} with the grows of frequency tuning out from resonance.

Furthermore there is one more possibility to distinguish contribution of $\vec{d_a}$ and $\vec{H}_{ind}(\vec{E})$ from that given by β^T_E. This can be done when the photon beam is incident orthogonally to the electric field $\overrightarrow{E}$. As it was shown above, the T-odd contribution to birefringence effect (depending on $\overrightarrow{d_a}$ and $\overrightarrow{H}_{ind}(\overrightarrow{E})$) appears in this case. Only the symmetrical part of tensor of dynamical polarizability α_{ik} of an atom (molecule) (14) contributes in it and antisymmetric one does not. To observe the T-odd birefringence effect it's more convenient to study atom exposed both to electric and magnetic fields. In this case the effect magnitude is proportional to $(\overrightarrow{H}\overrightarrow{E})$ and one can easily pick out T-odd effect among T-even ones changing $\overrightarrow{E}$ direction with respect to $\overrightarrow{H}$. It should be noted that two effects contribute in T-odd birefringence: 1. levels splitting, 2. mixing of ground state and excited state energy levels of atom in external fields.

Let us now estimate the magnitude of the effect of the T-odd photon plane rotation due to β^T_{mix}. According to the analysis [10, 11, 12], based on the calculations of the value of T- and P-noninvariant interactions given by [15], the ratio $\frac{\langle V^T_w\rangle}{\langle V^P_w\rangle} \leq 10^{-3} \div 10^{-4}$, where $\left\langle V^T_w\right\rangle$ is T and P-odd matrix element, $\left\langle V^P_w\right\rangle$ is P-odd T-even matrix element.

The P-odd T-even polarizability β^P_s is proportional to the electric dipole matrix element, the magnetic dipole matrix element and $\left\langle V^P_w\right\rangle$: $\beta^P_s \sim \langle d\rangle\langle\mu\rangle\left\langle V^P_w\right\rangle$ [15]. At the same time $\beta^T_{mix} \sim \left\langle d(\overrightarrow{E})\right\rangle\left\langle d(\overrightarrow{E})\right\rangle\left\langle V^T_w\right\rangle$. As a result,

$$\frac{\beta^T_{mix}}{\beta^P_s} \sim \frac{\left\langle d(\overrightarrow{E})\right\rangle\left\langle d(\overrightarrow{E})\right\rangle\left\langle V^T_w\right\rangle}{\langle d\rangle\langle\mu\rangle\left\langle V^P_w\right\rangle}. \tag{22}$$

Let us study the T-odd phenomena of the photon polarization plane rotation in an electric field $\overrightarrow{E}$ for the transition $n_0 \to f$ between the levels n_0 and f which have the same parity at $\overrightarrow{E} = 0$. The matrix element $\left\langle n_0, \overrightarrow{E}\right| d_\pm \left| f, \overrightarrow{E}\right\rangle$ does not equal to zero only if $\overrightarrow{E} \neq 0$. Let the energy of interaction of an atom with an electric field, $V_E = -\overrightarrow{d}\overrightarrow{E}$, be much smaller than the spacing Δ of the energy levels, which are mixed by the field $\overrightarrow{E}$. Then one can use the perturbation theory for the wave functions $\left|f, \overrightarrow{E}\right\rangle$:

$$\left|f, \overrightarrow{E}\right\rangle = |f\rangle + \sum_m \frac{\langle m| - d_z E\,|f\rangle}{E_f - E_m}\,|m\rangle\,, \tag{23}$$

where $z \parallel \overrightarrow{E}$. As a result, the matrix element $\left\langle n_0, \overrightarrow{E}\right| d_\pm \left| f, \overrightarrow{E}\right\rangle$ can be rewritten as:

$$\begin{aligned}\left\langle n_0, \overrightarrow{E}\right| d_\pm \left| f, \overrightarrow{E}\right\rangle &= \\ &= -\left\{\sum_m \frac{\langle n_0|\, d_\pm\,|m\rangle\langle m|\, d_z\,|f\rangle}{E_f - E_m} + \right. \\ &\quad \left. + \sum_p \frac{\langle n_0|\, d_z\,|p\rangle\langle p|\, d_\pm\,|f\rangle}{E_{n_0} - E_p}\right\} E.\end{aligned} \tag{24}$$

One can see that the matrix element $\left\langle d(\overrightarrow{E})\right\rangle \sim \frac{\langle d\rangle E}{\Delta}\langle d\rangle$ in this case. The other matrix elements in (20) can be evaluated at $\overrightarrow{E} = 0$. This gives the estimate as follows:

$$\beta^T_{mix} \sim \langle d\rangle\langle d\rangle\frac{\langle dE\rangle}{\Delta}\left\langle V^T_w\right\rangle \quad . \tag{25}$$

and, consequently, ratio (22) can be written as

$$\frac{\beta^T_{mix}}{\beta^P_s} \sim \frac{\langle d\rangle \langle d\rangle \frac{\langle dE\rangle}{\Delta} \left\langle V^T_w \right\rangle}{\langle d\rangle \langle \mu\rangle \left\langle V^P_w \right\rangle}. \tag{26}$$

Taking into account that the matrix element $\langle \mu\rangle \sim \alpha \langle d\rangle$ [20, 21], where $\alpha = \frac{1}{137}$ is the fine structure constant, equation (26) can be reduced to:

$$\frac{\beta^T_{mix}}{\beta^P_s} \sim \alpha^{-1} \frac{\langle dE\rangle}{\Delta} \frac{\left\langle V^T_w \right\rangle}{\left\langle V^P_w \right\rangle} \tag{27}$$

For the case $\frac{\langle dE\rangle}{\Delta} \sim 1$, ratio (27) gives

$$\frac{\beta^T_{mix}}{\beta^P_s} \sim \alpha^{-1} \frac{\left\langle V^T_w \right\rangle}{\left\langle V^P_w \right\rangle} \lesssim 10^{-1} \div 10^{-2} \tag{28}$$

Such condition can be realized, for example, for exited states of atoms and for two-atom molecules (TlF, BiS, HgF) which have a pair of nearly degenerate opposite parity states. As one can see, the ratio $\frac{\beta^T_{mix}}{\beta^P_s}$ is two orders larger as compared with the simple estimation $\frac{\left\langle V^T_w \right\rangle}{\left\langle V^P_w \right\rangle} \leq 10^{-3} \div 10^{-4}$ due to the fact that β^T_{mix} is determined by only the electric dipole transitions , while β^P_s is determined by both the electric and magnetic dipole transitions.

2. Time-violating photon polarization plane rotation by a diffraction grating.

As it has been shown in [10, 11], the energy of atom interaction with two coherent electromagnetic waves depends on the T-violating scalar polarizability β_t^T. Interaction of an atom (molecule) with two waves can be considered as a process of re-scattering of one wave into another and vice versa. Then, as it follows from an expression for the effective interaction energy, the amplitude $f\left(\vec{k}',\vec{k}\right)$ of the photon scattering by an unpolarized atom (molecule) at a non-zero angle is given by [11]:

$$f\left(\vec{k}',\vec{k}\right)=f_{ik}e_i^{\prime *}e_k=\frac{\omega^2}{c^2}\left(\alpha_s\vec{e}^{\,\prime *}\vec{e}+i\frac{1}{2}\beta_s^P\left(\vec{n}'+\vec{n}\right)\left[\vec{e}^{\,\prime *}\vec{e}\right]+\beta_s^T\left(\vec{n}'-\vec{n}\right)\left[\vec{e}^{\,\prime *}\vec{e}\right]\right), \tag{29}$$

where $\vec{k}$ is the wave vector of a scattered photon, $\vec{n}=\frac{\vec{k}}{k}$, $\vec{n}'=\frac{\vec{k}'}{k}$, α_s is the scalar P,T-invariant polarizability of an atom (molecule). Expression (29) holds true in the absence of external electric and magnetic fields.

The elastic scattering amplitude (29) can be derived from the general principles of symmetry. Indeed, there are four independent unit vectors: $\vec{\nu}_1=\frac{\vec{k}'+\vec{k}}{\left|\vec{k}'+\vec{k}\right|}$, $\vec{\nu}_2=\frac{\vec{k}'-\vec{k}}{\left|\vec{k}'-\vec{k}\right|}$, $\vec{e}$ and $\vec{e}^{\,\prime}$, which completely describe geometry of the elastic scattering process. The elastic scattering amplitude $f\left(\vec{k}',\vec{k}\right)$ depends on these vectors and therewith is a scalar. Obviously, one can compose three independent scalars from these vectors: $(\vec{e}^{\,\prime}\vec{e})$, $(\vec{\nu}_1\left[\vec{e}^{\,\prime *}\vec{e}\right])$, $(\vec{\nu}_2\left[\vec{e}^{\,\prime *}\vec{e}\right])$. As a result, the scattering amplitude can be written as:

$$f\left(\vec{k}',\vec{k}\right)=f_s\left(\vec{k}',\vec{k}\right)\vec{e}^{\,\prime *}\vec{e}+if_s^P\left(\vec{k}',\vec{k}\right)\vec{\nu}_1\left[\vec{e}^{\,\prime *}\vec{e}\right]+f_s^T\left(\vec{k}',\vec{k}\right)\vec{\nu}_2\left[\vec{e}^{\,\prime *}\vec{e}\right], \tag{30}$$

where f_s is the P-,T- invariant scalar amplitude, f_s^P is the P-violating scalar amplitude, and f_s^T is the P-,T- violating scalar amplitude.

It can easily be found from (29,30) that the term proportional to $\beta_s^T\left(f_s^T\right)$ vanishes in the case of forward scattering $\left(\vec{n}'\to\vec{n}\right)$. Vice versa, in the case of back scattering $\left(\vec{n}'\to-\vec{n}\right)$ the term proportional to $\beta_s^P\left(f_s^P\right)$ gets equal to zero.

Thus, one can conclude that the T-violating interactions manifest themselves in the processes of scattering by atoms (molecules). However, the scattering processes are usually incoherent and their cross sections are too small to hope for observation of the T-violating effect. Another situation takes place for diffraction gratings in the vicinity of the Bragg resonance where the scattering process is coherent. As a result, the intensities of scattered waves strongly increase: for instance, in the Bragg (reflection) diffraction geometry the amplitude of the diffracted-reflected wave may reach the unity. It gives us an opportunity to study the T-violating scattering processes [11] (the detailed discussion see at [22]).

To include the P, T violating processes into the diffraction theory, let us consider the microscopic Maxwell equations:

$$\begin{aligned} curl\vec{E} &= -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \quad , \quad curl\vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{j} \quad , \\ div\vec{E} &= 4\pi\rho \quad , \quad div\vec{B} = 0 \quad , \quad \frac{\partial \rho}{\partial t} + div\vec{j} = 0 \quad . \end{aligned} \tag{31}$$

where $\vec{E}$ is the electric field strength and $\vec{B}$ is the magnetic field induction, ρ and $\vec{j}$ are the microscopic densities of the electrical charge and the current induced by an electromagnetic wave, c is the speed of light. The Fourier transformation of these equations (i.e. $\vec{E}\left(\vec{r},t\right) = \frac{1}{2\pi^4}\int \vec{E}\left(\vec{k},\omega\right) e^{i\vec{k}\vec{r}} e^{-i\omega t} d^3k d\omega$ and so on) yields to equation for $\vec{E}\left(\vec{k},\omega\right)$:

$$\left(-k^2 + \frac{\omega^2}{c^2}\right)\vec{E}\left(\vec{k},\omega\right) = -\frac{4\pi i\omega}{c^2}\left[\vec{j}\left(\vec{k},\omega\right) - \frac{c^2k^2}{\omega^2}\vec{n}\left(\vec{n}\vec{j}\left(\vec{k},\omega\right)\right)\right], \tag{32}$$

where $\vec{n} = \frac{\vec{k}}{k}$.

In linear approximation, the current $\vec{j}\left(\vec{r},\omega\right)$ is coupled with $\vec{E}\left(\vec{r},\omega\right)$ by the well-known dependence: $j_i\left(\vec{r},\omega\right) = \int d^3r' \sigma_{ij}\left(\vec{r},\vec{r}',\omega\right) E_j\left(\vec{r}',\omega\right)$ with $\sigma_{ij}\left(\vec{r},\vec{r}',\omega\right)$ as the microscopic conductivity tensor being a sum of the conductivity tensors of the atoms (molecules) constituting the diffraction grating: $\sigma_{ij}\left(\vec{r},\vec{r}',\omega\right) = \sum_{A=1}^{N}\sigma_{ij}^{A}\left(\vec{r},\vec{r}',\omega\right)$, here σ_{ij}^{A} is the conductivity tensor of the A-type scatterers. The summation is done over all atoms (molecules) of the grating. In a diffraction grating, the tensor $\sigma_{ij}\left(\vec{r},\vec{r}',\omega\right)$ is a spatially periodic function. Therefore, $j_i\left(\vec{k},\omega\right)$ can be written as follows:

$$j_i\left(\vec{k},\omega\right) = \frac{1}{V_0}\sum_{\vec{\tau}}\sigma_{ij}^{c}\left(\vec{k},\vec{k}-\vec{\tau},\omega\right)E_j\left(\vec{k}-\vec{\tau},\omega\right) \tag{33}$$

where σ_{ij}^{c} is the Fourier transform of the conductivity tensor of a grating's elementary cell, $\vec{\tau}$ is the reciprocal lattice vector of the diffraction grating.

Using current representation (33), one can obtain from (32):

$$\left(-k^2 + k_0^2\right)E_i\left(\vec{k},\omega\right) = -\frac{\omega^2}{c^2}\sum_{\vec{\tau}}\hat{\chi}_{ij}\left(\vec{k},\vec{k}-\vec{\tau}\right)E_j\left(\vec{k}-\vec{\tau}\right) \tag{34}$$

Tensor of the diffraction grating susceptibility is given by

$$\hat{\chi}_{ij}\left(\vec{k},\vec{k}-\vec{\tau}\right) = \left(\delta_{il} - n_i n_l\right)\chi_{lj}\left(\vec{k},\vec{k}-\vec{\tau}\right) \tag{35}$$

with

$$\chi_{lj}\left(\vec{k},\vec{k}-\vec{\tau}\right) = \frac{4\pi i}{V_0\omega}\sigma_{lj}\left(\vec{k},\vec{k}-\vec{\tau}\right) = \frac{4\pi c^2}{V_0\omega^2}F_{lj}\left(\vec{k},\vec{k}-\vec{\tau}\right).$$

Here $F_{lj}\left(\vec{k},\vec{k}-\vec{\tau}\right) = \frac{i\omega}{c^2}\sigma_{lj}\left(\vec{k},\vec{k}-\vec{\tau}\right)$ is the amplitude of coherent elastic scattering of an electromagnetic wave by a grating elementary cell from a state with the wave vector $(\vec{k}-\vec{\tau})$ to a state with the wave vector $\vec{k}$.

The amplitude F_{lj} is obtained by summation of atomic (molecular) coherent elastic scattering amplitudes over a grating's elementary cell:

$$F_{lj}\left(\vec{k}' = \vec{k} + \vec{\tau}, \vec{k}\right) = \left\langle \sum_{A=1}^{N_c} f_{lj}^{A}\left(\vec{k}' = \vec{k} + \vec{\tau}, \vec{k}\right) e^{-i\vec{\tau}\vec{R}_A} \right\rangle, \tag{36}$$

where f_{lj}^{A} is the coherent elastic scattering amplitude by an A-type atom (molecule), $\vec{R}_A$ is the gravity center coordinate of the A-type atom (molecule) , N_c is the number of the atoms (molecules) in an elementary cell, angular brackets denote averaging over the coordinate distribution of scatterers in a grating's elementary cell. The amplitude f_{lj} is given by equation (29,30).

From (35), (36) and (30) one can obtain an expression for the susceptibility χ_{lj} of the elementary cell of an optically isotropic material:

$$\chi_{lj}\left(\vec{k}, \vec{k} - \vec{\tau}\right) = \chi_{s\vec{\tau}}\delta_{lj} + i\chi_{s\vec{\tau}}^{P}\varepsilon_{ljf}\nu_{1f}^{\vec{\tau}} + \chi_{s\vec{\tau}}^{T}\varepsilon_{ljf}\nu_{2f}^{\vec{\tau}} \tag{37}$$

where

$$\chi_{s\vec{\tau}}^{(P,T)} = \frac{4\pi c^2}{V_0\omega^2}\left\langle \sum_{A=1}^{N_c} f_s^{A(P,T)}\left(\vec{k}, \vec{k} - \vec{\tau}\right) e^{-i\vec{\tau}\vec{R}_A} \right\rangle,$$

$\chi_{s\vec{\tau}}$ is the scalar P-, T- invariant susceptibility of an elementary cell, $\chi_{s\vec{\tau}}^{P}$ is the P-violating, T- invariant susceptibility of the elementary cell, and $\chi_{s\vec{\tau}}^{T}$ is the P- and T-violating susceptibility of the elementary cell,

$$\vec{\nu}_1^{\vec{\tau}} = \frac{2\vec{k} - \vec{\tau}}{\left|2\vec{k} - \vec{\tau}\right|} \quad , \quad \vec{\nu}_2^{\vec{\tau}} = \frac{\vec{\tau}}{\tau}$$

Then, using (34,35,37) one can derive a set of equations describing the P and T violating interaction of an electromagnetic wave with a diffraction grating

$$\begin{aligned}\left(-\frac{k^2}{k_0^2} + 1\right) E_i\left(\vec{k}\right) &= -\left(\delta_{ij} - n_i n_j\right)\chi_{s0}E_j\left(\vec{k}\right) - i\chi_{s0}^{P}\left(\delta_{il} - n_i n_l\right)\varepsilon_{ljf}n_f E_j\left(\vec{k}\right) - \\ &\quad - \sum_{\vec{\tau}\neq 0}\{\left(\delta_{ij} - n_i n_j\right)\chi_{s\vec{\tau}}E_j\left(\vec{k} - \vec{\tau}\right) + \\ &\quad + i\chi_{s\vec{\tau}}^{P}\left(\delta_{il} - n_i n_l\right)\varepsilon_{ljf}\nu_{1f}^{\vec{\tau}}E_j\left(\vec{k} - \vec{\tau}\right) + \\ &\quad + \chi_{s\vec{\tau}}^{T}\left(\delta_{il} - n_i n_l\right)\varepsilon_{ljf}\nu_{2f}^{\vec{\tau}}E_j\left(\vec{k} - \vec{\tau}\right)\},\end{aligned} \tag{38}$$

where $k_0 = \frac{\omega}{c}$

Assuming the interaction to be P, T invariant $\left(\chi_s^{P} = \chi_s^{T} = 0\right)$, eqns. (38) are reduced to the conventional set of equations of dynamic diffraction theory [19]. The detailed analysis of these equations was done in [12].

According to [12] the angle of the photon polarization plane rotation out of Bragg conditions is defined by

$$\vartheta = -k_0 Re\chi_s^{P}\left(0\right)L + 2k_0\alpha_{\tau}^{-1}Re\left[\chi_{1s}\left(\vec{\tau}\right)\chi_{2s}^{T}\left(\vec{\tau}\right) - \chi_{2s}\left(\vec{\tau}\right)\chi_{1s}^{T}\left(\vec{\tau}\right)\right]L \tag{39}$$

So, the T-violating rotation arises in the case of nonzero odd part of the susceptibility: $\chi_2(\vec{\tau}) \neq 0$. Such a situation is possible if an elementary cell of the diffraction grating does not posses the center of symmetry.

In accordance with (39), the angle of the T-violating rotation grows at $\alpha_\tau \to 0$. However, the condition $\alpha_\tau |\chi_s(\vec{\tau})| \ll 1$ is violated at $\alpha_\tau^{-1} \to 0$, when the amplitude of diffracted and transmitted waves are comparable: $E\left(\vec{k} - \vec{\tau}\right) \simeq E\left(\vec{k}\right)$ and, consequently, the perturbation theory gets unapplicable. A rigorous dynamical diffraction theory should be applied in this case.

Let the Bragg condition is fulfilled only for the diffracted wave. It allows us to use the two-wave approximation of the dynamical diffraction theory [19]. Then, the set of equations (32) is reduced to two coupled equations, which for the back-scattering diffraction scheme $\left(\vec{k}_0 \parallel \vec{\tau}\right)$ take the form [12]

$$\begin{aligned}\left(\frac{k^2}{k_0^2} - 1\right) E_j\left(\vec{k}\right) &= \chi_s(0) E_j\left(\vec{k}\right) + i\chi_s^P(0)\,\varepsilon_{jmf} E_m\left(\vec{k}\right) n_f + \\ &\quad + \chi_s(\vec{\tau}) E_j\left(\vec{k} - \vec{\tau}\right) + \chi_s^T(\vec{\tau})\,\varepsilon_{jmf} E_m\left(\vec{k} - \vec{\tau}\right) \nu_{2f}^{\vec{\tau}}, \\ \left(\frac{\left(\vec{k} - \vec{\tau}\right)^2}{k_0^2} - 1\right) E_j\left(\vec{k} - \vec{\tau}\right) &= \chi_s(0) E_j\left(\vec{k} - \vec{\tau}\right) +\end{aligned} \tag{40}$$

$$i\chi_s^P(0)\,\varepsilon_{jmf} n_f\left(\vec{k} - \vec{\tau}\right) E_m\left(\vec{k} - \vec{\tau}\right) + \chi_s(-\vec{\tau}) E_j\left(\vec{k}\right) + \chi_s^T(-\vec{\tau})\,\varepsilon_{jmf} \nu_{2f}^{-\vec{\tau}} E_m\left(\vec{k}\right),$$

$\vec{n}\left(\vec{k} - \vec{\tau}\right) = \frac{\vec{k} - \vec{\tau}}{\left|\vec{k} - \vec{\tau}\right|}$.

These set of equations can be diagonalized for the photon with a certain circular polarization. Let the right-circularly polarized photon $(\vec{e}_+)$ be incident on the diffraction grating. Then, the diffraction process yields to the appearance of a back-scattered photon with the left circular polarization $\left(\vec{e}_-^{\vec{\tau}}\right)$ (this is because the momentum of the back-scattered photon $\vec{k}' = \vec{k} - \vec{\tau}$ is antiparallel to the momentum $\vec{k}$ of the incident one). And visa versa, the left-circularly polarized photon produces a right-circularly polarized back-scattered one.

Thus, for circularly polarized photons the set of vector equations (40) can be split into two independent sets of scalar equations [12]. The explicit solution of these equations yield to the following expression for the transmitted wave amplitude [12] (all the symbols are defined in [12]):

$$\vec{E}_\pm = \vec{e}_\pm (-1)^b e^{i\varphi_\pm}, \text{ where } \varphi_\pm = k_0 \left[\frac{1}{2}\varepsilon^\pm(\alpha_{1,2}) - \frac{k_0(\alpha_{1,2} - 2\chi_s(0)) L}{8\pi b}\Delta^\pm\right] L$$

Using this equation one can find the angle of the polarization plane rotation

$$\vartheta = Re(\varphi_+ - \varphi_-) = \vartheta^P + \vartheta^T_{1,2},$$

where $\vartheta^P = -k_0 Re\chi_s^P(0)L$ - defines the P-violating T-invariant rotation angle and $\vartheta_{1,2}^T$ corresponds to the T-violating rotation:

$$\begin{aligned}\vartheta_{1,2}^T(\alpha_{1,2}) &= \mp\frac{k_0^3L^3}{8\pi^2b^2}\sqrt{4\left(\chi_{1s}^2+\chi_{2s}^2\right)+\left(\frac{4\pi b}{k_0L}\right)^2}\times \\ &\times\left[\chi_{1s}(\vec{\tau})Re\chi_{2s}^T(\vec{\tau})-\chi_{2s}(\vec{\tau})Re\chi_{1s}^T(\vec{\tau})\right],\end{aligned} \tag{41}$$

the sign $(-)$ corresponds to α_1 , the sign $(+)$ corresponds to α_2.

The imaginary part of the T-violating polarizability $Im\chi_{s1,2}^T$ is responsible for the T-violating circular dichroism. Due to that process, a linearly polarized photon gets a circular polarization at the diffraction grating's output. The degree of the circular polarization of the photon is determined by the relation:

$$\delta_{1,2} = \frac{\left|\vec{E}_+\right|^2-\left|\vec{E}_-\right|^2}{\left|\vec{E}_+\right|^2+\left|\vec{E}_-\right|^2} \simeq Im\varphi_- - Im\varphi_+ = k_0 Im\chi_s^P(0)L\pm \tag{42}$$

$$\pm\frac{k_0^3L^3}{8\pi^2b^2}\sqrt{4\left(\chi_{1s}^2+\chi_{2s}^2\right)+\left(\frac{4\pi b}{k_0L}\right)^2}\left[\chi_{1s}(\vec{\tau})Im\chi_{2s}^T(\vec{\tau})-\chi_{2s}(\vec{\tau})Im\chi_{1s}^T(\vec{\tau})\right]$$

It should be pointed out that the resonance transmission condition is satisfied at a given b for two different values of α. This is because there is a possibility to approach to the Brilluan (the total Bragg reflection) bandgap both from high and low frequencies. The T-violating parts of the rotation angle are opposite in sign for α_1 and for α_2. It gives the addition opportunity to distinguish the T-violating rotation from the P-violating T-invariant rotation. Indeed, the P-violating rotation does not depend on the back Bragg diffraction in the general case because the P-violating scattering amplitude equals to zero for back scattering (see [12]). In accordance with (19,20) the T-violating rotation and dichroism grow sharply in the vicinity of the resonance Bragg transmission. At the first view, one could expect for ϑ^T the dependence $\vartheta^T \sim k_0 Re\chi_{s1,2}^T(\vec{\tau})L$ (see (39)). However, in the vicinity of resonance, the rotation angle ϑ^T turns out to be multiplied by the factor $B = (8\pi^2b^2)^{-1}k_0\sqrt{4\left(\chi_{1s}^2+\chi_{2s}^2\right)+\left(\frac{4\pi b}{k_0L}\right)^2}Lk_0\chi_{s1,2}L$ which provides the above mentioned growth (for example, $B \sim 10^5$ at $\chi_s \simeq 10^{-1}$, $k_0 \simeq 10^4 \div 10^5 cm^{-1}$, $L = 1cm$, $b = 1$).

Now, let us estimate the effect magnitude. To do this we must determine (in accordance with (41)) the T-violating susceptibility $\chi_{s1,2}^T$, which is proportional to the T-violating polarizability β_s^T of atom. The estimate carried out by [10, 11, 15] gives $\beta_s^T \sim 10^{-3} \div 10^{-4}\beta_s^P$, where β_s^P is the P-violating T-invariant scalar polarizability. The polarizability β_s^P was studied both theoretically and experimentally [15]. Particularly, the theory gives $\beta_s^P \cong 10^{-30}cm^3$ for atoms analogous to Bi, Tl, Pb. It yields to the estimate $\cong 10^{-33} \div 10^{-34}cm^3$ for the T-violating atomic polarizability. The polarizability β_s^P causes the P-violating rotation of the polarization plane by the angle $\vartheta^P = kRe\chi_s^P(0)L \cong 10^{-7}$ rad/cm×L for the gas density $\rho = 10^{16} \div 10^{17}cm^{-3}$. As a result, in our case the parameter $\varphi = k\chi_s^T(\tau)\,L$ turns out to be $\varphi \cong 10^{-10} \div 10^{-11}$ rad/cm×L and can be even less by the factor h/d, where h is the corrugation amplitude of the diffraction grating while d is the distance between waveguide's mirrors. Assuming this factor to be $\sim 10^{-1}$, we shall find $\varphi \cong 10^{-11} \div 10^{-12}$ rad/cm×L. Thus, the final estimate of the T-violating rotation angle ϑ^T is

$$\vartheta^T \cong 10^{-11} \div 10^{-12} \frac{rad}{cm} {k_0}^2 \chi_s^2(\tau) L^3 \tag{43}$$

In real situation the susceptibility of a grating $\chi_s(\tau)$ can exceed the unity. However, our analysis has been performed under the assumption $\chi_s \ll 1$. Suppose $\chi_s = 10^{-1}$, $k_0 = 10^4$ then $\vartheta^T \simeq 10^{-6} \div 10^{-7} L^3$ and, consequently, for L= 1 cm one can get the rotation angle $\vartheta^T \simeq 10^{-6} \div 10^{-7} rad$.

Obviously, the obtained the T-violating rotation angle ϑ^T is of the same order as compared with ϑ^P. It makes possible experimental observation of the phenomenon of the T-violating polarization plane rotation.

It should be noted that the manufacturing of diffraction gratings for the wave lengths longer than visible light can be simpler. That is why we would like to attract attention to the possibility of studying of the T-violating polarization plane rotation in the vicinity of frequencies of atomic (molecular) hyperfine transitions; for example, for Ce (the transition wavelength is $\lambda = 3.26$ cm) and Tl ($\lambda = 1.42$ cm).

Thus, we have shown that the phenomenon of the T-violating polarization plane rotation appears when the photon is scattered by a volume diffraction grating. The phenomenon grows sharply in the vicinity of the resonance transmission condition. An experimental scheme based on a waveguide, containing a diffraction grating and gas, has been proposed that enables real experiments on observation of the T-violating polarization plane rotation to be performed. The rotation angle has been shown to be $\vartheta^T = 10^{-6} \div 10^{-7} L^3$, where L is the waveguide length (thickness of the equivalent volume diffracting grating).

3. The possibility to observe the phenomena experimentally.

The possibility to observe the phenomena experimentally can be discussed now. In accordance with (6) the angle of the T-odd rotation in electric field can be evaluated as follows:

$$\vartheta^T_{mix} \sim \frac{2\pi\rho\omega}{c}\beta^T_{mix} L \sim \frac{\beta^T_{mix}}{\beta^P_S}\vartheta^P \sim \alpha^{-1}\frac{\langle dE\rangle}{\Delta}\frac{\langle V^T_w\rangle}{\langle V^P_w\rangle}\vartheta^P. \tag{44}$$

According to the experimental data [23, 24] being well consistent with calculations [15] the typical value of ϑ^P is $\vartheta^P \sim 10^{-6} rad$ (for the length L being equal to the several absorption lengths of the light propagating through a gas L_a).

For the electric field $E \sim 10^4 V \cdot cm^{-1}$ the parameter $\frac{\langle dE\rangle}{\Delta}$ can be estimated as $\frac{\langle dE\rangle}{\Delta} \sim 10^{-5}$ for Cs, Tl and $\frac{\langle dE\rangle}{\Delta} \sim 10^{-4}$ for Yb and lead. Therefore, one can obtain $\vartheta^T_{mix} \sim 10^{-13} rad$ for Cs, Tl and $\vartheta^T_{mix} \sim 10^{-12} rad$ for Yb and lead. For the two-atom molecules (TlF, BiS, HgF) the angle ϑ^T_{mix} can be larger, because they have a pair of degenerate opposite parity states.

It should be noted that the classical up-to-date experimental techniques allow to measure angles of light polarization plane rotation up to $4,3 \cdot 10^{-11} rad$ [25].

A way to increase the rotation angle ϑ^T is to increase the length L of the path of a photon inside a medium (see (6)). It can be done, for example, by placing a medium (gas in an electric field or non-center-symmetrical diffraction grating) in a resonator or inside a laser gyroscope (Fig.6). This becomes possible due to the fact that in contrast with the phenomenon of P-odd rotation of the polarization plane of photon the T-odd rotation in an electric field (as well as in a diffraction grating) is accumulated while photon is moving both in the forward and backward directions. Use of resonator gives a great advantage: even several non-center-symmetrical elementary cells (Fig.7) placed in it can provide the effect equivalent to that provided by the full-length diffraction grating (Fig.8,9).

For the first view the re-reflection of the wave in resonator (or light multiple passing over circle resonator of a laser gyroscope) can not provide the significant increase of the photon path length L in comparison with the absorption length L_a because of the absorption of photons in a medium. Nevertheless this difficulty can be overcome when the part of resonator is filled by the amplifying medium (for example, inverse medium). As a result, the electromagnetic wave being absorbed by the investigated gas is coherently amplified in the amplifier and then is refracted to the gas again. Consequently, under the ideal conditions the light pulse can exist in such resonator-amplifier for arbitrarily long time. And if, for example, the polarization plane of the wave rotates around the $\overrightarrow{E}$ direction, the peculiar "photon trap" appears (phase difference of waves with right (left) circular polarization moving in the opposite directions in a laser gyroscope or phase difference of waves with orthogonal plane polarizations for birefringence effect increases in time). The angle of rotation $\vartheta^T_t = \Omega^T \cdot t$, where Ω^T is the frequency of the photon polarization plane rotation around the $\overrightarrow{E}$ direction, t is the time of electromagnetic wave being in a "trap". It is easy to find the frequency Ω^T from (6): $\Omega^T = \frac{\vartheta^T}{L}c = 2\pi\rho\omega\beta^T_E$. From the estimates of ϑ^T it is evident that for $\vartheta^T \sim 10^{-12}$ rad (Lead, Yb) the frequency Ω^T appears to be $\Omega^T = \frac{\vartheta^T}{L_a}c \sim 10^{-4}\sec^{-1}$.

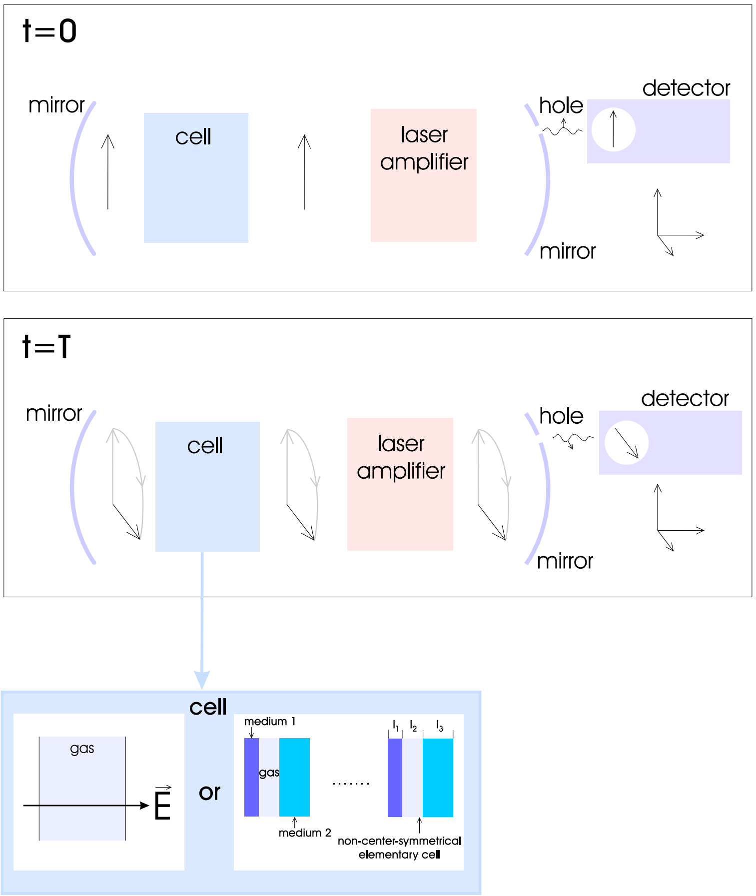


Figure 6:

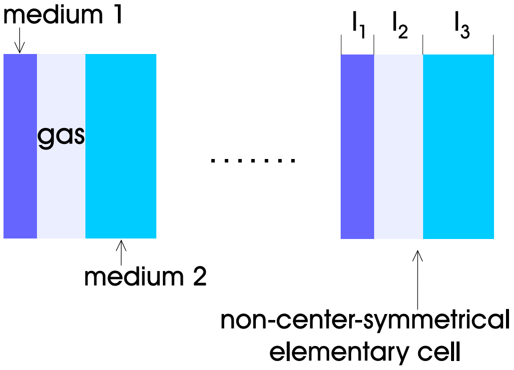


Figure 7:

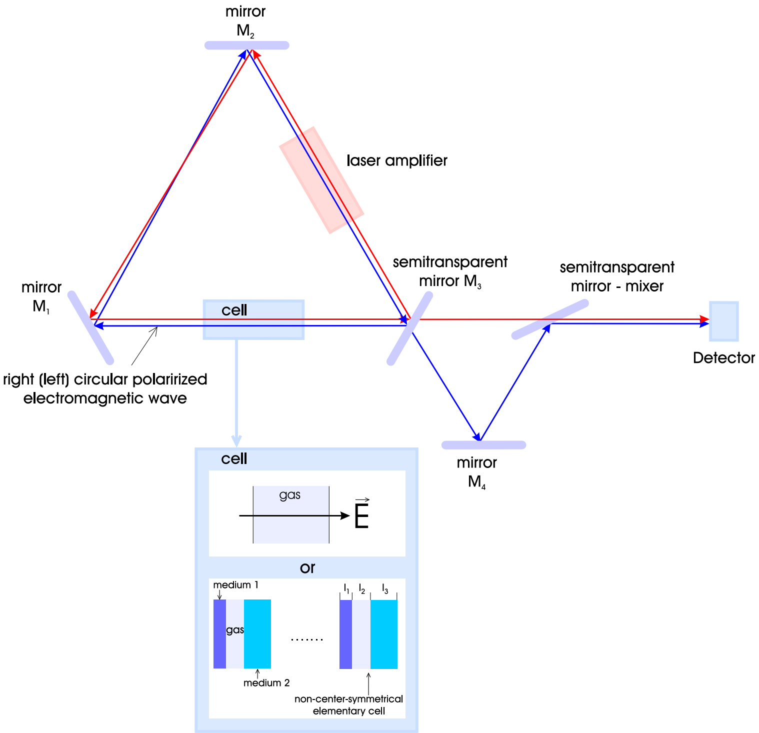


Figure 8:

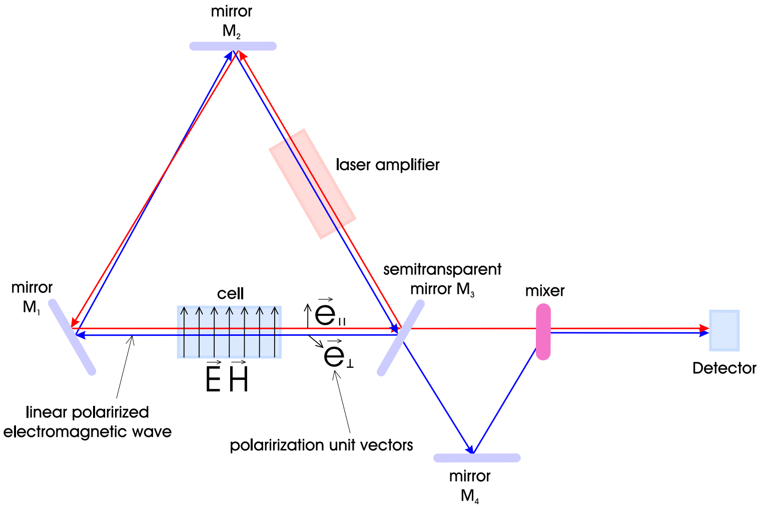


Figure 9:

Therefore $\vartheta_t^T \sim 10^{-4}t$ and for the time t of about 3 hours the angle ϑ_t^T becomes ~ 1 *rad*. The similar estimates for the atoms Cs, Tl ($\vartheta^T \sim 10^{-13}$ *rad*) give that for the same time the angle $\vartheta_t^T \sim 10^{-1}$ *rad*.

The time t is limited, in particular, by spontaneous radiation of photons in an amplifier that gradually leads to the depolarization of photon gas in resonator. Surely, it is the ideal picture, but here is the way to further increase of the experiment sensitivity. The achieved sensitivity in measurements of phase incursion in laser gyroscope makes possible to observe the effect in laser gyroscope, too. Laser interferometers used as gravitational wave detectors also can provide neccessary sensitivity.

Requiring to measure rotation angle $\sim 10^{-6} rad$ in "photon trap" and taking into consideration that existing technique allows to measure much less angles one can expect to observe effect of the order $\frac{V_T}{V_P} \sim 10^{-9} \div 10^{-10}$.

Let us consider this question from other point of view. Suppose effects of polarization plane rotation and birefringence be caused only by atom EDM one can estimate the possible sensitivity of EDM measurement in such experiments. Suppose we will measure rotation angle with sensitivity about $10^{-6} rad/hour$. Phase difference is $\delta\varphi = k(n_{\parallel} - n_{\perp})L = k(n_{\parallel} - n_{\perp})cT$, where T is the observation time (here T is supposed to be $T = 1$ *hour*). Representing $\delta\varphi$ in the form

$$\delta\varphi = \frac{\rho c T \lambda^2}{2\pi} \frac{\Gamma_e d_a E}{\hbar \Gamma^2} \tag{45}$$

where ρ is the atoms density, Γ_e is the level radiation width, Γ is the atom level width (including Doppler widening), E is the electric field strength one can estimate d_a as

$$d_a = \frac{2\pi\hbar\Gamma^2}{\rho c T \lambda^2 \Gamma_e E} \delta\varphi \approx 10^{-33} e \tag{46}$$

(here $\lambda \sim 10^{-4} cm$, $E = 10^2 CGSE$, $\rho = 10^{17}$ $atoms/cm^3$).

For comparison it is interesting to note that the best expected EDM measurement limit in recent publications [4] is about $d_a \approx 10^{-28} e$, so the advantages of the proposed method becomes evident.

All the said can be applied not only for the optical range but for the radio frequency range as well where the observation of the mentioned phenomenon is also possible by the use of the mentioned atoms and molecules [12].

Thus, we have shown that the T-odd and P-odd phenomena of photon polarization plane rotation and circular dichroism in an electric field are expected to be observable experimentally. The similar situation appears at the use of non-center-symmetrical diffraction grating.

It should be noted, that the new T-odd and P-odd phenomenon of photon polarization plane rotation (circular dichroism) in an electric field has general meaning. Due to quantum electrodynamic effects of electron-positron pair creation in strong electric, magnetic or gravitational fields, the vacuum is described by the dielectric permittivity tensor ε_{ik} depending on these fields[21, 26]. The theory of ε_{ik} [21, 26] does not take into account the weak interaction of electron and positron with each other. Considering the weak interaction between electron and positron in the process of pair creation in an electric (gravitational) field one can obtain that the permittivity tensor of vacuum in strong electric (gravitational) field contains the term $\varepsilon_{ik}^{vac} \sim i\beta^{T}_{vac\overrightarrow{E}}\varepsilon_{ikl}n_{lE}$ ($\varepsilon_{ik}^{vac} \sim i\beta^{T}_{vac\overrightarrow{g}}\varepsilon_{ikl}n_{lg}, \overrightarrow{n_g} = \frac{\overrightarrow{g}}{g}$, $\overrightarrow{g}$ is the free fall acceleration), and as a result, the polarization plane rotation (circular dichroism) phenomena exist for photons moving in an electric (gravitational) field in vacuum. And visa versa γ-quanta appearing under single-photon electron-positron annihilation in an electric (gravitational) field have the admixture of circular polarization, caused by T-odd P-odd weak interactions.

4. Phenomenon of the time-reversal violating magnetic field generation by a static electric field in a medium and vacuum.

As it was shown P- and T-odd interactions cause mixing of opposite parity levels of atom (molecule) that yields to the appearance of P- and T-odd terms of the atom (molecule) polarizability [10]. This makes possible to observe various optical phenomena, for example, photon polarization plane rotation (birefringence and circular dichroism) in an optically homogeneous medium placed to an electric field.

The energy of atom (molecule) in external electromagnetic field includes the term caused by the time reversal violating interactions [10]:

$$\Delta U = -\frac{1}{2}\beta_S^T \vec{E}\vec{H}, \tag{47}$$

where β_S^T is the scalar T-noninvariant polarizability of atom (molecule), $\vec{E}$ is the external electric field, $\vec{H}$ is the external magnetic field.

It's well known [20] that when the external field frequency $\omega \to 0$ the polarizabilities describe the processes of magnetization of medium by a static magnetic field and electric polarization of a medium by a static electric field

The energy of interaction of magnetic moment $\vec{\mu}$ with magnetic field $\vec{H}$

$$W_H = -\vec{\mu}\vec{H} \tag{48}$$

Comparison of (47) and (48) let one to conclude that the action of stationary electric field on an atom (molecule) induces the magnetic moment of atom

$$\overrightarrow{\mu_{ind}}(\vec{E}) = \frac{1}{2}\beta_S^T \vec{E} \tag{49}$$

On the other hand, the energy of interaction of electric dipole moment $\vec{d}$ with electric field $\vec{E}$

$$W_E = -\vec{d}\vec{E}. \tag{50}$$

As it follows from (47) and (50), magnetic field induces the electric dipole moment of atom

$$\overrightarrow{d_{ind}}(\vec{H}) = \frac{1}{2}\beta_S^T \vec{H} \tag{51}$$

As appears from the above, atom (molecule) being placed to static electric field gets the induced magnetic moment which in its part produces magnetic field. And similarly, if atom (molecule) is exposed to magnetic field the induced electric dipole moment yields to the appearance of its associated electric field.

Let us consider the simplest possible experiment. Suppose that homogeneous isotropic matter (liquid or gas) is exposed to an electric field $\vec{E}$. From the above it follows that the time reversal violation yields to the appearance of magnetic field $\vec{H}_T = 4\pi\rho\vec{\mu}(\vec{E})$ parallel to $\vec{E}$ in this area (ρ is the number of atoms (molecules) of matter per cm^3). And vice versa, the electric field $\overrightarrow{E}_T = 4\pi\rho\overrightarrow{d}(\overrightarrow{H})$ appears under matter placement to the area occupied by a magnetic field $\overrightarrow{H}$. Let us estimate the effect value. It is easy to do by β_S^T evaluation. The general case explicit expression for polarizabilities for time dependent fields were derived in [10] (see eqns. (12)-(20) therein).

Briefly the calculation technique is as follows. Let us suppose that atom is placed to the arbitrary periodic in time electric and magnetic fields. The energy of interaction of an atom (molecule) with these fields has the routine form

$$W = -\widehat{\overrightarrow{d}}\overrightarrow{E} - \widehat{\overrightarrow{\mu}}\overrightarrow{H} + ... \tag{52}$$

where $\widehat{\overrightarrow{d}}$ is the operator of atom electric dipole moment and $\widehat{\overrightarrow{\mu}}$ is the operator of atom magnetic dipole moment

$$\overrightarrow{E} = \frac{1}{2}\left\{\overrightarrow{E}_0\, e^{-i\omega t} + \overrightarrow{E}_0^*\, e^{i\omega t}\right\},\ \overrightarrow{H} = \frac{1}{2}\left\{\overrightarrow{H}_0\, e^{-i\omega t} + \overrightarrow{H}_0^*\, e^{i\omega t}\right\} \tag{53}$$

The Shrödinger equation describing atom interaction with electromagnetic field is as follows:

$$i\hbar\frac{\partial\psi(\xi,t)}{\partial t} = [H_A(\xi) + W(\xi,t)]\psi(\xi,t), \tag{54}$$

where $H_A(\xi)$ is the atom Hamiltonian taking into account the weak interaction of electrons with nucleus in the center of mass of the system, ξ is the space and spin variable of electron and nucleus, W is the energy of interaction of atom with electromagnetic field of frequency ω

$$\begin{aligned} W &= Ve^{-i\omega t} + V^+e^{i\omega t}, \qquad (55)\\ V &= -\frac{1}{2}(\overrightarrow{d}\overrightarrow{E_0} + \overrightarrow{\mu}\overrightarrow{H_0}), V^+ = -\frac{1}{2}(\overrightarrow{d}\overrightarrow{E_0}^* + \overrightarrow{\mu}\overrightarrow{H_0}^*) \end{aligned}$$

Let us perform the transformation $\psi = \exp(-\frac{iH_At}{\hbar})\varphi$. Suppose $H_A\psi_n = E_n\psi_n$ ($E_n = E_n^{(0)} - \frac{1}{2}i\Gamma_n$, $E_n^{(0)}$ is the atom level energy, Γ_n is the atom level width), then $\varphi = \sum_n b_n(t)\psi_n$. Therefore it follows from (55)

$$\begin{aligned} i\hbar\frac{\partial b_n(t)}{\partial t} = \sum_f \{\langle n|V|f\rangle \exp[i(E_n - E_f - \hbar\omega)t/\hbar] + \qquad (56)\\ + \langle n|V^+|f\rangle \exp[i(E_n - E_f + \hbar\omega)t/\hbar]\big\}\, b_f(t),\ \langle\psi_n|\psi_m\rangle \ll 1. \end{aligned}$$

Suppose b_{n0} be the ground state amplitude. Let us substitute the amplitude b_f describing the excited atom state into the equation for b_{n0} and study this equation at time $t \gg \tau_f = \hbar/\Gamma_f$ (or $\tau_f = \hbar/\Delta E$); $\Delta E = E_f^{(0)} - E_{n0} - \hbar\omega$; $\Gamma_f \gg |\langle n|V|f\rangle|$ (or $\Delta E \gg |\langle n|V|f\rangle|$). Therefore b_{n0} is defined by equation

$$i\hbar\frac{\partial b_{n0}(t)}{\partial t} = \widehat{U}_{eff}\, b_{n0},\ where$$

$$\widehat{U}_{eff} = -\sum_f \left(\frac{\langle n_0|V|f\rangle \, \langle f|V^+|n_0\rangle}{E_f - E_{n0} + \hbar\omega} + \frac{\langle n_0|V^+|f\rangle \, \langle f|V|n_0\rangle}{E_f - E_{n0} - \hbar\omega} \right) \tag{57}$$

Substituting V and V^+ into (57) one can obtain

$$\widehat{U}_{eff} = -\frac{1}{2}\widehat{g}^{E}_{ik} E_{0i} E^*_{0k} - \frac{1}{2}\widehat{g}^{H}_{ik} H_{0i} H^*_{0k} - \frac{1}{2}\widehat{g}^{EH}_{ik} E_{0i} H^*_{0k} - \frac{1}{2}\widehat{g}^{HE}_{ik} H_{0i} E^*_{0k}, \tag{58}$$

where the polarizability of atom (molecule) is:

$$\begin{aligned}
\widehat{g}^{E}_{ik} &= -\frac{1}{2}\left(\sum_f \frac{\langle n_0|d_i|f\rangle \, \langle f|d_k|n_0\rangle}{E_f - E_{n0} + \hbar\omega} + \frac{\langle n_0|d_k|f\rangle \, \langle f|d_i|n_0\rangle}{E_f - E_{n0} - \hbar\omega} \right) \\
\widehat{g}^{H}_{ik} &= -\frac{1}{2}\left(\sum_f \frac{\langle n_0|\mu_i|f\rangle \, \langle f|\mu_k|n_0\rangle}{E_f - E_{n0} + \hbar\omega} + \frac{\langle n_0|\mu_k|f\rangle \, \langle f|\mu_i|n_0\rangle}{E_f - E_{n0} - \hbar\omega} \right) \\
\widehat{g}^{EH}_{ik} &= -\frac{1}{2}\left(\sum_f \frac{\langle n_0|d_i|f\rangle \, \langle f|\mu_k|n_0\rangle}{E_f - E_{n0} + \hbar\omega} + \frac{\langle n_0|\mu_k|f\rangle \, \langle f|d_i|n_0\rangle}{E_f - E_{n0} - \hbar\omega} \right) \\
\widehat{g}^{HE}_{ik} &= -\frac{1}{2}\left(\sum_f \frac{\langle n_0|\mu_i|f\rangle \, \langle f|d_k|n_0\rangle}{E_f - E_{n0} + \hbar\omega} + \frac{\langle n_0|d_k|f\rangle \, \langle f|\mu_i|n_0\rangle}{E_f - E_{n0} - \hbar\omega} \right)
\end{aligned}$$

It should be noted that $\widehat{g}^{E}_{ik}$ and $\widehat{g}^{H}_{ik}$ are the P- and T-invariant electric and magnetic polarizability tensors and $\widehat{g}^{EH}_{ik}$ and $\widehat{g}^{HE}_{ik}$ are the P- and T-noninvariant polarizability tensors

Let an atom be placed at the static ($\omega \to 0$) magnetic and electric fields $\overrightarrow{E}$ and $\overrightarrow{H}$ of the same direction. Then it's perfectly easy to obtain the effective energy of P- and T-odd interaction of an atom with these fields.

$$\widehat{U}^{T,P}_{eff} = -\frac{1}{2}\left(\sum_f \frac{\langle n_0|d_z|f\rangle \, \langle f|\mu_z|n_0\rangle + \langle n_0|\mu_z|f\rangle \, \langle f|d_z|n_0\rangle}{E_f - E_{n_0}} \right) EH \tag{59}$$

Axis z is supposed to be parallel to $\overrightarrow{E}$. Thus from (47)

$$\beta^T_S = \sum_f \frac{\langle n_0|d_z|f\rangle \, \langle f|\mu_z|n_0\rangle + \langle n_0|\mu_z|f\rangle \, \langle f|d_z|n_0\rangle}{E_f - E_{n_0}} \tag{60}$$

Let us estimate the β^T_S order of magnitude. The atom state $|f\rangle$ does not possess the certain parity because of weak T-odd interactions. And over the weakness of V_T the state $|f\rangle$ is mixed with the opposite parity state by factor of $\eta_T = \frac{V^T_W}{E_f - E_n}$. According to the above

$$\beta^T_S \sim \frac{\langle d\rangle \, \langle \mu\rangle}{E_f - E_{n_0}} \eta_T \tag{61}$$

For the heavy atoms the mixing coefficient can attain the value $\eta_T \approx 10^{-14}$. Taking into account that matrix element $\langle\mu\rangle \sim \alpha\langle d\rangle$ (where $\alpha = \frac{1}{137}$ is the fine structure constant) one can obtain $\beta_S^T \sim \eta_T \, \alpha \frac{\langle d \rangle^2}{\Delta} \approx 10^{-16} \cdot \frac{8 \cdot 10^{-36}}{10^{-12}} \approx 10^{-40}$. Therefore, the electric field $E = 10^2\ CGSE$ induces magnetic moment $\mu_T \approx 10^{-38}$. Then, the magnetic field in the liquid target can be estimated as follows

$$H = 4\pi \rho \mu_T \approx 10^{23} \cdot 10^{-38} = 10^{-15}\ gauss \tag{62}$$

The magnitude of magnetic field strength can be increased, for example, by tightening of the magnetic field with superconductive shield. In this way the measured field strength can be increased by four orders when one collect the field from the area 1 m^2 to the area 1 cm^2 (Fig.5).

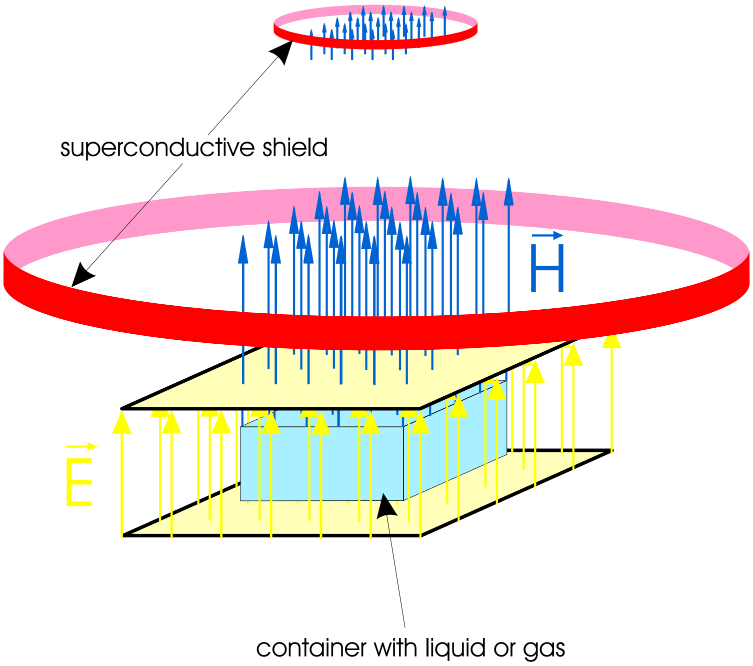


Figure 10:

The induced magnetic moment produces magnetic field at the electron (nucleus) of the atom. This field $H^T(E) \sim \mu \, \langle \frac{1}{r^3} \rangle \sim 10^{-38} \cdot 10^{26} = 10^{-12}\ gauss$. Therefore, the frequency of precession of atom magnetic moment μ_A in the magnetic field induced by an external electric field

$$\Omega_E \sim \frac{\mu_A \, \beta \, E \, \langle 1/r^3 \rangle}{\hbar} = \frac{10^{-20} \cdot 10^{-12}}{10^{-27}} = 10^{-5}\ \sec^{-1} \tag{63}$$

It should be reminded that to measure the electric dipole moment the shift of precession frequency of atom spin in the presence of both magnetic and electric fields is investigated. Then, the T-odd shift of precession frequency of atom spin includes two terms: frequency shift conditioned by interaction of atom electric dipole moment with electric field $\omega_E \sim d_A E/\hbar$ and frequency shift $\Omega \sim \mu H^T(E)\ /\hbar$ defined above. This aspect should be considered when interpreting the similar experiments. One should take note of the mixing coefficient η_T essential increase when the opposite parity levels are closed to each other or even degenerate. Then the effect can grow up as much as several orders $10^5 \div 10^6$ (this occurs, for example, for Dy, TlF, BiS, HgF).

The similar phenomenon of magnetic field induction by electric field can occur in vacuum too.

Due to quantum electrodynamic effect of electron-positron pair creation in strong electric, magnetic or gravitational field, the vacuum is described by the dielectric ε_{ik} and magnetic μ_{ik} permittivity tensors depending on these fields. The theory of ε_{ik} [21] does not take into account the weak interaction of electron and positron with each other. Considering the T- and P-odd weak interaction between electron and positron in the process of pair creation in an electric (magnetic, gravitational) field one can obtain the density of electromagnetic energy of vacuum contains term $\beta_v^T(\vec{E}\vec{H})$ similar (47) (in the case of vacuum polarization by a stationary gravitational field $\beta_g^T(\vec{H}\vec{n_g})$, $\vec{n_g} = \frac{\vec{g}}{g}$, $\vec{g}-$ gravitational acceleration).

As a result both electric and magnetic fields (directed along the electric field) could exist around an electric charge. But in this case $\oint \vec{B}\,d\vec{S} \neq 0$ ($\vec{B}$ is the magnetic induction) that is impossible in the framework of classic electrodynamics. The existence of such field would means the existence of induced magnetic monopole. If the condition $\oint \vec{B}\,d\vec{S} = 0$ is fulfilled then for the spherically symmetrical case the field appears equal to zero. Surely, the value of this magnetic field is extremely small, but the possibility of its existence is remarkable itself.

The above result can be obtained in the framework of general Lagrangian formalism. Lagrangian density can depend only on the field invariants. Two invariants are known for the quasistatic electromagnetic field: $(\vec{E}\vec{H})$ and $(E^2 - H^2)$. In conventional T-invariant theory these invariants are included in the Lagrangian L only as $(E^2 - H^2)$ and $(\vec{E}\vec{H})^2$, i.e. $L = L(E^2 - H^2, (\vec{E}\vec{H})^2)$ [21]. But while taking into account the T-odd interactions the Lagrangian can include invariant $(\vec{E}\vec{H})$ raising to the odd power, i.e.

$$L_T = L_T(E^2 - H^2, (\vec{E}\vec{H})^2, (\vec{E}\vec{H})) \tag{64}$$

Expanding (64) by weak interaction one can obtain

$$L_T = L(E^2 - H^2, (\vec{E}\vec{H})^2) + \beta_T(\vec{E}\vec{H}), \tag{65}$$

where L is the density of Lagrangian in P- and T-invariant electrodynamics, $\beta_T = \beta_T(E^2 - H^2, (\vec{E}\vec{H})^2)$ is the constant can be found in certain theory. The explicit form of L is cited in [21].

The additions caused by the vacuum polarization can be described by the field dependent dielectric and magnetic permittivity of vacuum. According to [21] the electric induction vector $\vec{D}$ and magnetic induction vector $\vec{B}$ are defined as:

$$\vec{D} = \frac{\partial L}{\partial \vec{E}}, \; \vec{B} = -\frac{\partial L}{\partial \vec{H}} \tag{66}$$

Similarly the electric polarization $\vec{P}$ and magnetization $\vec{M}$ of vacuum can be found

[21]:

$$\overrightarrow{P} = \frac{\partial(L_T - L_0)}{\partial\overrightarrow{E}},\ \overrightarrow{M} = -\frac{\partial(L_T - L_0)}{\partial\overrightarrow{H}}, \qquad (67)$$

$$\overrightarrow{D} = \overrightarrow{E} + 4\pi\overrightarrow{P},\ \overrightarrow{B} = \overrightarrow{H} + 4\pi\overrightarrow{M}. \qquad (68)$$

In accordance with the above, the T-noninvariance yields to the appearance of additional P- and T-odd terms to the electric polarization $\overrightarrow{P}$ and magnetization $\overrightarrow{M}$. There are the addition to the vector of electric polarization $\overrightarrow{P}$ proportional to the magnetic field strength $\overrightarrow{H}$ and the addition to the vector of magnetization $\overrightarrow{M}$ proportional to the electric field strength $\overrightarrow{E}$

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