Time-reversal violating rotation of polarization plane of light in gas placed in the electric field

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Abstract

The rotation of polarization plane of light in a gas placed in the electric field is considered. Different mechanisms that cause this phenomenon are investigated. Angle of polarization plane rotation for transition $6S_{1/2} \rightarrow 7S_{1/2}$ in cesium ($\lambda = 539$ nm) is estimated. The possibility to observe this effect in experiment is discussed.

1 Introduction

Violation of time reversal invariance was discovered more then 30 years ago in K meson decay [1, 2]. Up to now it remains one of the great unsolved problems in elementary particle physics. Many experiments were devoted to the search of any other manifestation of time reversal noninvariance. Among them for example are measurements of electrical dipole moment (EDM) of neutrons [3], atoms and molecules [4, 5, 6]. No EDM was found but these experiments impose strong restrictions on theory. In particular search of EDM in heavy atoms set a tight limits on parameters of electron - nucleon PT violating interactions and value of electron EDM.

At present more precise schemes of experiment are actively discussed. One of them is the observation of the light polarization plane rotation caused by pseudo-Zeeman splitting of magnetic sub-levels of atom with nonzero EDM in electric field. This effect arises due to interaction $W = -\vec{d_a}\vec{E}$ of atomic EDM with external electric field [7, 8, 9]. We should note here that discussions of these experiments [7, 8, 9] take into account only static EDM of atom. According to [10, 11] atom has another PT noninvariant characteristic that describes its response to the external electric field. It is a P and T - odd polarizability β_E^{PT} that arises in electric field due to interference of PT - odd and Stark - induced transition amplitudes. As was shown in [10] PT- odd polarizability also leads to the rotation of photon polarization plane and circular dichroism of atomic gas in external electric field. This contribution should exist even in a hypothetical case when atomic EDM in a ground and excited states are occasionally equal to zero and pseudo-Zeeman splitting of atomic levels is absent. Unlike rotation of polarization plane due to atomic EDM that manifest Macaluso-Corbino dependence of angle on light frequency, rotation caused by β_E^{PT} is a kinematic analog of Faraday rotation in a Van-Vleck paramagnetic.

Moreover PT noninvariant polarizability β_E^{PT} cause magnetization of atom in electric field [11, 12]. This magnetization in turn induce magnetic field H_{ind} . The energy of interaction of magnetic moment of atom with this field is $W_H = -\vec{\mu}_a \vec{H}_{ind}(E)$. Therefore the total splitting of atomic levels that cause rotation of polarization plane of light is equal to $W_H = -\vec{d}_a \vec{E} - \vec{\mu}_a \vec{H}_{ind}(E)$. This splitting appears even if atomic EDM d_a is equal to zero.

In this paper the mechanisms of rotation of polarization plane of light due to both PT noninvariant electronnucleon interactions and nonzero electron EDM are considered. Estimates of expected angle of polarization plane rotation near highly forbidden magnetic dipole transition $6S_{1/2} \rightarrow 7S_{1/2}$ in cesium ($\lambda = 539$ nm) are performed. Possible experimental schemes to observe this rotation are discussed.

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2 PT - odd mixing

We start with the simplest case. Let us place an atom in a ground state $s_{1/2}$ to the electric field. If we take into account admixture of the nearest $p_{1/2}$ state due to P and T odd interactions and interaction with the electric field, then the wave function of atom takes the form.

$$|\tilde{s}_{1/2}\rangle = \frac{1}{\sqrt{4\pi}} (R_0(r) - R_1(r)(\vec{\sigma}\vec{n})\eta - R_1(r)(\vec{\sigma}\vec{n})(\vec{\sigma}\vec{E})\delta)|\chi_{1/2}\rangle$$
(1)

Here $\vec{\sigma}$ - are the Pauli matrices, $\vec{n} = \vec{r}/r$ is the unit vector along the direction of \vec{r} , R_0 and R_1 are radial parts of $s_{1/2}$ and $p_{1/2}$ wave functions respectively, $|\chi_{1/2}\rangle$ is the spin part of wave function, η and δ are the mixing coefficients due to P and T noninvariant interactions and electric field respectively.

Let us consider orientation of electron spin in atom. In order to find the spatial distribution of spin direction we can calculate the matrix element of electron spin operator in respect to the spin part of atomic wave function. Only terms proportional to the product of electrical field strength \vec{E} and PT odd mixing coefficient η are important for our consideration because only they cause PT - odd rotation of polarization plane of light. The change of spin direction due to these terms is

$$\Delta \vec{s}(\vec{r}) = \frac{\eta \delta}{8\pi} R_1^2 \left\langle \chi_{1/2} | (\vec{\sigma}\vec{n}) \vec{\sigma}(\vec{\sigma}\vec{n}) (\vec{\sigma}\vec{E}) + (\vec{\sigma}\vec{E}) (\vec{\sigma}\vec{n}) \vec{\sigma}(\vec{\sigma}\vec{n}) | \chi_{1/2} \right\rangle$$
$$= \frac{\eta \delta R_1^2}{8\pi} \left(4\vec{n}(\vec{n}\vec{E}) - 2\vec{E} \right)$$
(2)

The vector field $4\vec{n}(\vec{n}\vec{E}) - 2\vec{E}$ is shown in Fig. 1. Since $\Delta \vec{s}$ does not depend on initial direction of atomic spin, this spin structure appears even in non-polarized atom. Let us note that the spin vector averaged over spatial variables differs from zero and is directed along the vector of electric field strength \vec{E} . The photons with directions of angular moment parallel and antiparallel to the electric field will interact with such spin structure in a different ways causing rotation of polarization plane of light.

Polarization of atoms produce magnetic moment of gas [12]. Thus we have another interesting PT - odd effect. If we place gas in electric field, small magnetic field will appear. Magnetic field in turn will interact with magnetic moment of atom giving another contribution to the rotation of the polarization plane of light [11].

According to [10] if light propagates along the direction of electric field, then the amplitude of elastic coherent forward scattering of light by nonpolarized atoms has the form

$$f_{ik}(0) = f_{ik}^{ev} + \frac{\omega^2}{c^2} (i\beta_s^P \epsilon_{ikl} n_{\gamma l} + i\beta_E^{PT} \epsilon_{ikl} n_{El})$$
(3)

Here f_{ik}^{ev} is P and T invariant part of scattering amplitude, β_s^P is the P - odd but T - even scalar atomic polarizability, β_E^{PT} is P and T - odd scalar polarizability of atom, $\vec{n}_{\gamma} = \vec{k}/k$ is the unit vector along the direction of photon propagation, $\vec{n}_E = \vec{E}/E$ is the unit vector along the direction of electric field, ϵ_{ijk} is the third rank antisymmetric tensor.

The refraction index of gas can be written as

$$n = 1 + \frac{2\pi N}{k^2} f(0) \tag{4}$$

where N is the number of atoms per cm^3 , k is the photon wave vector. Using (3) expression (4) can be rewritten as follows

$$n_{\pm} = 1 + \frac{2\pi N}{k^2} (f^{ev}(0) \mp \frac{\omega^2}{c^2} [\beta_s^P + \beta_E^{PT}(\vec{n}_E \vec{n}_{\gamma})])$$
(5)

Indices + and - stands for left and right circular polarization of incident light respectively. Angle of rotation of polarization plane has the form

$$\phi = \frac{1}{2}kRe(n_{+} - n_{-}) = -\frac{2\pi N\omega}{c}(\beta_{s}^{P} + \beta_{E}^{PT}(\vec{n}_{E}\vec{n}_{\gamma}))$$
(6)



Figure 1: Vector field $4\vec{n}(\vec{n}\vec{E}) - 2\vec{E}$. Vectors on figure shows direction of atomic spin in $s_{1/2}$ state if we take into account admixture of $p_{1/2}$ state due to PT noninvariant interactions and external electric field.

Term proportional to β_s^P describes well known phenomenon of P - odd but T - even rotation of polarization plane of light. Term proportional to β_E^{PT} describes P and T noninvariant rotation of polarization plane of light about the direction of electric field. P and T - odd rotation of polarization plane of light change sign with the reversal of electric field direction in contrast to P odd but T even rotation. This allow us to distinguish PT - odd rotation from the other possible mechanisms of rotation of polarization plane.

Refraction index of gas has both real and imaginary parts. Since imaginary part of refraction index for left and right circularly polarized photons are also different due to P and T odd interactions, the admixture of circular polarization to the linearly polarized light traveling in gas (circular dichroism) appears.

Let us consider PT noninvariant polarizability β_E^{PT} . According to [11, 10] the tensor of dynamical polarizability of an atom (molecule) has the form

$$\alpha_{ik}^{n} = \sum_{m} \left\{ \frac{\langle \tilde{g}_{n} | d_{i} | \tilde{e}_{m} \rangle \langle \tilde{e}_{m} | d_{k} | \tilde{g}_{n} \rangle}{E_{em} - E_{gn} - \hbar\omega} + \frac{\langle \tilde{g}_{n} | d_{k} | \tilde{e}_{m} \rangle \langle \tilde{e}_{m} | d_{i} | \tilde{g}_{n} \rangle}{E_{em} - E_{gn} + \hbar\omega} \right\}$$
(7)

where $|\tilde{g}_n \rangle$ and $|\tilde{e}_m \rangle$ are wave functions of atom in ground and excited states perturbed by electric field and PT - noninvariant interactions, d is the operator of dipole transition, ω is the frequency of incident light, E_{em} and E_{gn} are the energies of atom in states $|\tilde{g}_n \rangle$ and $|\tilde{e}_m \rangle$ respectively.

In general case atoms are distributed to the sub - levels of ground state g_n with the probability P(n). Therefore, α_{ik}^n should be averaged over all states n. As a result, the polarizability can be written

$$\alpha_{ik} = \sum_{n} P(n)\alpha_{ik}^{n} \tag{8}$$

The tensor α_{ik} can be decomposed into the irreducible parts as

$$\alpha_{ik} = \alpha_0 \delta_{ik} + \alpha^s_{ik} + \alpha^a_{ik}. \tag{9}$$

Here $\alpha_0 = \frac{1}{3} \sum_i \alpha_{ii}$ is the scalar, $\alpha_{ik}^s = \frac{1}{2} (\alpha_{ik} + \alpha_{ki}) - \alpha_0 \delta_{ik}$ is the symmetric tensor of rank two, $\alpha_{ik}^a = \frac{1}{2} (\alpha_{ik} - \alpha_{ki})$ is the antisymmetric tensor of rank two,

$$\alpha_{ik}^{a} = \frac{\omega}{\hbar} \sum_{n} P(n) \sum_{m} \left\{ \frac{\langle \tilde{g}_{n} | d_{i} | \tilde{e}_{m} \rangle \langle \tilde{e}_{m} | d_{k} | \tilde{g}_{n} \rangle - \langle \tilde{g}_{n} | d_{k} | \tilde{e}_{m} \rangle \langle \tilde{e}_{m} | d_{i} | \tilde{g}_{n} \rangle}{\omega_{em,gn}^{2} - \omega^{2}} \right\}$$
(10)

where $\omega_{em,gn} = (E_{em} - E_{gn})/\hbar$.

Let atoms (molecules) be nonpolarized. The antisymmetric part of polarizability (10) is equal to zero in the absence of T- and P- odd interactions [11]. It should be reminded that for the P-odd and T-even interactions the antisymmetric part of polarizability differs from zero only for both the electric and magnetic dipole transitions [13].

We can evaluate the antisymmetric part α_{ik}^a of the tensor α_{ik} of dynamical polarizability of atom (molecule), and, as a result, obtain the expression for β_E^{PT} in the following way. According to (5) the magnitude of the PT-odd effect is determined by the polarizability β_E^{PT} or by the amplitude $f_{\pm}(0)$ of elastic coherent scattering of a photon by an atom (molecule). If $\vec{\pi}_E \parallel \vec{\pi}_{\gamma}$ the amplitude $f_{\pm}(0)$ in the dipole approximation can be written as $f_{\pm} = \omega^2 \alpha_{ik} e_i^{(\pm)} e_k^{*(\pm)}/c^2 = \mp \omega^2 \beta_E^{PT}/c^2$. As a result, in order to obtain the amplitude f_{\pm} , the polarizability (7) for photon polarization states $\vec{e} = \vec{e}_{\pm}$ should be found. Using (7) we can present the polarizability β_E^{PT} as follows:

$$\beta_E^{PT} = \frac{\omega}{\hbar} \sum_n P(n) \sum_m \left\{ \frac{\langle \widetilde{g}_n | d_- | \widetilde{e}_m \rangle \langle \widetilde{e}_m | d_+ | \widetilde{g}_n \rangle - \langle \widetilde{g}_n | d_+ | \widetilde{e}_m \rangle \langle \widetilde{e}_m | d_- | \widetilde{g}_n \rangle}{\omega_{em,gn}^2 - \omega^2} \right\}$$
(11)

For further analysis the more detailed expressions for atom (molecule) wave functions are necessary. The constants of PT noninvariant interactions are very small. Therefore we can use the perturbation theory. Let $|\overline{g}\rangle$ and $|\overline{e}\rangle$ be the wave function of ground and excited states of atom (molecule) interacting with an electric field \vec{E} in the absence of PT - odd interactions. Switch on PT noninvariant interaction ($H_T \neq 0$). According to the perturbation theory the wave functions $|\tilde{g}\rangle$ and $|\tilde{e}\rangle$ can be written in this case as

$$|\widetilde{g}\rangle = |\overline{g}\rangle + \sum_{n} |n\rangle \frac{\langle n|H_{T}|\overline{g}\rangle}{E_{g} - E_{n}}$$

$$|\widetilde{e}\rangle = |\overline{e}\rangle + \sum_{n} |n\rangle \frac{\langle n|H_{T}|\overline{e}\rangle}{E_{e} - E_{n}}$$
(12)

where H_T is Hamiltonian of T noninvariant interactions.

It should be mentioned that both numerator and denominator of (11) contain H_T . Suppose H_T to be small one can represent the total polarizability β_E^{PT} as the sum of two terms

$$\beta_E^T = \beta_{mix}^T + \beta_{split}^T. \tag{13}$$

Here

$$\beta_{mix}^{PT} = \frac{\omega}{\hbar} \sum_{n} P(n) \sum_{m} \left\{ \frac{\langle \tilde{g}_n | d_- | \tilde{e}_m \rangle \langle \tilde{e}_m | d_+ | \tilde{g}_n \rangle - \langle \tilde{g}_n | d_+ | \tilde{e}_m \rangle \langle \tilde{e}_m | d_- | \tilde{g}_n \rangle}{\omega_{\overline{e}m,\overline{g}n}^2 - \omega^2} \right\}$$
(14)

where $\omega_{\overline{e}m,\overline{g}n}$ does not include the *PT* noninvariant shift of atomic levels, and

$$\beta_{split}^{PT} = \frac{\omega}{\hbar} \sum_{n} P(n) \sum_{m} \left\{ \frac{\langle \overline{g}_{n} | d_{-} | \overline{e}_{m} \rangle \langle \overline{e}_{m} | d_{+} | \overline{g}_{n} \rangle - \langle \overline{g}_{n} | d_{+} | \overline{e}_{m} \rangle \langle \overline{e}_{m} | d_{-} | \overline{g}_{n} \rangle}{\omega_{em,gn}^{2} - \omega^{2}} \right\}$$
(15)
$$\omega_{em,gn} = (E_{em}(\vec{E}) - E_{gn}(\vec{E}))/\hbar$$

It should be reminded that energy levels $E_{e,m}(\vec{E})$ and $E_{g,n}(\vec{E})$ contain shifts caused by interaction of electric dipole moment of atom with electric field \vec{E} and magnetic moment of atom with T-odd induced magnetic field $\vec{H}_{ind}(\vec{E})$.

Below we will consider nonpolarized atoms and small detuning of radiation frequency from atomic transition. Therefore (14) and (15) can be written as follows.

$$\beta_{mix}^{PT} = \frac{1}{2\hbar(2j_g+1)} \sum_{n,m} \left\{ \frac{\langle \tilde{g}_n | d_- | \tilde{e}_m \rangle \langle \tilde{e}_m | d_+ | \tilde{g}_n \rangle - \langle \tilde{g}_n | d_+ | \tilde{e}_m \rangle \langle \tilde{e}_m | d_- | \tilde{g}_n \rangle}{\omega_{\overline{e}m,\overline{g}n} - \omega} \right\}$$
(16)

$$\beta_{split}^{PT} = \frac{1}{2\hbar(2j_g+1)} \sum_{m,n} \left\{ \frac{\langle \overline{g}_n | d_- | \overline{e}_m \rangle \langle \overline{e}_m | d_+ | \overline{g}_n \rangle - \langle \overline{g}_n | d_+ | \overline{e}_m \rangle \langle \overline{e}_m | d_- | \overline{g}_n \rangle}{\omega_{em,gn} - \omega} \right\}$$
(17)

In this section we will study only rotation of polarization plane associated with β_{mix} . The rotation associated with β_{split} will be considered in the next section.

Due to Doppler shift resonance frequency of transition for a single atom depends on velocity of atom in a gas. In order to obtain expressions for absorption length and angle of polarization plane rotation we should average (14) over Maxwell distribution of atomic velocity. After standard calculations (see e. q. [14]) expressions take the form.

$$\phi = \frac{2\pi N\omega}{c} \langle \operatorname{Re}\beta_{mix}^{PT} \rangle_{v} = -\pi N l \frac{\omega}{\Delta_{D}\hbar c} g(u,v) [|\overline{A^{+}}|^{2} - |\overline{A^{-}}|^{2}]$$

$$L^{-1} = 2k \langle \operatorname{Im}n_{\pm} \rangle_{v} = 4\pi N \frac{\omega}{\Delta_{D}\hbar c} f(u,v) |\overline{A^{\pm}}|^{2}$$
(18)

where $\langle \rangle_v$ denotes the averaging over atomic velocity, $|\overline{A^+}|^2$ and $|\overline{A^-}|^2$ are the squares of transition amplitudes for left and right circularly polarized photons averaged over atomic polarization.

$$|\overline{A^{\pm}}|^2 = \frac{1}{(2j_g+1)} \sum_{m_g} \langle \tilde{g} | d^{\pm} | \tilde{e} \rangle \langle \tilde{e} | d^{\mp} | \tilde{g} \rangle$$
(19)

 $\Delta_D = \omega_0 \sqrt{2kT/Mc^2}$ is Doppler linewidth, f(u, v) and g(u, v) are equal to

$$\begin{cases} g(u,v) \\ f(u,v) \end{cases} = \begin{bmatrix} \mathrm{Im} \\ \mathrm{Re} \end{bmatrix} \sqrt{\pi} e^{-w^2} \left(1 - \Phi(-iw)\right)$$

$$(20)$$

here w = u + iv, $\Phi(z) = \frac{2}{\sqrt{\pi}} \int_0^z dt e^{-t^2}$, $u = (\omega - \omega_0)/\Delta_D$, $v = \Gamma/2\Delta_D$, Γ is the recoil linewidth,

Let us assume that electric field is small enough and we can use first order perturbation theory. Perturbed states $|\tilde{g}\rangle$ and $|\tilde{e}\rangle$ in this case have the form

$$|\tilde{g}\rangle = |g\rangle + \sum_{n} |n\rangle \frac{\langle n|H_{T}|g\rangle}{E_{g} - E_{n}} + \sum_{m} |m\rangle \frac{\langle m| - \vec{d}\vec{E}_{z}|g\rangle}{E_{g} - E_{m}}$$
$$|\tilde{e}\rangle = |e\rangle + \sum_{n} |n\rangle \frac{\langle n|H_{T}|e\rangle}{E_{e} - E_{n}} + \sum_{n} |m\rangle \frac{\langle m| - \vec{d}\vec{E}_{z}|e\rangle}{E_{e} - E_{m}}$$
(21)

Here H_T is Hamiltonian of PT noninvariant interactions, $|g\rangle$ and $|e\rangle$ are unperturbed ground and excited states of atom. Only terms proportional to products of H_T and $-\vec{dE}$ leads to phenomenon of interest.

Using (21) we can write.

$$|\overline{A^+}|^2 - |\overline{A^-}|^2 = \frac{2}{2j_g + 1} \operatorname{Re} \sum_{m_g} \left(\left\langle g | d_+^{PT} | e \right\rangle \left\langle e | d_-^{St} | g \right\rangle - \left\langle g | d_-^{PT} | e \right\rangle \left\langle e | d_+^{St} | g \right\rangle \right)$$
(22)

where d_i^{PT} is the admixture of E1 amplitude due to PT - odd interactions

$$\langle g|d_i^{PT}|e\rangle = \sum_m \frac{\langle g|H_T|m\rangle \langle m|d_i|e\rangle}{E_m - E_g} + \frac{\langle g|d_i|m\rangle \langle m|H_T|e\rangle}{E_m - E_e}$$
(23)

and $d_i^{St} = \Lambda_{ik} E_k$ is the Stark - induced amplitude.

$$\left\langle g|d_i^{St}|e\right\rangle = E_k\Lambda_{ik} = E_k\sum_n \frac{\left\langle g|d_k|n\right\rangle \left\langle n|d_i|e\right\rangle}{E_n - E_g} + \frac{\left\langle g|d_i|n\right\rangle \left\langle n|d_k|e\right\rangle}{E_n - E_e}$$
(24)

Here Λ_{ik} is the tensor of transition atomic polarizability.

According to [15] Stark - induced E1 amplitude has the form

$$\left\langle e|d_{\epsilon}^{St}|g\right\rangle = \sum_{q,q'} \Lambda_{q,q'} E_{-q} \epsilon_{-q'} = \sum_{K,Q} (-1)^Q \Lambda_Q^K (E \otimes \epsilon)_{-Q}^K, \tag{25}$$

where E_q is external electric field strength, $\epsilon_{q'}$ is the strength of electric field in a laser wave, Λ_Q^K and $(E \otimes \epsilon)_{-Q}^K$ are the components of irreducible spherical tensors.

Using Wigner - Ekhard theorem we can represent Λ_Q^K as follows.

$$\Lambda_Q^K = (-1)^{j_e - m_e} \begin{pmatrix} j_e & K & j_g \\ -m_e & Q & m_g \end{pmatrix} \Lambda^K$$

Reduced matrix elements Λ^{K} (K = 0, 1, 2) are proportional to scalar, vector and tensor transition polarizability respectively. Due to orthogonality of 3*j*-symbols only terms proportional to the vector part remains in (22) after summation over magnetic sub-levels.

In order to obtain the angle of polarization plane rotation and absorption length for atoms with the nuclear spin, we should take into account hyperfine structure of atomic levels. After necessary transformations (see e. q. [14]) equation (18) can be rewritten using reduced matrix elements of corresponding transitions.

$$\phi = -4\pi N_F l \frac{\omega}{\hbar c \Delta_D} g(u, v) \frac{1}{3(2F_g + 1)} K^2 Re(\langle g || d^{PT} || e \rangle \langle e || d^{St} || g \rangle)$$

$$L^{-1} = 4\pi N_F \frac{\omega}{\hbar c \Delta_D} f(u, v) \frac{1}{3(2F_g + 1)} K^2 |\langle g || A || e \rangle|^2$$
(26)

Here F_g , F_e are total angular moments of atom in ground and exited states, j_g , j_e are the total angular moments of electrons in these states, i is the nuclear spin,

$$N_F = N \frac{2F_g + 1}{(2i+1)(2j_g + 1)}$$

is the density of atoms with total moment F_q ,

$$K^{2} = (2F_{g} + 1)(2F_{e} + 1) \left\{ \begin{array}{ll} i & j_{g} & F_{g} \\ 1 & F_{e} & J_{e} \end{array} \right\}$$

and $\langle g || d^{St} || e \rangle$ is proportional to the Λ^1 . We assume that electric field is parallel to the direction of light propagation.

3 Rotation of polarization plane due to atomic EDM

Presence of EDM in ground or excited state of atom also cause rotation of polarization plane of light. We can derive the expression for the angle of polarization plane rotation performing the calculations similar to those described in section 2, but using β_{split} instead of β_{mix} . But in this case the calculations can be greatly simplified if we note that the mechanism of *PT* noninvariant rotation caused by atomic EDM is analogous to the Faraday rotation of polarization plane in a week magnetic field. Indeed according to [14, 16] application of weak magnetic field to the atomic gas affect the refractive index in two ways: through the changes in the energies of the magnetic sub-levels and through the mixing of hyperfine states.

If we consider only terms proportional to the magnetic field strength H and neglect the higher order terms then the levels shift becomes [16]

$$\Delta E_i = -H\langle i|\mu_z|i\rangle$$

The magnetic field H mixes states of the same F_z but different F, so the state $|j\rangle$ becomes

$$|\overline{j}\rangle = |j\rangle - \sum_{k \neq j} H_z \frac{|k\rangle \langle k|\mu_z|j\rangle}{E_k - E_j}$$

If atom has an EDM then applied electric field affect the refraction index in the same way (see (15)). It shifts the atomic levels

$$\Delta E_i = -E\langle i|d_z|i\rangle$$

and mixes the hyperfine states of atom with the same F_z but different F

$$|\overline{j}\rangle = |j\rangle - \sum_{k \neq j} E_z \frac{|k\rangle \langle k|d_z|j\rangle}{E_k - E_j}$$

Therefore after substitutions $E \to H$, $\mu_g \to d_g$, $\mu_e \to d_e$ where d_e , d_g are EDM of atom in ground and excited states, μ_i is the magnetic moment of state *i*, we can use in calculations the expression of [14, 16] for rotation of polarization plane of light in a weak magnetic field.

If we take into account only dipole transitions (it is possible for example for $6s_{1/2} \rightarrow 7s_{1/2}$ transition in cesium), then the angle of polarization plane rotation has the form

$$\phi = \frac{2\pi Nl}{(2i+1)(2j_g+1)} \frac{\omega}{\Delta_D \hbar c} \frac{E_z}{\hbar \Delta_D} |A|^2 \left(\frac{\partial g(u,v)}{\partial u} \delta_1 + 2g(u,v)\gamma_1\right).$$
(27)

where A is the reduced matrix element of transition amplitude. The expressions for parameters γ_1 and δ_1 are given below

$$\begin{split} \gamma_1 &= \frac{(2F_g+1)(2F_e+1)}{\sqrt{6}}(-1)^i \left\{ \begin{array}{l} i & j_g & F_g \\ 1 & F_e & j_e \end{array} \right\} [d_e(-1)^{j_e+F_g} \sqrt{\frac{(j_e+1)(2j_e+1)}{j_e}} \\ & \left(\frac{\Delta_D}{\Delta_{hf}(F_e,F_e-1)}(2F_e-1) \left\{ \begin{array}{l} i & j_g & F_g \\ 1 & F_e-1 & j_e \end{array} \right\} \left\{ \begin{array}{l} i & j_e & F_e \\ 1 & F_e-1 & j_e \end{array} \right\} \left\{ \begin{array}{l} F_g & 1 & F_e \\ 1 & F_e-1 & j_e \end{array} \right\} \left\{ \begin{array}{l} F_g & 1 & F_e \\ 1 & F_e-1 & 1 \end{array} \right\} \\ & + \frac{\Delta_D}{\Delta_{hf}(F_e,F_e+1)}(2F_e+3) \left\{ \begin{array}{l} i & j_g & F_g \\ 1 & F_e+1 & j_e \end{array} \right\} \left\{ \begin{array}{l} i & j_e & F_e \\ 1 & F_e+1 & j_e \end{array} \right\} \left\{ \begin{array}{l} I & F_e+1 & j_e \end{array} \right\} \left\{ \begin{array}{l} F_g & 1 & F_e \\ 1 & F_e+1 & j_e \end{array} \right\} \left\{ \begin{array}{l} F_g & 1 & F_e \\ 1 & F_e+1 & 1 \end{array} \right\}) \\ & - \left(j_e \leftrightarrow j_g, F_e \leftrightarrow F_g, d_e \leftrightarrow d_g, \right)] \end{split}$$

and

$$\delta_{1} = \frac{(2F_{g}+1)(2F_{e}+1)}{\sqrt{6}}(-1)^{i} \begin{cases} i & j_{g} & F_{g} \\ 1 & F_{e} & j_{e} \end{cases}^{2} \left[d_{e}(-1)^{j_{e}+F_{g}} \sqrt{\frac{(j_{e}+1)(2j_{e}+1)}{j_{e}}} \right] \\ (2F_{e}+1) \begin{cases} i & j_{e} & F_{e} \\ 1 & F_{e} & j_{e} \end{cases} \end{cases} \begin{cases} F_{g} & F_{e} & 1 \\ 1 & 1 & F_{e} \end{cases} + \left(j_{e} \leftrightarrow j_{g}, F_{e} \leftrightarrow F_{g}, d_{e} \leftrightarrow d_{g}, \right) \end{cases}$$

Here Δ_{hf} is the hyperfine level splitting, Δ_D is the Doppler linewidth, j_g and j_e are the angular moments of electrons in atom, i is the nuclear spin, F_g and F_e are total angular moments of atom in a ground and excited states respectively.

First term in (27) arise from level splitting in electric field. It describes effect similar to Macaluso - Corbino rotation of polarization plane in magnetic field. Second term appears due to mixing of hyperfine levels with the different total moment F but the same F_z in electric field. It describes the T noninvariant analog of polarization plane rotation due to Van-Vleck mechanism.

4 Estimates

Let us compare the magnitude of PT - odd polarization plane rotation for different transitions. If spin of atomic nucleus is zero than the angle of rotation of polarization plane per absorption length due to PT - odd level mixing according to (18) has the form

$$\phi(L_{abs}) = \frac{g(u,v)}{4f(u,v)} \frac{|\overline{A^+}|^2 - |\overline{A^-}|^2}{|\overline{A^\pm}|^2}$$
(28)

If detuning $\Delta \sim \Delta_D$ then $g \sim f \sim 1$.

As in the case of P odd but T even interaction [14], we will discuss the rotation of polarization plane of light near magnetic dipole transitions. We can estimate the angle of polarization plane rotation per absorption length as follows.

$$\phi(L_{abs}) \sim \frac{\left(\langle d \rangle^2 \langle H_T \rangle / \Delta E \right) \left(\langle d \rangle E_z / \Delta E \right)}{\langle \mu \rangle^2} \sim \frac{\langle H_T \rangle}{\alpha^2 \Delta E} \frac{\langle d \rangle E_z}{\Delta E}$$
(29)

Here $\langle d \rangle \sim ea_0$, $\langle \mu \rangle \sim \alpha \langle d \rangle$ are the values of E1 and M1 transition amplitudes, $\langle H_T \rangle$ is the matrix element of PT noninvariant interaction, $\Delta E \sim Ry$ is the typical space between energies of opposite parity states, a_0 is the Bohr radius, $\alpha = 1/137$ is the fine structure constant.

Near strongly forbidden magnetic dipole transitions (e. q. $6s_{1/2} \rightarrow 7s_{1/2}$ in Cs) Stark-induced E1 amplitude is several orders of magnitude greater than M1 one. Absorption of light here depends primarily on Stark-induced amplitude. We can write for angle of polarization plane rotation per absorption length the following expression

$$\phi(L_{abs}) \sim \frac{(\langle d \rangle^2 \langle H_T \rangle / \Delta E)(\langle d \rangle E_z / \Delta E)}{(\langle d \rangle^2 E_z / \Delta E)^2} \sim \frac{\langle H_T \rangle}{\Delta E} \frac{\Delta E}{\langle d \rangle E_z}$$
(30)

Usually $\langle d \rangle E_z / \Delta E$ is less than $10^{-3} \sim 10^{-4}$. Therefore angle of rotation per absorption length is higher for strongly forbidden M1 transition.

Angle of rotation per absorption length near allowed E1 transition is essentially lower than (29) and (30).

$$\phi(L_{abs}) \sim \frac{\left(\left\langle d \right\rangle^2 \left\langle H_T \right\rangle / \Delta E\right) \left(\left\langle d \right\rangle E_z / \Delta E\right)}{\left\langle d \right\rangle^2} \sim \frac{\left\langle H_T \right\rangle}{\Delta E} \frac{\left\langle d \right\rangle E_z}{\Delta E}$$
(31)

It should be noted here that the angle of rotation of polarization plane per unit length has the same order of magnitude in all three cases. The difference in angle of rotation per absorption length is caused by different absorption of light near the corresponding transition.

It is interesting to compare these estimates with the rotation of polarization plane caused by nonzero EDM of atom. In the absence of hyperfine structure the angle of rotation per absorption length caused by splitting of magnetic sub-levels in electric field can be estimated using (18) and (27) as follows

$$\phi(L_{abs}) = \frac{1}{2(2j_g + 1)} \frac{E_z \delta}{f \hbar \Delta_D} \frac{\partial g}{\partial u} \sim \frac{d_{at} E_z}{\hbar \Delta_D} \sim \frac{\langle d \rangle E}{\hbar \Delta_D} \frac{\langle H_T \rangle}{\Delta E}, \tag{32}$$

where $d_{at} \sim \langle d \rangle \langle H_T \rangle / \Delta E$ is the EDM of atom, $\Delta_D \sim (10^{-5} \sim 10^{-6}) \Delta E$ is the Doppler linewidth. The value $\phi(L_{abs})$ here does not depend on magnitude of transition amplitude A and has the same order of magnitude for all kinds of transition considered above.

5 P and T - odd interactions in atom

Several mechanisms can cause the PT noninvariant interactions in atom. According to [14] they include PT - odd weak interactions of electron and nucleon, the interaction of electric dipole moment of electron with the electric field inside the atom, interaction of electrons with electric dipole and magnetic quadrupole moments of nucleus and PT odd electron - electron interaction.

Below we will consider two kinds of PT - odd interactions that according to [14] give the dominant contribution in our case. This is PT - odd electron nucleon interaction and interaction of electron EDM with the electric field inside atom.

According to [14, 17, 18] Hamiltonian of T - violating interaction between electron and hadron has the form

$$H_T = C_s \frac{G}{\sqrt{2}} (\bar{e}i\gamma_5 e)(\bar{n}n) + C_t \frac{G}{\sqrt{2}} (\bar{e}i\gamma_5 \sigma_{\mu\nu} e)(\bar{n}\sigma^{\mu\nu}n)$$
(33)

where $G = 1.055 \cdot 10^{-5} m_p^{-2}$ is Fermi constant, e and n are electron and hadron field operators respectively, C_s and C_t are dimensionless constants that characterize the strengths of T - violating interactions relative to usual T - conserving weak interaction. The first term in (33) describes scalar hadronic current coupling to pseudoscalar electronic current, and the second one describes tensor hadron current coupling to the pseudotensor electron current.

Matrix elements for this T - odd Hamiltonians according to [14] is equal to

$$\left\langle s_{1/2} || H_{\text{Todd}} || p_{1/2} \right\rangle = \frac{G m_e^2 \alpha^2 Z^2 R}{2\sqrt{2}\pi} \frac{\mathbf{R} \mathbf{y}}{\sqrt{\nu_s \nu_p^3}} 2\gamma C_s A \tag{34}$$

where m_e is the electron mass, Ry = 13.6 eV is Rydberg energy constant, ν_i is the effective principal quantum number of state *i*, *A* is the atomic number, *R* is the relativistic factor (R = 2.8 for cesium), $\gamma = \sqrt{(j+1/2)^2 - Z^2 \alpha^2}$ and *j* is the total angular moment of atom. We neglect here tensor part of interaction for simplicity.

Hamiltonian of interaction of electron EDM and electric field inside atom that mix opposite parity atomic states has the form [14]

$$H_d = \sum_k (\gamma_{0k} - 1) \vec{\Sigma}_k \vec{E}_k \tag{35}$$

where E_k is the electric field strength acting upon electron k. When summation in (35) is performed over one valence electron only and electric field strength near the nucleus approximately equals to $\vec{E} = Z\alpha \vec{r}/r^3$, matrix element of operator H_d can be written as follows [14]

$$\langle j, l = j + 1/2 || H_d || j, l' = j - 1/2 \rangle = -\frac{4(Z\alpha)^3}{\gamma(4\gamma^2 - 1)(\nu_l \nu_{l'})^{3/2} a_0^2}$$
(36)

where l and l' are the orbital angular moments, a_0 is the Bohr radius.

6 Estimates for $6s_{1/2} \rightarrow 7s_{1/2}$ transition in cesium

Let us estimate the PT - odd rotation of polarization plane for highly forbidden M1 transition $6s_{1/2} \rightarrow 7s_{1/2}$ in cesium. The schemes of atomic levels of cesium is shown in Fig. 2.

6.1 Rotation of polarization plane of light due electron nucleon interactions

In order to obtain the angle of polarization plane rotation for the highly forbidden M1 transition $6s_{1/2} \rightarrow 7s_{1/2}$ due to P and T noninvariant interactions between electron and nucleus we can use the well known results for P - odd but T - even weak interactions. Matrix elements for T - even Hamiltonian H_w has the form [14].

$$\langle s_{1/2} || H_w || p_{1/2} \rangle = i \frac{G m_e^2 \alpha^2 Z^2 R}{2\sqrt{2\pi}} \frac{\mathbf{R} \mathbf{y}}{\sqrt{\nu_s \nu_p}^3} Q_w$$
(37)

 $Q_w = -N + Z(1 - 4\sin\theta_W)$ is the week nuclear charge, N and Z are the number of neutrons and protons in nucleus respectively, $\sin^2\theta_W$ is the Weinberg angle.



Figure 2: Scheme of atomic levels for cesium. Energy of atomic levels is given in cm^{-1} .

Comparing this expressions with (34) and using the wave functions of $6s_{1/2}$ and $7s_{1/2}$ cesium states perturbed by P - odd but T - even interactions [14]

$$|\widetilde{6s_{1/2}}\rangle = |6s_{1/2}\rangle + i10^{-11} \left(-\frac{Q_w}{N}\right) (1.17|6p_{1/2}\rangle + 0.34|7p_{1/2}\rangle) |\widetilde{7s_{1/2}}\rangle = |7s_{1/2}\rangle + i10^{-11} \left(-\frac{Q_w}{N}\right) (0.87|6p_{1/2}\rangle - 1.33|7p_{1/2}\rangle)$$
(38)

one can obtain atomic wave functions perturbed by P and T noninvariant interactions

$$\widetilde{|6s_{1/2}\rangle} = |6s_{1/2}\rangle + 10^{-11} \left(2\gamma \frac{A}{N}C_s\right) (1.17|6p_{1/2}\rangle + 0.34|7p_{1/2}\rangle)$$

$$\widetilde{|7s_{1/2}\rangle} = |7s_{1/2}\rangle + 10^{-11} \left(2\gamma \frac{A}{N}C_s\right) (0.87|6p_{1/2}\rangle - 1.33|7p_{1/2}\rangle).$$
(39)

After simple calculations reduced matrix element of PT odd E1 transition can be written as follows

$$\langle 6s_{1/2} || d^{PT} || 7s_{1/2} \rangle = -\sqrt{3} 10^{-11} \left(2\gamma \frac{A}{N} C_s \right) \left(0.87 \left\langle 6s_{1/2} || d || 6p_{1/2} \right\rangle \right. \\ \left. -1.17 \left\langle 6p_{1/2} || d || 7s_{1/2} \right\rangle - 0.34 \left\langle 7p_{1/2} || d || 7s_{1/2} \right\rangle \right)$$

Using values of radial integrals [14] $\rho(6s_{1/2}, 6p_{1/2}) = -5.535$, $\rho(7s_{1/2}, 6p_{1/2}) = 5.45$, $\rho(7s_{1/2}, 7p_{1/2}) = -12.30$ we obtain

$$\left< 6s_{1/2} || d^{PT} || 7s_{1/2} \right> = 1.27 \cdot 10^{-10} |e| a_0 C_s \tag{40}$$

The matrix element of Stark - induced $6s_{1/2} \rightarrow 7s_{1/2}$ transition in cesium has traditionally been written in form [19]

$$\langle 6s_{1/2}, m' | d_i^{St} | 7s_{1/2}, m \rangle = \alpha E_i \delta_{mm'} + i\beta \epsilon_{ijk} E_j \langle m' | \sigma_k | m \rangle$$

where *m* and *m'* are magnetic quantum numbers of ground and excited states of cesium, E_i is the electric field strength, σ_k is the Pauli matrix, α and β are the scalar and vector transition polarizability (aee also (25)). The value of $\langle 6s_{1/2} || d^{St} || 7s_{1/2} \rangle$ introduced in (26) can be expressed for cesium via the vector transition polarizability β as follows. $\langle 6s_{1/2} || d^{St} || 7s_{1/2} \rangle = \sqrt{6\beta E}$. Value of β is well known from theoretical calculations [20] as well as from experiment [21]. According to [20] it is equal to $\beta = 27.0a_0^3$. Therefore

$$\langle 7s_{1/2} || d^{St} || 6s_{1/2} \rangle = 1.28 \cdot 10^{-8} |e| a_0 E(V/cm)$$
(41)

When temperature is T = 750K pressure of Cs vapor is p = 10 kPa [22], concentration of atoms is $N = 10^{18} cm^{-3}$ and Doppler linewidth is $\Delta_D/\omega_0 = 10^{-6}$. For transition between hyperfine levels $F_g = 4 \rightarrow F_e = 4$, where coefficient K^2 is maximal ($K^2 = 15/8$) when detuning $\Delta \sim \Delta_D$, $v = \Gamma/2\Delta_D \simeq 0.1$ and $f \approx 1$, $g \approx 0.7$, absorption length in longitudinal electric field E is equal to $L_{abs} = 7 \cdot 10^{10}/E^2 (V/cm)$ (length is measured in centimeters) and angle of PT noninvariant rotation of polarization plane is

$$\phi = 1.0 \cdot 10^{-13} C_s l E.$$

If $E = 10^4 \text{V/cm}$ then $L_{abs} = 7\text{m}$. The best signal to noise ratio is achieved when $l = 2L_{abs}$ [14]. In this case $|\phi| = 1.3 \cdot 10^{-6} C_s$. The lowest limit to the parameters of electron-nucleon interaction $C_s < 4 \cdot 10^{-7}$ was set in [4]. Corresponding limit to the angle of rotation of polarization plane is $|\phi| < 0.5 \cdot 10^{-12}$ rad.

6.1.1 Cesium EDM

Using wave functions (39) we can obtain EDM in $6s_{1/2}$ and $7s_{1/2}$ states of Cs

$$d_{6s_{1/2}} = -1.35 \cdot 10^{-10} C_s |e| a_0$$

$$d_{7s_{1/2}} = -4.39 \cdot 10^{-10} C_s |e| a_0.$$

Under the same condition of experiment as before expression (27) yields the angle of polarization plane rotation due to level splitting in electric field

$$|\phi_1| = 1.4 \cdot 10^{-24} l C_s E_z^3 (V/cm) < 8 \cdot 10^{-16}$$

and the angle of rotation due to hyperfine levels mixing

$$|\phi_2| = 2.1 \cdot 10^{-24} l C_s E_z^3 (\text{V/cm}) < 1.2 \cdot 10^{-15}.$$

(We assume here that for detuning $\Delta \sim \Delta_D$ functions $g(u, v) \simeq 0.7$, $\partial g(u, v)/\partial u \simeq 1.1$). These angles are two orders of magnitude lower than one arising from interference of Stark-induced and PT noninvariant transition amplitudes.

6.2 Rotation of polarization plane of light due to electron EDM

If the PT noninvariant interaction in atom is caused by interaction of electron EDM with the electric field of nucleus then wave functions of 6s and 7s states of cesium take the form

$$|\tilde{n}s_{1/2}\rangle = |ns_{1/2}\rangle + \sum_{m} |mp_{1/2}\rangle + \frac{\langle mp_{1/2}|H_d|ns_{1/2}\rangle}{E_g - E_n}$$
(42)

where matrix element of operator H_d is given in (36).

We should take into account admixture of states $6p_{1/2}$ and $7p_{1/2}$ to $6s_{1/2}$ and $7s_{1/2}$ states of cesium. After necessary calculations perturbed wave functions of atom can be written as follows

$$\begin{aligned} |\tilde{6}s_{1/2} \rangle &= |6s_{1/2} \rangle - (35|6p_{1/2} \rangle + 10.5|7p_{1/2} \rangle) d_e / (ea_0) \\ |\tilde{7}s_{1/2} \rangle &= |7s_{1/2} \rangle + (27.7|6p_{1/2} \rangle - 36.2|7p_{1/2} \rangle) d_e / (ea_0) \end{aligned}$$
(43)

Using Eq. (43) we can obtain reduced matrix element of electric dipole transitions between $\tilde{6}s_{1/2}$ and $\tilde{7}s_{1/2}$ states

$$\langle 6s_{1/2} || d^{PT} || 7s_{1/2} \rangle = \sqrt{2/3} d_e (35\rho(6p_{1/2}, 7s_{1/2}) + 10.5\rho(7p_{1/2}, 7s_{1/2}) + 27.7\rho(6p_{1/2}, 6s_{1/2})) = -72 d_e$$

$$(44)$$

and electric dipole moment of cesium in ground state $d_{6s_{1/2}}$ and excited state $d_{7s_{1/2}}$

$$\begin{aligned} d_{6s_{1/2}} &= 131d_e \\ d_{7s_{1/2}} &= 400d_e, \end{aligned}$$
 (45)

where d_e is the electron EDM.

As we mention before two effects induce T - noninvariant rotation of polarization plane in electric field. First of them is the interference of PT - odd and Stark induced transition amplitudes and second is the interaction of atomic EDM with electric field. After substitution of (44) to equation (26) one can obtain the angle of rotation due to interference of amplitudes $|\phi| < 0.6 \cdot 10^{-12}$ under the same experimental conditions as before.

The rotation due to atomic EDM is a sum of two contributions. Using first term in Eq. (27) and Eq. (45) one can obtain for the rotation induced by splitting of magnetic sub-levels in electric field the angle $|\phi_1| < 1.3 \cdot 10^{-15}$. The mixing of hyperfine components (second term in equation (27)) gives the contribution $|\phi_2| < 2 \cdot 10^{-15}$.

For estimates we use experimental limit on electron EDM from [4] $|d_e| < 4 \cdot 10^{-27} |e|$ cm. Limits on angles quoted above are close to those obtained for PT - odd electron nucleon interactions.

7 Discussion of experiment

The simplest experimental scheme to observe the pseudo - Faraday rotation of polarization plane of light in electric field consist of cell with atomic gas placed in the electric field and sensitive polarimeter. In the case of large absorption length one can place this cell in resonator or delay line optical cavity to reduce the size of experimental setup (see e. q. [23]).

Several schemes was proposed to increase the sensitivity of measurements. One of them is based on the nonlinear magneto - optic effect (NMOE) [8, 9]. Due to ultra - narrow width of dispersion like shaped Faraday rotation caused by NMOE the sensitivity of this experiments to the PT noninvariant rotation of polarization plane (the change of rotation angle with the change of applied electric field) is several orders of magnitude higher than in the conventional scheme. The authors of [9] hopes to achieve the sensitivity to the cesium EDM $d_{Cs} < 10^{-26} |e|$ cm. The corresponding limit to the electron EDM is $d_{Cs} < 10^{-28} |e|$ cm.

Even higher sensitivity can probably provide the method of measurements of polarization plane rotation proposed in [11]. This method is based on observation of evolution of polarization of light in a cell with atomic vapor and amplifying media with inverse population of atomic levels placed in the resonator. According to [11] the compensation of absorption of light in the cell allows to increase the observed angle of polarization plane rotation.

8 Conclusion

In the present article we have considered phenomenon of rotation of polarization plane of light in gas placed in electric field. Calculations of angle of polarization plane rotation were performed for $6S_{1/2} \rightarrow 7S_{1/2}$ transition in atomic cesium. Two mechanisms that cause this effect are considered. They are interference of Stark-induced and PT noninvariant transition amplitude and atomic EDM. Both of them can be induced by PT noninvariant interaction between electrons and atomic nucleus and by interaction of electron EDM with electric field inside atom.

For the highly forbidden M1 transition $6S_{1/2} \rightarrow 7S_{1/2}$ in cesium we can expect the angle of polarization plane rotation per absorption length due to PT - odd atomic polarizability $\beta_{mix}^{PT} \phi < 10^{-12}$. This angle is three orders of magnitude greater than one caused by atomic EDM for this transition.

Angle of polarization plane rotation can be significantly greater for other atoms, for example rare-earth elements, where additional amplification goes from close levels of opposite parity. The interesting example is the transition $6s^2 {}^1S_0 \rightarrow 6s5d {}^3D_1$, ($\lambda = 408$ nm) in Yb [24], where the value of PT odd angle can be two orders of magnitude higher then for cesium due to larger PT noninvariant amplitude.

Therefore we can hope that experimental measurement of described phenomenon can achieve sensitivity in measurements of parameters of PT noninvariant interactions between electron and nucleus and electron EDM, comparable with the current atomic EDM experiments.

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