23. *Kaczorek T., Marchenko V., Sajewski L.* Solvability of 2D hybrid linear systems - comparison of three different methods // Acta Mechanica et Automatica. 2008. V. 2. No. 2. P. 59–66.
24. *Narendra K.S., Shorten R.* Hurwitz stability of Metzler matrices // IEEE Trans. Autom. Contr. 2010. V. 55. No. 6. P. 1484–1487.
25. *Sajewski L.* Solution of 2D singular hybrid linear systems // Kybernetes. 2009. V. 38. N. 7/8. P. 1079–1092.
26. *Xiao Y.* Stability test for 2-D continuous-discrete systems // Proc. 40th IEEE Conf. on Decision and Control, 2010. V. 4. P. 3649–3654.
27. *Xiao Y.* Stability, controllability and observability of 2-D continuous-discrete systems // Proc. Int. Symp. on Circuits and Systems, 2003. V. 4. P. 468–471.

CONVEX SIP PROBLEMS WITH FINITELY REPRESENTABLE COMPACT INDEX SETS

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Inroduction. Semi-Infinite Programming (SIP) deals with extremal problems that involve infinitely many constraints in a finite dimensional space. Due to the numerous theoretical and practical applications, today semi-infinite optimization is a topic of a special interest. Since the most efficient methods for solving optimization problems are usually based on optimality conditions that permit not only to test the optimality of a given feasible solution, but also to find the better direction to optimality, the study of these conditions is essential. Usually the optimality conditions are formulated under certain assumptions that are called Constraint Qualifications (CQ) [1, 2]. On the other hand, the optimality conditions that do not use too strong additional assumptions are of special interest since they are more universal and have more applications.

1. Problem statement. Consider a convex Semi-Infinite Programming problem in the form

$$(P): \qquad \min_{x\in\mathbb{R}^n} c(x) \quad \text{s.t.} \quad f(x,t)\le 0 \ \ \forall\, t\in T,$$

with a finitely representable compact index set $T=\{t\in\mathbb{R}^s:\ g_k(t)\le 0,\ k\in K\},\ |K|<\infty$.

We suppose that the objective function $c(x), x \in \mathbb{R}^n$, is convex; for all $t \in T$, the constraint function $f(x,t),\ x \in \mathbb{R}^n$, is convex w.r.t. x; functions $f(x,t),\ g_k(t), t \in \mathbb{R}^s, k \in K$, are twice continuously differentiable w.r.t. t. Our main aim is to formulate and test new CQ-free optimality conditions for the convex SIP problems (P) on the base of approach proposed in [3].

2. Definitions and main result. Denote by X the feasible set of problem (P): $X = \{x \in \mathbb{R}^n : \ f(x,t) \le 0, \forall t \in T\}$.

Definition 1. *An index $\bar{t} \in T$ is called an immobile one if $f(x,\bar{t}) = 0$ for all $x \in X$.*

Denote by T^* the *set of all immobile indices* in problem (P). Let $\bar{t} \in T^*$. It is evident that for all $x \in X$ the vector $\bar{t}$ solves the following *lower level problem*:

$$LLP(x): \qquad \max_t f(x,t) \quad \text{s.t.} \quad g_k(t) \le 0,\ k \in K.$$

Given $t \in T$, denote by $K_a(t) \subset K$ the set of indices that are active at t, and let $L(t)$ be the linearized cone of feasible directions in the index set T at t:

$$K_a(t) := \{k \in K : \ g_k(t) = 0\}, \qquad (1)$$
$$L(t) := \{l \in \mathbb{R}^s : \ l'\partial g_k(t)/\partial t \le 0,\ k \in K_a(t)\}.$$

Assumption 1. *The constraints of $LLP(x)$ satisfy Mangasarian-Fromovitc constraint qualification [4] at every $\bar{t} \in T^*$.*

For $x \in X, t \in T,\ l \in L(t)$, let us define the following functions with whose help one can formulate optimality conditions for problem $LLP(x)$:

$$F_1(x,t,l) := l'\partial f(x,t)/\partial t, F_2(x,t,l) := l'(\partial^2 f(x,t)/\partial t^2)l + val(LP(x,t,l)).$$

Here $val(LP(x,t,l))$ is the optimal value of the cost function in the problem

$$LP(x,t,l): \qquad \max_{\omega} \ \omega'\frac{\partial f(x,t)}{\partial t}, \quad \text{s.t.} \quad \omega'\frac{\partial g_k(t)}{\partial t} \le l'\frac{\partial^2 g_k(t)}{\partial t^2}l, k \in K_a(t).$$

Definition 2. *Given an immobile index $\bar{t} \in T^*$ and a nontrivial feasible direction $\bar{l} \in L(\bar{t})$, let us define the immobility order $q(\bar{t},\bar{l})$ of $\bar{t}$ along $\bar{l}$ as follows:*
1) $q(\bar{t},\bar{l}) = 0$ if $\exists\ \bar{x} = x(\bar{t},\bar{l}) \in X$ such that $F_1(\bar{x},\bar{t},\bar{l}) < 0$;
2) $q(\bar{t},\bar{l}) = 1$ if $F_1(x,\bar{t},\bar{l}) = 0$ for all $x \in X$ and $\exists\ \bar{x} = x(\bar{t},\bar{l}) \in X$ such that $F_2(\bar{x},\bar{t},\bar{l}) < 0$;
3) $q(\bar{t},\bar{l}) > 1$ if $F_1(x,\bar{t},\bar{l}) = 0$ and $F_2(\bar{x},\bar{t},\bar{l}) = 0$ for all $x \in X$.

Assumption 2. *Suppose that* $q(l,t) \leq 1, \ \forall l \in L(t) \setminus \{0\}, \ \forall t \in T^*$.

Assumption 2 implies that the set T^* consists of a finite number of elements:

$$T^* = \{t_j^*, \ j \in J_*\} \text{ with some finite index set } J_*.$$

Given $t_j^* \in T^*$, consider the set $L(j) := L(t_j^*)$, where $L(t)$ is defined in (1). The set $L(j)$ is a polyhedral cone in $\mathbb{R}^s$. Then, according to the known results on the polyhedral cone's decomposition, there exist vectors

$$b_i(j), \ i \in P(j), \quad a_i(j), i \in I(j), \quad |P(j)| + |I(j)| < \infty,$$

such that the set $L(j)$ admits a finite representation in the parametric form as follows:

$$L(j) = \{l \in \mathbb{R}^s : \ l = \sum_{i \in P(j)} \beta_i b_i(j) + \sum_{i \in I(j)} \alpha_i a_i(j), \ \alpha_i \geq 0, \ i \in I(j)\}.$$

Denote $\quad I_*(j) : \ = \{i \in I(j) : q(t_j^*, a_i(j)) = 0\}$ and $I_0(j) := I(j) \backslash I_*(j)$;

$$L(x,j) : \ = \{l \in L(j) : \alpha_i = 0, \ i \in I_*(j); \ \ F_2(x, t_j^*, l) = 0\}.$$

Theorem 1. *Let Assumptions 1, 2 be fulfilled for the convex SIP problem* (P). *Then a vector* $x^0 \in X$ *is optimal in this problem if and only if there exist indices and directions*

$$t_j \in \{t \in T : f(x^0, t) = 0\} \setminus T^*, j \in J_a;$$
$$l_j^{(k)} \in L(x^0, j), \ k = \overline{1, k_j}, j \in J_*, \ \sum_{j \in J_*} k_j + |J_a| \leq n,$$

such that the vector x^0 *is optimal in the following convex Nonlinear Programming (NLP) auxiliary problem:*

$$\min_{x \in \mathbb{R}^n} c(x)$$

s.t.
$$f(x, t_j^*) = 0, \ F_1(x, t_j^*, b_i(j)) = 0, \ i \in P(j);$$
$$F_1(x, t_j^*, a_i(j)) = 0, \ i \in I_0(j);$$
$$F_1(x, t_j^*, a_i(j)) \leq 0, \ i \in I_*(j); \ \ F_2(x, t_j^*, l_j^{(k)}) \leq 0, \ k = \overline{1, k_j}, \ j \in J^*,$$
$$f(x, t_j) \leq 0, j \in J_a.$$

Study of properties of the NLP auxiliary problem permits us to formulate new CQ-free optimality conditions for the SIP problem under consideration. These conditions have the form of explicit optimality criteria, taking the form of sufficient optimality conditions when

Assumptions 1, 2 are relaxed. Thanks to their constructive nature, the conditions obtained can be easily verified. Since the Assumptions assumed in this paper are weaker than the known from literature constraint qualifications for SIP problems, the new optimality conditions can be applied for a more general case of SIP problems.

References

1. *Goberna M.A., Lópes M.A.* Semi-Infinite Programming: recent advances. Kluwer, Dordrecht. 2001.
2. *Ruckmann J.-J., Shapiro A.* Second-order optimality conditions in Generalized Semi-Infinite Programming// Set-Valued Analysis. 2001. No. 9. P. 169—186.
3. *Kostyukova O.I., Tchemisova T.V.* Implicit Optimality Criterion for convex SIP problem with box constrained index set// TOP. 2012. V. 20, No. 2, P. 475–502.
4. *Mangasarian O.L., Formovitz S.* The Fritz-John necessary optimality conditions in presence of equality and inequality constraints// J. Math. Anal. Appl. 1967. V. 17. P. 37–47.

NONLINEAR POSITIONAL DIFFERENTIAL GAME IN THE CLASS OF MIXED STRATEGIES

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The feedback control problem for a nonlinear dynamic system under lack of information on disturbances is considered. The problem on minmax-maxmin of ensured result for a given positional quality index is formalized into an antagonistic two-player differential game in the framework of the concept of the Sverdlovsk (Ekaterinburg) school on the theory of control and differential games. The problem is solved in the class of mixed positional strategies. The existence of a solution for considered differential game – of the value of the game and the saddle point – is determined. The solution of a problem is based on application of the appropriate models-leaders, the so-called methods of minimax and maximin extremal shift [2] and the method of upped convex hulls [1]. Although we use probabilistic mechanisms in formation of control, the final result is guaranteed with probability arbitrary close to one. Results of the study are applied to the control model [3] of a mechanical device. It simulates a controller in the space equipment used for docking and landing of modules. Simulation outputs are presented.