For finitely sets $(A_i)_{i\in I}$ a generalization of the separation law holds and it can be shown, that a separating set can be constructed from Demyanov-differences of the sets A_i.

We consider conditional minimality: A pair $(A, B) \in \mathcal{K}^2(X)$ is called *convex* if $A \cup B$ is a convex set and a convex pair $(A, B) \in \mathcal{K}^2(X)$ is called *minimal convex* if for any convex pair $(C, D) \in [A, B]$ the relation $(C, D) \leq (A, B)$ implies that $(A, B) = (C, D)$.

It is possible to consider the problem pairs of convex sets in the more general frame of a commutative semigroup S which is ordered by a relation $\leq$ and which satifies the condition: if $as \leq bs$ for some $s \in S$, then $a \leq b$. Then $(a, b) \in S^2 = S \times S$ corresponds to a *fraction* $a/b \in S^2$ and *minimality* to a relative prime representation of $a/b \in S^2$.

References

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DEMYANOV DIFFERENCE IN INFINITE DIMENSIONAL SPACES

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We generalize the Demyanov difference to the case of real Hausdorff topological vector spaces.

For $A, B \subset X$ we define *upper difference* $\mathcal{E}_{A,B}$ as the family $\mathcal{E}_{A,B} = \{C \in \mathcal{C}(X) | A \subset \overline{B + C}\}$, where $\mathcal{C}(X)$ is the family of all nonempty closed convex subsets of the topological vector space X. We denote the family of inclusion minimal elements of $\mathcal{E}_{A,B}$ by $m\mathcal{E}_{A,B}$. We define a new subtraction by $A \overset{D}{-} B = \overline{\mathrm{conv}} \bigcup m\mathcal{E}_{A,B}$. We show that $A \overset{D}{-} B$ is a generalization of Demyanov difference.

We prove some clasical properties of the Demyanov difference.

For a locally convex vector space X and compact sets $A, B, C \in \mathcal{C}(X)$ the Demyanov-Difference has the following properties:

(D1) If $A = B + C$, then $C = A \overset{D}{-} B$.

(D2) $(A \overset{D}{-} B) + B \supset A$.

(D3) If $B \subset A$, then $0 \in A \overset{D}{-} B$.

(D4) $(A \overset{D}{-} B) = -(B \overset{D}{-} A)$

(D5) $A \overset{D}{-} C \subset (A \overset{D}{-} B) + (B \overset{D}{-} C)$.

In the proofs we use a new technique which is based on the properties given in the following lemma.

Let X be a Hausdorff topological vector space, A be closed convex, B bounded subset of X. Then for every bounded subset M we have

$$\overline{A + M} = \bigcap_{C \in \mathcal{E}_{A,B}} \overline{B + C + M}.$$

We also give connections between Minkowski subtraction and the union of upper differences.

Let X be a Hausdorff topological vector space, A be closed convex, B bounded subset of X. Then $A \dot{-} B = \bigcap \mathcal{E}_{A,B}$ where $A \dot{-} B = \{x \in X | B + x \subset A\}$.

We show that in the case of normed spaces the Demyanov difference coincides with classical definitions of Demyanov subtraction.

COMPLETENESS IN MINKOWSKI-RÅDSTRÖM-HÖRMANDER SPACES

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A Minkowski-Rådström-Hörmander space $\widetilde{X}$ is a quotient space over the family $\mathcal{B}(X)$ of all nonempty bounded closed convex subsets of a Banach space X. We prove in that a metric d_{BP} (Bartels-Pallaschke metric) is the strongest of all complete metrics in the cone $\mathcal{B}(X)$ and Hausdorff metric d_H is the coarsest of them. Our results follow from for more general case of a quotient space over an abstract convex cone S with complete metric d. We also extend a definition of Demyanov's difference (related to Clarke's subdifferential) of finite dimensional convex sets $A \overset{D}{-} B$ to infinite dimensional Banach space X and we prove in that Demyanov's metric generated by such extension, is complete.