$$\int_0^\ell |q(x)|\, dx < +\infty. \tag{2}$$

The solution of the problem (1) is reduced to Volterra type integral equation:

$$y(x,\delta) = \alpha_1 e^{i\delta x} + \alpha_2 e^{-i\delta x} + \frac{1}{\delta}\int_0^\ell \sin\delta(\xi - x) q(\xi) y(\xi,\delta) d\xi.$$

It is solved by the sequential approximations method. Considering condition (2), their convergence to the exact solution of the problem is proved.

Influence of volume content of bubbles and contraction on wave propagation velocity is revealed.

THE AVERAGING IN THE MULTIFREQUENCY SYSTEM OF DIFFERENTIAL EQUATIONS WITH LINEARLY TRANSFORMED ARGUMENTS AND NOETHER BOUNDARY CONDITIONS

Ya.I. Bihun, I.V. Krasnokutska

Chernivtsi National University, 2 Kotsyubynskyi Str., 58012, Chernivtsi, Ukraine
yaroslav.bihun@gmail.com, Nicka176@yandex.ru

Introduction. Quite often mathematical models of oscillation processes are systems of differential equations, in which part of variables evolves with time slowly (slow or amplitude variables), while the others – fast (fast or phase variables). In many cases, by carrying in a small parameter $\varepsilon > 0$, this systems can be written as

$$\frac{da}{d\tau} = X(\tau, a, \varphi, \varepsilon), \quad \frac{d\varphi}{d\tau} = \frac{\omega(\tau, a)}{\varepsilon} + Y(\tau, a, \varphi, \varepsilon),$$

where $a \in D \subset R^n$, $\varphi \in R^m$; $m \geq 2$; ε is a small positive parameter, the vector functions X and Y are periodic in the variables φ with the period 2π. The main problem arising in the study of properties of solutions of the system is the problem of resonance relations between the components of the variable frequency vector $\omega(\tau, a)$ [?].

Noether boundary problems for ordinary differential equations and equations with lateness were researched in the work [2].

In this work we substantiate the method of averaging for the initial and boundary-value problem with linearly transformed argument and the frequency vector depending on slow variable. The obtained results are used for the investigation of the existence of the method of averaging for some boundary-value problems.

1. Problem Setting. This paper presents the conditions of existence of solution the system with slow and fast variables on the form

$$\frac{da}{d\tau} = X(\tau, a_\Lambda, \varphi_\Theta), \tag{1}$$

$$\frac{d\varphi}{d\tau} = \frac{\omega(\tau)}{\varepsilon} + Y(\tau, a_\Lambda, \varphi_\Theta), \tag{2}$$

where $\tau \in [0, L]$, small parameter $\varepsilon \in (0, \varepsilon_0]$, $\varepsilon_0 \ll 1$, $x \in D \subset \mathbb{R}^m$, $\varphi \in \mathbb{R}^m$; λ_i i θ_j – numbers from semi-interval $(0, 1]$, $0 < \lambda_1 < \dots < \lambda_{r_1} \leq 1$, $0 < \theta_1 < \dots < \theta_{r_2} \leq 1$, $a_{\lambda_i}(\tau) = a(\lambda_i \tau)$, $\varphi_{\theta_j}(\tau) = \varphi(\theta_j \tau)$, $a_\Lambda = (a_{\lambda_1}, \dots, a_{\lambda_{r_1}})$, $\varphi_\Theta = (\varphi_{\theta_1}, \dots, \varphi_{\theta_{r_2}})$.

Let us set the system (1), (2) of boundary conditions

$$A_0 a|_{\tau=0} + A_1 a|_{\tau=L} + \int_0^L f(s, a_\Lambda(s), \varphi_\Theta(s))ds = d, \tag{3}$$

$$B_0 \varphi|_{\tau=0} + B_1 \varphi|_{\tau=L} + \int_0^L B(s)\varphi(s)ds = g_0 a|_{\tau=0} + g_1 a|_{\tau=1} + g_2 \int_0^L a(s)ds, \tag{4}$$

where f – preset n-measurable function, 2π-periodic with components φ_Θ, A_0, A_1 – constant $(n \times n)$ matrixes, B_0, B_1 – constant $q \times m$-matrixes, and B is vector-function of the same extension, d – preset n-vector, g_0, g_1, g_2 – constant $(q \times n)$-matrixes.

We constructed the averaged system on fast variables on form:

$$\frac{d\overline{a}}{d\tau} = X_0(\tau, \overline{a}_\Lambda), \tag{5}$$

$$\frac{d\overline{\varphi}}{d\tau} = \frac{\omega(\tau)}{\varepsilon} + Y_0(\tau, \overline{a}_\Lambda). \tag{6}$$

Averaged boundary conditions:

$$A_0 \overline{a}|_{\tau=0} + A_1 \overline{a}|_{\tau=L} + \int_0^L f_0(s, \overline{a}_\Lambda(s))ds = d, \tag{7}$$

$$B_0 \overline{\varphi}|_{\tau=0} + B_1 \overline{\varphi}|_{\tau=L} + \int_0^L B(s)\overline{\varphi}(s)ds = g_0 \overline{a}|_{\tau=0} + g_1 \overline{a}|_{\tau=1} + g_2 \int_0^L \overline{a}(s)ds. \tag{8}$$

We propose and justify the averaging for systems with delay with multipoint or integral boundary conditions. The average is also applied to integral boundary conditions. For slow and fast variables the averaging method was constructed on the interval $[0, L]$ and the estimate of error $O(\varepsilon^{\alpha})$, $\alpha \in (0, 1/(2m)]$ was obtained [3].

2. The Averaging Method. In the paper [4] was proved next theorem.

Теорема 1. *Let us suppose, that:*

1. all needed conditions are solved [3];

2. the unique solution of noether problem (5), (7) with $\overline{a} = \overline{a}(\tau, \overline{y})$, $\overline{a}(0, \overline{y}) = \overline{y}$ *exists, and it lays in* D *together with certain* ρ*-neighbourhood;*

3. matrix $M_1 = A_0 + A_1 \dfrac{\partial \overline{a}(L, \overline{y})}{\partial \overline{y}} + \displaystyle\int_0^L f_0(s, \overline{a}(s, \overline{y})) \dfrac{\partial \overline{a}(s, \overline{y})}{\partial \overline{y}} ds$ – *invertible, and*

$$M_2 = B_0 + B_1 + \int_0^L B(s) ds \text{ – } (q \times m)\text{- full rank matrix, } q \geq m.$$

Then there will be found constants $c > 0$, $\varepsilon_1 > 0$ *such that for every* $\varepsilon \in (0, \varepsilon_1]$ *the unique solution of the boundary problem (1)–(4) exists, moreover, for fast variables* φ *as pseudo solution, and for all* $\tau \in [0, L]$ *and* $\varepsilon \in (0, \varepsilon_1]$ *estimation performs*

$$\|a(\tau, y, \psi, \varepsilon) - \overline{a}(\tau, \overline{y})\| + \|\varphi(\tau, y, \psi, \varepsilon) - \overline{\varphi}(\tau, \overline{y}, \overline{\psi}, \varepsilon)\| \leq c\varepsilon^{\alpha},$$

$$\alpha \in (0, 1/(mq)].$$

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