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CONVERTIBILITY OF EXHAUSTERS OF CONTINUOUS POSITIVELY HOMOGENEOUS FUNCTIONS

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In [1] a family $\Phi := \{\varphi\}$ of sublinear functions $\varphi : \mathbb{R}^n \to \mathbb{R}$ was called *a primal upper exhauster* of a positively homogeneous function $p : \mathbb{R}^n \to \mathbb{R}$ if

$$p(x) = \inf_{\varphi \in \Phi} \varphi(x) \text{ for all } x \in \mathbb{R}^n. \quad (1)$$

Similarly, a family $\Psi := \{\psi\}$ of superlinear functions $\psi : \mathbb{R}^n \to \mathbb{R}$ was called *a primal lower exhauster* of a positively homogeneous function $p : \mathbb{R}^n \to \mathbb{R}$ if

$$p(x) = \sup_{\psi \in \Psi} \psi(x) \text{ for all } x \in \mathbb{R}^n. \quad (2)$$

The primal exhausters were introduced by A.M. Rubinov (see [2]) and were entitled the exhaustive families of upper convex (lower concave) approximations. The term "exhauster" was invented by V.F. Demyanov [3]. In Demyanov's terminology an upper exhauster of p is the family of subdifferentials $\{\partial\varphi \,|\, \varphi \in \Phi\}$ corresponding to a family of sublinear functions Φ that satisfies (1). In [1] the family $\{\partial\varphi \,|\, \varphi \in \Phi\}$, where Φ is a primal upper exhauster, was called *a dual upper exhauster* and the family $\{\partial\psi \,|\, \psi \in \Psi\}$, where Φ is a primal lower exhauster, was called *a dual lower exhauster.*

In [2] was shown that a positively homogeneous function $p : \mathbb{R}^n \to \mathbb{R}$ is continuous on $\mathbb{R}^n$ if and only if it admits both an upper exhauster and a lower one.

The equalities (1) and (2) show that an upper exhauster as well as a lower one completely characterize the continuous positively homogeneous function p. Therefore if one of the exhausters is known (for instance, an upper one) than it is natural to expect that it is possible to transform it into another (lower) one. A procedure of such transformation is called [4] *an exhauster conversion* or, shortly, *a convertor.*

V.F. Demyanov [4] developed a convertor for exhausters of Lipshitzian positively homogeneous functions. In case of exhausters of continuous positively homogeneous functions which are not Lipshitzian his method of conversion makes it possible to construct only so called a generalized exhauster containing sublinear (or superlinear) functions with values in the extended reals $\overline{\mathbb{R}}$.

In this report we will present the method of exhauster conversion which transforms any initial (upper or lower) exhauster of continuous positively homogeneous functions into a regular (lower or upper) exhauster consisting of only sublinear (or superlinear) functions with values in $\mathbb{R}$.

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QUASIDIFFERENTIABLE CALCULUS AND MINIMAL PAIRS OF COMPACT CONVEX SETS

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The quasidifferential calculus developed by V.F. Demyanov and A.M. Rubinov [1] almost 30 years ago provides a complete analogon to the classical calculus of differentiation for a wide class of non-smooth functions.