ESTIMATES FOR SLOW CONTROLS

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Consider the abstract linear control system

$$y' = Ay + Bu,$$

where A generates a C_0-semigroup on X and $B \in \mathcal{L}(U, X)$, U and X being Banach spaces. The minimum energy to bring $x \in X$ to zero in time $t > 0$ is

$$\mathcal{E}(t, x) = \inf\{\|u\|; u \in L^\infty(0, t; U), y(t, x, u) = 0\}.$$

We study the behavior of $\mathcal{E}(t, x)$, when $t \to \infty$. In fact, we get estimation of the type

$$\mathcal{E}(t, x) \leq \gamma(t)\|x\|,$$

with explicit $\gamma(t)$, for t large. Our results are related to [2], [1], and [3].

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OPTIMALITY CONDITIONS IN THE ALTERNANCE FORM

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The most important tool used in optimization is that of directional derivative or different generalizations of it. For the class of directionally differentiable functions in the n-dimensional space, a necessary condition for an unconstrained minimum is $f'(x^*, g) \geq 0 \quad \forall g \in \mathbb{R}^n$ while a necessary condition for an unconstrained maximum is $f'(x^*, g) \leq 0 \quad \forall g \in \mathbb{R}^n$ where $f'(x, g)$ is the derivative of the function f at a point $x \in \mathbb{R}^n$ in a direction g.

The above stated conditions become efficient for special classes of directionally differentiable functions. For example, in the case of convex and max-type functions, $f'(x, g)$ takes the form $f'(x, g) = \max_{v \in \partial f(x)}(v, g)$, where $\partial f(x)$ is the subdifferential of the function f at a point $x \in \mathbb{R}^n$. Then the above necessary condition for a minimum is equivalent to the inclusion $0_n \in \partial f(x^*)$. If a point $x_0 \in \mathbb{R}^n$ is given, to check this condition, one can find $\min_{z \in \partial f(x_0)} ||z|| = ||z(x_0)||$. If $||z(x_0)|| = 0$, it means that x_0 satisfies the necessary condition.

In the case of an arbitrary directionally differentiable function, by means of the notions of upper and lower exhausters (see [2]), the problem of verification of optimality of a given point is also reduced to that of checking the condition $0 \in C$ for one or several convex sets C. In turn, the problem of verifying this condition is reduced to that of finding the point of C which is the nearest to the origin. If the origin does not belong to C, we easily find a descent direction (and after testing all sets C, one find a steepest descent direction). Then it is possible to construct a numerical method. This approach was developed in [1,2,3].

Another approach is based on the so-called alternance idea coming back to P.L.Chebyshev. In [4] this approach was extended to study mathematical programming problems.

For the classical Chebyshev approximation problem (the problem of approximating a function $f(t) : G \to \mathbb{R}$ by a polynomial $P(t)$), the condition for a minimum takes the so-called alternance form: for a polynomial $P^*(t)$ to be a solution to the Chebyshev approximation problem, a collection of points $\{t_i \mid t_i \in G\}$ should exist at which the difference $P^*(t) - f(t)$ attains its maximal absolute value with alternating signs. This condition can easily be verified, and if it does not hold, one can find a "better" polynomial. In the present talk, it will be demonstrated that the alternance form of the necessary conditions for a minimum is valid not only for Chebyshev approximation problems, but also in the general case of directionally differentiable functions. Both unconstrained and constrained optimization problems are discussed. In many cases a constrained optimization problem can be reduced (via Exact Penalization Techniques) to an unconstrained one.

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CONVERTIBILITY OF EXHAUSTERS OF CONTINUOUS POSITIVELY HOMOGENEOUS FUNCTIONS

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In [1] a family $\Phi := \{\varphi\}$ of sublinear functions $\varphi : \mathbb{R}^n \to \mathbb{R}$ was called *a primal upper exhauster* of a positively homogeneous function $p : \mathbb{R}^n \to \mathbb{R}$ if

$$p(x) = \inf_{\varphi \in \Phi} \varphi(x) \text{ for all } x \in \mathbb{R}^n. \quad (1)$$

Similarly, a family $\Psi := \{\psi\}$ of superlinear functions $\psi : \mathbb{R}^n \to \mathbb{R}$ was called *a primal lower exhauster* of a positively homogeneous function $p : \mathbb{R}^n \to \mathbb{R}$ if

$$p(x) = \sup_{\psi \in \Psi} \psi(x) \text{ for all } x \in \mathbb{R}^n. \quad (2)$$

The primal exhausters were introduced by A.M. Rubinov (see [2]) and were entitled the exhaustive families of upper convex (lower concave) approximations. The term "exhauster" was invented by V.F. Demyanov [3]. In Demyanov's terminology an upper exhauster of p is the family of subdifferentials $\{\partial\varphi \,|\, \varphi \in \Phi\}$ corresponding to a family of sublinear functions Φ that satisfies (1). In [1] the family $\{\partial\varphi \,|\, \varphi \in \Phi\}$, where Φ is a primal upper exhauster, was called *a dual upper exhauster* and the family $\{\partial\psi \,|\, \psi \in \Psi\}$, where Φ is a primal lower exhauster, was called *a dual lower exhauster.*

In [2] was shown that a positively homogeneous function $p : \mathbb{R}^n \to \mathbb{R}$ is continuous on $\mathbb{R}^n$ if and only if it admits both an upper exhauster and a lower one.