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VECTOR SPACES OF CONVEX SETS: REPRESENTATION OF ITS ELEMENTS AND APPLICATION TO CRYSTAL GROWTH DESCRIPTION

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The quasidifferential calculus [1], developed by V.F. Demyanov and A.M. Rubinov about 30 years ago provides a complete analogon to the classical calculus of differentiation for a wide class of non-smooth functions. Although this looks at the first glance as a generalized subgradient calculus for pairs of subdifferentials it turns out that, after a more detailed analysis, the quasidifferential calculus is a kind of Fréchet-differentiations whose gradients are elements of a suitable Minkowski–Rådström–Hörmander space. Since the elements of the Minkowski–Rådström–Hörmander space are not uniquely determined, we mainly focused our attention to smallest possible representations of quasidifferentials, i.e. to minimal representations. For this purpose we investigated Minkowski addition of convex sets, an inverse operation of subtraction and a quotient vector space of differences of sets. The main results of our research are published in the monograph [3] and the survey paper [4] and concern the following topics:

- Existence and characterization of minimal representations for pairs of compact sets [3, 4].
- Uniqueness of minimal pairs (up to translations) in two-dimensional spaces [3, 4].
- The general translation property and non-uniqueness of minimal pairs in higher dimensional spaces [3, 4].
- Criteria of minimality and methods of reduction [3, 4].

Moreover significant results concerning minimal representation of the difference of polygons and polyhedra are obtained.

In [3] and [4] the basic concept (fundament) of the theory of minimal pairs is presented. Further studies concern the existence of minimal pairs in non-reflexive Banach spaces. An other interesting problem is to find necessary and sufficient condition of minimality in at least three-dimensional spaces and to improve methods of reduction for higher dimensional spaces. Further directions of research concern the structure of continuous piecewise linear functions, separation of convex sets by sets with application to data-classification and the investigation of possible areas of implementation of vector spaces of convex sets and minimal representations of its elements. An important example in this area is the crystal growing process. In particular we were able to give a formula of crystal growth if faces of a crystal grow with constant velocity and to state necessary and sufficient condition for crystal face disappearing [2] .

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ON A SOLUTION OF THE OPTIMAL CONTROL PROBLEM WITH MULTIPOINT BOUNDARY CONDITIONS WITH SWEEP METHOD

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Assume that the considered process is described by the equation

$$\dot{x}(t) = F(t)x(t) + G(t)u(t) + v(t), t \in [0, T] \tag{1}$$

with multipoint boundary conditions

$$\sum_{i=0}^{p} \Phi_i x(t_i) = q, t_i \in (0, T). \tag{2}$$