Birefringence (rotation of polarization plane and spin dichroism) of deuterons in carbon target

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Birefringence phenomenon for deuteron with energy up 20 MeV in carbon target is considered. The estimation for spin dichroism and for angle of rotation of polarization plane of deuterons is presented. It is shown that magnitude of the phenomenon strongly depends on behavior of the deuteron wave functions on small distance between nucleon in deuteron.

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I. INTRODUCTION

It was shown in [1], [2] that birefringence effect arise for deuterons (in general for all particles with spin $S \geqslant 1$) passing through unpolarized isotropic matter. According to [1], [2] this phenomenon is caused by difference of spindependent forward scattering amplitude for deuterons with spin projection $m = 0$ and $m = \pm 1$ on coordinate axis parallel to deuteron wave vector (m is magnetic quantum number). As the result there is appears possibility to study real and imaginary part of spindependent forward scattering amplitude at experiment. The first experimental study of deuteron spin dichroism in carbon target was carried out at the electrostatic HVEC tandem Van-de-Graaff accelerator with deuterons of up to 20 MeV (Institut für Kernphysik of Universität zu Köln) [3], [4]. According to experimental results appearance of tensor polarization in the transmitted deuterons beam was observed [3], [4]. As a result spin dichroism of deuteron beam passing through unpolarized carbon target was discovered. Later in 2007 spin dichroism was measured for 5.5 GeV/c deuterons in carbon target on Nuclotron in Dubna [5].

In this paper the difference of spindependet forward scattering amplitude for deuteron with energy up to 20 MeV on carbon target on the base of eikonal approximation is considered. The estimation for angle of rotation of polarization plane and for deuteron spin dichroism is done. It is shown that magnitude of the phenomenon strongly depends on behavior of the deuteron wave functions for small distance between nucleon in deuteron.

II. EIKONAL APPROXIMATION FOR DEUTERON SPINDEPENDET FORWARD SCATTERING AMPLITUDE ON CARBON TARGET

Let us discuss a possible magnitude of the deuteron birefringence effect. According to [1], [2] birefringence effect depends on amplitudes of zero-angle elastic coherent scattering of a deuteron by a nucleus $f(m = \pm 1)$ and $f(m = 0)$. Let us consider theory of birefringence effect [1], [2] briefly.

The Hamiltonian H can be written as

$$H = H_D(\vec{r_p}, \vec{r_n}) + H_D(\vec{r}) + H_N(\{\xi_i\}) + V_{DN}(\vec{r_p}, \vec{r_n}, \{\xi_i\}) \tag{1}$$

where H_D is the deuteron Hamiltonian; H_N the nuclear Hamiltonian; V_{DN} stands for the energy of deuteron-nucleus nuclear and Coulomb interaction; $r_p(r_n)$ is the coordinate of the proton (neutron) entering into the deuteron $\{\xi_i\}$ is a set of coordinates of the nucleons. Introducing the deuteron center-of-mass coordinate $\vec{R}$ and the relative distance between the proton and neutron in the deuteron $\vec{r} = \vec{r_p} - \vec{r_n}$, we write (1) as

$$H = -\frac{\hbar^2}{2m_D}\Delta(\vec{R}) + H_D(\vec{r}) + H_N(\{\xi_i\}) + V_{DN}^N(\vec{R}, \vec{r}, \{\xi_i\}) + V_{DN}^C(\vec{R}, \vec{r}, \{\xi_i\}) \tag{2}$$

Let us consider scattering of deuterons with energy above deuterons binding energy ε_d. For deuterons with energy 10 MeV time of nuclear deuteron-carbon interaction is $\tau^N \simeq 5 \cdot 10^{-22}\, s$. whereas the characteristic period of oscillation of nucleus in the deuteron is $\tau \simeq 2\pi\hbar/\varepsilon_d \simeq 2 \cdot 10^{-21}\, s$. As a result we can apply the impulse approximation. In this

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approximation we can neglect of binding energy of nucleons in deuteron i. e. neglect of $H_D(\vec{r})$ in (2). As a result,

$$H = -\frac{\hbar^2}{2m_D}\Delta(\vec{R}) + H_N(\{\xi_i\}) + V_{DN}^N(\vec{R},\vec{r},\{\xi_i\}) + V_{DN}^C(\vec{R},\vec{r},\{\xi_i\}) \tag{3}$$

As is seen, in the impulse approximation the problem of determining the scattering amplitude reduces to the problem of scattering by a nucleus of a structureless particle of the deuteron mass. In this case the coordinate r is a parameter. Therefore, the relations obtained for the cross section and the forward scattering amplitude should be averaged over the parameter mentioned. For fast deuterons the scattering amplitude can be found in the eikonal approximation [6] , [7]. The amplitude of coherent zero-angle scattering can be written in this approximation as follows

$$f(0) = \frac{k}{2\pi\ i}\int\left(e^{i\chi_D(\vec{b},\vec{r})} - 1\right) d^2b\,|\varphi(\vec{r})|^2\, d^3r \tag{4}$$

where k is the deuteron wavenumber, $\vec{b}$ is the impact parameter, $\varphi(\vec{r})$ is deuterons wave function. The phase shift due to the deuteron scattering by a carbon is

$$\chi_D = \chi_{pN} + \chi_{nN} + \chi_{pN}^C = -\frac{1}{\hbar v}\int_{-\infty}^{+\infty} V_{pN}\left(\vec{b},z',\vec{r}_\perp\right)dz' - \frac{1}{\hbar v}\int_{-\infty}^{+\infty} V_{nN}\left(\vec{b},z',\vec{r}_\perp\right)dz' - \frac{1}{\hbar v}\int_{-\infty}^{+\infty} V_{pN}^C\left(\vec{b},z',\vec{r}_\perp\right)dz', \tag{5}$$

where $\vec{r}_\perp$ is the $\vec{r}$ component perpendicular to the momentum of incident deuteron, v is the deuteron speed.

For the polarized deuteron under consideration the probability $|\varphi(\vec{r})|^2$ is differentiate for different spin states of deuteron. Thus for states with magnetic quantum number $m = \pm 1$, the probability is $|\varphi_{\pm1}(\vec{r})|^2$, whereas for $m = 0$, it is $|\varphi_0(\vec{r})|^2$. Owing to the additivity of phase shifts, equation (4) can be rewritten as

$$\begin{aligned} f(0) &= \frac{k}{\pi}\int\left\{t_{pN}\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right) + t_{nN}\left(\vec{b}-\frac{\vec{r}_\perp}{2}\right) + t_{pN}^C\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right) + 2it_{pN}\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right)t_{pN}^C\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right)\right\}|\varphi(\vec{r})|^2 d^2bd^3r \\ &+ \frac{k}{\pi}\int\left\{2it_{nN}\left(\vec{b}-\frac{\vec{r}_\perp}{2}\right)t_{pN}^C\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right) + 2it_{pN}\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right)t_{nN}\left(\vec{b}-\frac{\vec{r}_\perp}{2}\right)\right\}|\varphi(\vec{r})|^2 d^2bd^3r \\ &- \frac{k}{\pi}\int 4t_{pN}\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right)t_{nN}\left(\vec{b}-\frac{\vec{r}_\perp}{2}\right)t_{pN}^C\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right)|\varphi(\vec{r})|^2 d^2bd^3r, \end{aligned} \tag{6}$$

where

$$t_{nN(pN)}^{(C)} = \frac{e^{i\chi_{nN(pN)}^{(C)}} - 1}{2i}.$$

From (6) it follows

$$\begin{aligned} f(0) &= F_{pN}(0) + F_{nN}(0) + F_{pN}^C(0) + 2iF_{ppN}^C + \frac{2ik}{\pi}\int t_{nN}\left(\vec{b}-\frac{\vec{r}_\perp}{2}\right)t_{pN}^C\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right)|\varphi(\vec{r}_\perp,z)|^2 d^2bd^2r_\perp dz \\ &+ \frac{2ik}{\pi}\int t_{pN}\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right)t_{nN}\left(\vec{b}-\frac{\vec{r}_\perp}{2}\right)|\varphi(\vec{r}_\perp,z)|^2 d^2bd^2r_\perp dz \\ &- \frac{4k}{\pi}\int t_{pN}\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right)t_{nN}\left(\vec{b}-\frac{\vec{r}_\perp}{2}\right)t_{pN}^C\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right)|\varphi(\vec{r}_\perp,z)|^2 d^2bd^2r_\perp dz, \end{aligned} \tag{7}$$

where

$$F_{nN(pN)}^{(C)}(0) = \frac{k}{\pi}\int t_{nN(pN)}^{(C)}(\vec{\xi})d^2\xi = \frac{m_D}{m_{n(p)}}\ f_{n(p)}^{(C)}(0), \quad F_{ppN}^C(0) = \frac{k}{\pi}\int t_{pN}\ t_{pN}^C(\vec{\xi})d^2\xi$$

and $f_{n(n)}^{(C)}(0)$ is the nuclear and the Coulomb amplitude of the proton (neutron)-carbon zero-angle elastic coherent scattering. The expression (7) can be rewritten as

$$\begin{aligned} f(0) &= F_{pN}(0) + F_{nN}(0) + F_{pN}^C(0) + 2iF_{ppN}^C + \frac{2ik}{\pi}\int t_{nN}(\vec{\eta})\,t_{pN}^C\left(\vec{\xi}\right)\left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 d^2\xi d^2\eta dz \\ &+ \frac{2ik}{\pi}\int t_{pN}\left(\vec{\xi}\right)t_{nN}(\vec{\eta})\left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 d^2\xi d^2\eta dz - \frac{4k}{\pi}\int t_{pN}\left(\vec{\xi}\right)t_{nN}(\vec{\eta})\,t_{pN}^C\left(\vec{\xi}\right)\left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 d^2\xi d^2\eta dz. \end{aligned} \tag{8}$$

Then from (9)

$$\begin{aligned}
Ref(0) &= ReF_{pN}(0)+ReF_{nN}(0)+ReF^{C}_{pN}(0)-2ImF^{C}_{ppN}-\frac{2k}{\pi}Im\int t_{nN}\left(\vec{\eta}\right)t^{C}_{pN}\left(\vec{\xi}\right)\left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}d^{2}\xi d^{2}\eta dz\\
&- \frac{2k}{\pi}Im\int t_{pN}\left(\vec{\xi}\right)t_{nN}\left(\vec{\eta}\right)\left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}d^{2}\xi d^{2}\eta dz-\frac{4k}{\pi}Re\int t_{pN}\left(\vec{\xi}\right)t_{nN}\left(\vec{\eta}\right)t^{C}_{pN}\left(\vec{\xi}\right)\left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}d^{2}\xi d^{2}\eta dz,\\
Imf(0) &= ImF_{pN}(0)+ImF_{nN}(0)+ImF^{C}_{pN}(0)+2ReF^{C}_{ppN}+\frac{2k}{\pi}Re\int t_{nN}\left(\vec{\eta}\right)t^{C}_{pN}\left(\vec{\xi}\right)\left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}d^{2}\xi d^{2}\eta dz \qquad (9)\\
&+ \frac{2k}{\pi}Re\int t_{pN}\left(\vec{\xi}\right)t_{nN}\left(\vec{\eta}\right)\left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}d^{2}\xi d^{2}\eta dz-\frac{4k}{\pi}Im\int t_{pN}\left(\vec{\xi}\right)t_{nN}\left(\vec{\eta}\right)t^{C}_{pN}\left(\vec{\xi}\right)\left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}d^{2}\xi d^{2}\eta dz.
\end{aligned}$$

In accordance with [1]-[4] rotation of polarization plane is determined by the difference of the amplitudes $Ref(m=\pm1)$ and $Ref(m=0)$ and spin dichroism is determined by the difference of the amplitudes $Imf(m=\pm1)$ and $Imf(m=0)$. From (10) it follows that

$$\begin{aligned}
Re(d_1) &= -\frac{2k}{\pi}Im\int t_{nN}\left(\vec{\eta}\right)t^{C}_{pN}\left(\vec{\xi}\right)\left[\left|\varphi_{\pm1}\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}-\left|\varphi_{0}\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}\right]d^{2}\xi d^{2}\eta dz\\
&- \frac{2k}{\pi}Im\int t_{pN}\left(\vec{\xi}\right)t_{nN}\left(\vec{\eta}\right)\left[\left|\varphi_{\pm1}\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}-\left|\varphi_{0}\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}\right]d^{2}\xi d^{2}\eta dz\\
&- \frac{4k}{\pi}Re\int t_{pN}\left(\vec{\xi}\right)t_{nN}\left(\vec{\eta}\right)t^{C}_{pN}\left(\vec{\xi}\right)\left[\left|\varphi_{\pm1}\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}-\left|\varphi_{0}\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}\right]d^{2}\xi d^{2}\eta dz,\\
Im(d_1) &= \frac{2k}{\pi}Re\int t_{nN}\left(\vec{\eta}\right)t^{C}_{pN}\left(\vec{\xi}\right)\left[\left|\varphi_{\pm1}\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}-\left|\varphi_{0}\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}\right]d^{2}\xi d^{2}\eta dz\\
&+ \frac{2k}{\pi}Re\int t_{pN}\left(\vec{\xi}\right)t_{nN}\left(\vec{\eta}\right)\left[\left|\varphi_{\pm1}\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}-\left|\varphi_{0}\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}\right]d^{2}\xi d^{2}\eta dz\\
&- \frac{4k}{\pi}Im\int t_{pN}\left(\vec{\xi}\right)t_{nN}\left(\vec{\eta}\right)t^{C}_{pN}\left(\vec{\xi}\right)\left[\left|\varphi_{\pm1}\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}-\left|\varphi_{0}\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}\right]d^{2}\xi d^{2}\eta dz, \qquad (10)
\end{aligned}$$

where d_1 is the difference of spindependent forward scattering amplitudes.

At scattering of deuterons on light nucleus the characteristic radius of the deuteron is large comparing with the radius of nucleus. For this reason for estimation of effects, when integrating, we can suppose that the functions t_{pN} and t_{nN} act on φ as δ-function. Then

$$\begin{aligned}
Re(d_1) &= -2Im\left\{F_{nN}(0)\int t^{C}_{pN}\left(\vec{\xi}\right)\left[\left|\varphi_{\pm1}\left(\vec{\xi},z\right)\right|^{2}-\left|\varphi_{0}\left(\vec{\xi},z\right)\right|^{2}\right]d^{2}\xi dz\right\}\\
&- \frac{2\pi}{k}Im\left\{F_{pN}(0)F_{nN}(0)\int\left[\left|\varphi_{\pm1}\left(0,z\right)\right|^{2}-\left|\varphi_{0}\left(0,z\right)\right|^{2}\right]dz\right\}\\
&- \frac{4\pi}{k}Re\left\{F^{C}_{ppN}(0)F_{nN}(0)\int\left[\left|\varphi_{\pm1}\left(0,z\right)\right|^{2}-\left|\varphi_{0}\left(0,z\right)\right|^{2}\right]dz\right\},\\
Im(d_1) &= 2Re\left\{F_{nN}(0)\int t^{C}_{pN}\left(\vec{\xi}\right)\left[\left|\varphi_{\pm1}\left(\vec{\xi},z\right)\right|^{2}-\left|\varphi_{0}\left(\vec{\xi},z\right)\right|^{2}\right]d^{2}\xi dz\right\}\\
&+ \frac{2\pi}{k}Re\left\{F_{pN}(0)F_{nN}(0)\int\left[\left|\varphi_{\pm1}\left(0,z\right)\right|^{2}-\left|\varphi_{0}\left(0,z\right)\right|^{2}\right]dz\right\}\\
&- \frac{4\pi}{k}Im\left\{F^{C}_{ppN}(0)F_{nN}(0)\int\left[\left|\varphi_{\pm1}\left(0,z\right)\right|^{2}-\left|\varphi_{0}\left(0,z\right)\right|^{2}\right]dz\right\}. \qquad (11)
\end{aligned}$$

The magnitude of the birefringence effect is determined by difference $\left|\varphi_{\pm1}\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}-\left|\varphi_{0}\left(\vec{\xi}-\vec{\eta},z\right)\right|^{2}$ i.e. by the difference of distributions of nucleon density in the deuteron for different deuteron spin orientations. The structure of the wavefunction φ_m is well known:

$$\varphi_m=\frac{1}{\sqrt{4\pi}}\left\{\frac{u(r)}{r}+\frac{1}{\sqrt{8}}\frac{W(r)}{r}\hat{S}_{12}\right\}\chi_m \qquad (12)$$

where $u(r)$ is the deuteron radial wave function corresponding to the S-wave; $W(r)$ is the radial function corresponding to the D-wave; the operator $\hat{S}_{12}=6(\hat{\vec{S}}\vec{n}_r)^2-2\hat{\vec{S}}^2$; $\vec{n}_r=\frac{\vec{r}}{r}$; $\hat{\vec{S}}=\frac{1}{2}(\vec{\sigma}_1+\vec{\sigma}_2)$ and $\vec{\sigma}_{1(2)}$ ate the Pauli spin matrices describing proton(neutron) spin.

Using (12) we obtain

$$|\varphi_{\pm 1}|^2 - |\varphi_0|^2 = -\frac{3}{4\pi}\left\{\frac{1}{\sqrt{2}}\frac{u(r)W(r)}{r^2} - \frac{1}{4}\frac{W(r)^2}{r^2}\right\}\left(n_{rx}^2 + n_{ry}^2 - 2n_{rz}^2\right),$$

$$\left|\varphi_{\pm 1}\left(\vec{\xi}, z\right)\right|^2 - \left|\varphi_0\left(\vec{\xi}, z\right)\right|^2 = -\frac{3}{4\pi}\left\{\frac{1}{\sqrt{2}}\frac{u(r)W(r)}{r^2} - \frac{1}{4}\frac{W(r)^2}{r^2}\right\}\frac{\xi^2 - 2z^2}{r^2},$$

$$\int\left[|\varphi_{\pm 1}(0,z)|^2 - |\varphi_0(0,z)|^2\right]dz = \frac{3}{\pi}\int_0^\infty\left\{\frac{1}{\sqrt{2}}\frac{u(r)W(r)}{r^2} - \frac{1}{4}\frac{W(r)^2}{r^2}\right\}dr = \frac{3}{\pi}\,G, \tag{13}$$

where $r^2 = \xi^2 + z^2$.

Substituting equation (13) into (11) yields

$$\begin{aligned}
Re(d_1) &= \frac{3}{2\pi}Im\left\{F_{nN}(0)\int t_{pN}^C\left(\vec{\xi}\right)\left\{\frac{1}{\sqrt{2}}\frac{u(r)W(r)}{r^2} - \frac{1}{4}\frac{W(r)^2}{r^2}\right\}\frac{\xi^2 - 2z^2}{r^2}d^2\xi dz\right\}\\
&- \frac{6}{k}Im\left\{F_{pN}(0)F_{nN}(0)\right\}G - \frac{12}{k}Re\left\{F_{ppN}^C(0)F_{nN}(0)\right\}G,\\
Im(d_1) &= -\frac{3}{2\pi}Re\left\{F_{nN}(0)\int t_{pN}^C\left(\vec{\xi}\right)\left\{\frac{1}{\sqrt{2}}\frac{u(r)W(r)}{r^2} - \frac{1}{4}\frac{W(r)^2}{r^2}\right\}\frac{\xi^2 - 2z^2}{r^2}d^2\xi dz\right\}\\
&+ \frac{6}{k}Re\left\{F_{pN}(0)F_{nN}(0)\right\}G - \frac{12}{k}Im\left\{F_{ppN}^C(0)F_{nN}(0)\right\}G.
\end{aligned} \tag{14}$$

Now we can evaluate the deuteron spin dichroism and angle of rotation of polarization plane. Let unpolarized deuterons beam pass through carbon target. According to [1]-[4] spin dichroism A and tensor polarization can be written as

$$p_{zz} \approx -\frac{4}{3}\mathrm{A}, \quad p_{xx} = p_{yy} \approx \frac{2}{3}\mathrm{A}, \tag{15}$$

where

$$\mathrm{A} = \frac{I_0 - I_\pm}{I_0 + I_\pm} = \frac{N_a z}{2M_r}\left(\sigma_{\pm 1} - \sigma_0\right) = \frac{2\pi N_a z}{kM_r}Im(d_1) \tag{16}$$

I_0 is the intensity of the deuteron beam after the target if the deuteron beam before the target is in the spin state $m = 0$ and, similarly, $I_\pm$ is the intensity of the deuteron beam after the target if the deuteron beam before the target is in the spin state $m = \pm 1$, z is thickness of target in g/cm^2, N_a is Avogadro number, M_r is molar mass for targets matter, $\sigma_{\pm 1}$ and $\sigma_{\pm 1}$ are the deuteron total cross-section of scattering for spin state $m = \pm 1$ and $m = 0$ respectively.

According to [1]-[4] angle of rotation of polarization plane is

$$\vartheta = \frac{2\pi N_a z}{kM_r}Re(d_1). \tag{17}$$

For estimation of nucleon-carbon strong interaction in (5) lets consider optical Woods-Saxon potential for 5.25 MeV nucleons $V_{nN}(r) = V_{pN}(r) = \frac{-52.5-0.9i}{1+exp(2(r-3.045))}$. Total cross-section for n-C scattering at 5.25 MeV calculated by optical potential and eikonal approximation is about 1.2 barn that agree with experimental data. For Coulomb p-C interaction in (5) we consider Coulomb screening potential. For calculation of parameter G the deuterons wave functions from [8] was applied. Obtained value G is about 0.05.

In (14) the first items for $Re(d_1)$ and $Im(d_1)$ are describe contribution of interference of nuclear n-C and Coulomb p-C interactions (lets denote that as NC), the second items are describe contribution of interference of nuclear p-C and n-C interactions (NN) and the third items are describe contribution of interference of nuclear p-C, n-C and Coulomb p-C interactions (NNC). Dependencies on energy of contributions of every items to $\sigma_{\pm 1} - \sigma_0$ and $Re(d_1)$ are shown on fig.1.

So according (16) for carbon target with $z = 0.1\,g/cm^2$ and for energy conditions of experiment (6-13 MeV) [3], [4] dichroism is about 0.01. On the fig.2 is shown dependence of averaged effective difference of total cross-section $\sigma_{\pm 1} - \sigma_0$ on averaged deuteron energy inside carbon targets obtained in experiments [3], [4]. We would like to make a note that since $\sigma_0 \neq \sigma_{\pm 1}$, measuring of spin dichroism possible in spin-filtering experiment in COSY (investigation of different decreasing of intensity of transmitted proton beam after passing of deuteron target with longitudinal and transversal polarization relatively proton wave vector).

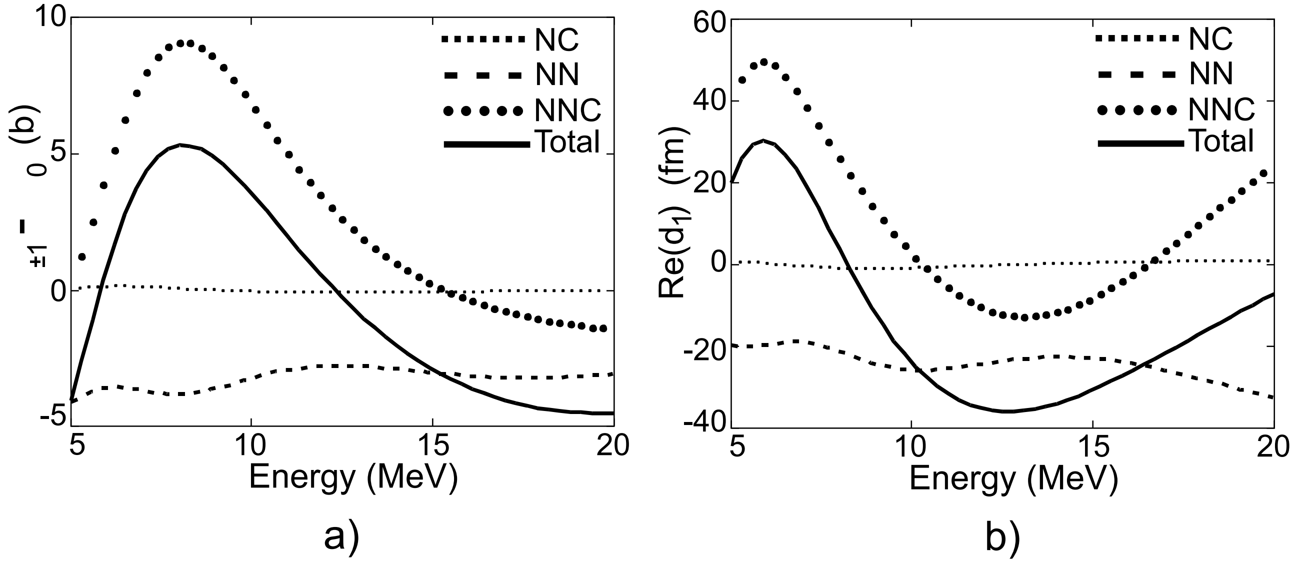

Fig. 1: Dependencies on deuteron energy of contributions of items NC, NN, NNC and their sum to a) $\sigma_{\pm 1} - \sigma_0$ and b) $Re(d_1)$.

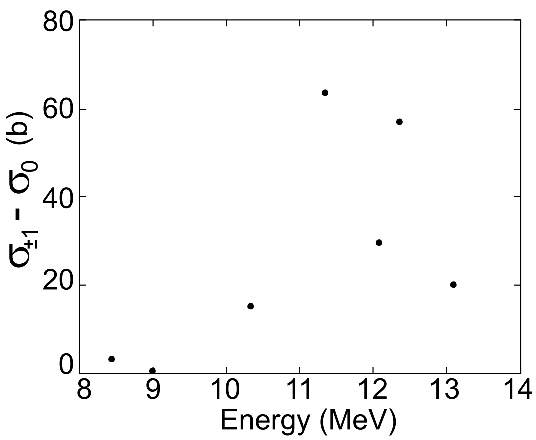

Fig. 2: Dependence of averaged effective difference of total cross-section $\sigma_{\pm 1} - \sigma_0$ on averaged deuteron energy inside carbon targets obtained in experiments [3], [4].

For same carbon target but for a little higher energy angle of rotation of polarization plane is about -0.5°. So that value of rotation can be measured experimentally on the installation described in [3], [4].

There are some reasons that can essentially increase birefringence effect. First of all, it is interaction of nucleon with carbon. On the fig.3 is shown the estimated total cross-section, calculated by simple Woods-Saxon potential and eikonal approximation in comparison with experimental total cross-section. Interaction of nucleon with carbon has a lot of resonances in energy region of carried out experiment. So experimental cross-section for some energy interval in 2-2.5 times more than estimated that can result in increasing of effects up to 4-6.25 times for that energy interval. At the second, parameter G is very sensitive to deuterons wave functions at small distances. At the third, the increasing of weight of D-state (in [8] it is 4.85%) is increase birefringence effects.

According to fig.1 Coulomb scattering play very important role in birefringence value and behavior. Position of peak, caused by Coulomb interaction is sensitive to Coulomb potential so it can be shifted for realistic interaction. Fig.1 and fig.3 give qualitative explanation of experimental results on fig.2 [3], [4]: sign of dichroism, strong dependence on energy, non-monotone and non-linear dependence of dichroism on target thickness.

Let's consider now another model of d-C interaction. At deuterons energy 10.5 MeV the characteristic time of Coulomb deuteron-carbon interaction caused by radius of Coulomb screening is $\tau^C \simeq 2 \cdot 10^{-18}\, s$. i. e. much more then characteristic period of nucleons oscillation in deuteron τ ($R^C \simeq 3 \cdot 10^{-9}\, cm \gg R^N \simeq 3 \cdot 10^{-13}\, cm$ where R^C and R^N are radiuses of Coulomb screening and nucleus of carbon respectively). So in this case we can't to neglect by

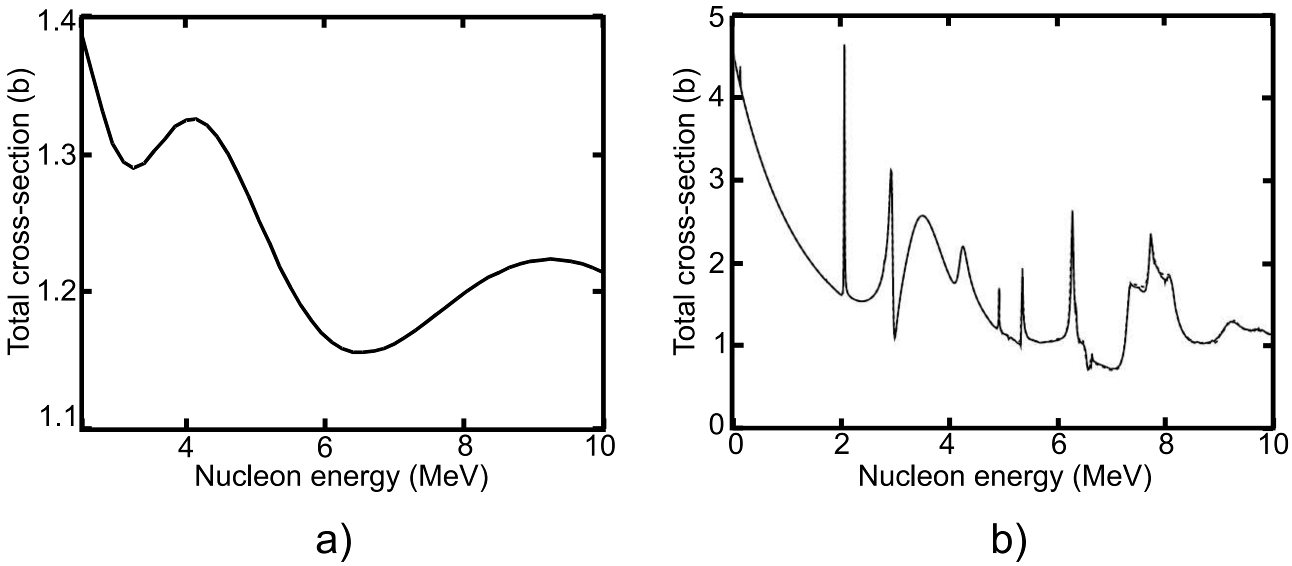

Fig. 3: Dependencies on nucleon energy of total nucleon-carbon cross-section calculated by a) eikonal approximation and b) from experimental data.

deuterons binding energy. Due to high frequency of oscillation of nucleons in carbon and in deuteron for deuteron-carbon Coulomb interaction we can use approximation for that in (2) Coulomb interaction is averaged by deuterons ground state wave functions. I this case Coulomb potential of p-C interaction will be shifted from the proton to the deuteron center of mass. Then (6) can be written as

$$\begin{aligned} f(0) &= \frac{k}{\pi}\int\left\{t_{pN}\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right)+t_{nN}\left(\vec{b}-\frac{\vec{r}_\perp}{2}\right)+t_{pN}^C(b)+2it_{pN}\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right)t_{pN}^C(b)\right\}|\varphi(\vec{r})|^2 d^2bd^3r \\ &+ \frac{k}{\pi}\int\left\{2it_{nN}\left(\vec{b}-\frac{\vec{r}_\perp}{2}\right)t_{pN}^C(b)+2it_{pN}\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right)t_{nN}\left(\vec{b}-\frac{\vec{r}_\perp}{2}\right)\right\}|\varphi(\vec{r})|^2 d^2bd^3r \\ &- \frac{k}{\pi}\int 4t_{pN}\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right)t_{nN}\left(\vec{b}-\frac{\vec{r}_\perp}{2}\right)t_{pN}^C(b)|\varphi(\vec{r})|^2 d^2bd^3r. \end{aligned} \tag{18}$$

From (18) it follows

$$\begin{aligned} f(0) &= F_{pN}(0)+F_{nN}(0)+F_{pN}^C(0)+\frac{2ik}{\pi}\int\left\{t_{pN}\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right)t_{pN}^C(b)+t_{nN}\left(\vec{b}-\frac{\vec{r}_\perp}{2}\right)t_{pN}^C(b)\right\}|\varphi(\vec{r}_\perp,z)|^2 d^2bd^2r_\perp dz \\ &+ \frac{2ik}{\pi}\int t_{pN}\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right)t_{nN}\left(\vec{b}-\frac{\vec{r}_\perp}{2}\right)|\varphi(\vec{r}_\perp,z)|^2 d^2bd^2r_\perp dz \\ &- \frac{4k}{\pi}\int t_{pN}\left(\vec{b}+\frac{\vec{r}_\perp}{2}\right)t_{nN}\left(\vec{b}-\frac{\vec{r}_\perp}{2}\right)t_{pN}^C(b)|\varphi(\vec{r}_\perp,z)|^2 d^2bd^2r_\perp dz. \end{aligned} \tag{19}$$

The expression (19) can be rewritten as

$$\begin{aligned} f(0) &= F_{pN}(0)+F_{nN}(0)+F_{pN}^C(0)+\frac{2ik}{\pi}\int\left\{t_{pN}\left(\vec{\xi}\right)t_{pN}^C\left(\frac{\vec{\xi}+\vec{\eta}}{2}\right)+t_{nN}(\vec{\eta})t_{pN}^C\left(\frac{\vec{\xi}+\vec{\eta}}{2}\right)\right\}\left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 d^2\xi d^2\eta dz \\ &+ \frac{2ik}{\pi}\int t_{pN}\left(\vec{\xi}\right)t_{nN}(\vec{\eta})\left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 d^2\xi d^2\eta dz-\frac{4k}{\pi}\int t_{pN}\left(\vec{\xi}\right)t_{nN}(\vec{\eta})t_{pN}^C\left(\frac{\vec{\xi}+\vec{\eta}}{2}\right)\left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 d^2\xi d^2\eta dz. \end{aligned} \tag{20}$$

Then from (20)

$$
\begin{aligned}
Ref(0) &= ReF_{pN}(0) + ReF_{nN}(0) + ReF^C_{pN}(0) \\
&- \frac{2k}{\pi} Im \int \left\{ t_{pN}\left(\vec{\xi}\right) t^C_{pN}\left(\frac{\vec{\xi}+\vec{\eta}}{2}\right) + t_{nN}\left(\vec{\eta}\right) t^C_{pN}\left(\frac{\vec{\xi}+\vec{\eta}}{2}\right) \right\} \left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 d^2\xi d^2\eta dz \\
&- \frac{2k}{\pi} Im \int t_{pN}\left(\vec{\xi}\right) t_{nN}\left(\vec{\eta}\right) \left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 d^2\xi d^2\eta dz - \frac{4k}{\pi} Re \int t_{pN}\left(\vec{\xi}\right) t_{nN}\left(\vec{\eta}\right) t^C_{pN}\left(\frac{\vec{\xi}+\vec{\eta}}{2}\right) \left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 d^2\xi d^2\eta dz, \\
Imf(0) &= ImF_{pN}(0) + ImF_{nN}(0) + ImF^C_{pN}(0) \\
&+ \frac{2k}{\pi} Re \int \left\{ t_{pN}\left(\vec{\xi}\right) t^C_{pN}\left(\frac{\vec{\xi}+\vec{\eta}}{2}\right) + t_{nN}\left(\vec{\eta}\right) t^C_{pN}\left(\frac{\vec{\xi}+\vec{\eta}}{2}\right) \right\} \left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 d^2\xi d^2\eta dz \qquad (21) \\
&+ \frac{2k}{\pi} Re \int t_{pN}\left(\vec{\xi}\right) t_{nN}\left(\vec{\eta}\right) \left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 d^2\xi d^2\eta dz - \frac{4k}{\pi} Im \int t_{pN}\left(\vec{\xi}\right) t_{nN}\left(\vec{\eta}\right) t^C_{pN}\left(\frac{\vec{\xi}+\vec{\eta}}{2}\right) \left|\varphi\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 d^2\xi d^2\eta dz.
\end{aligned}
$$

From (22) for spin-dependent part of forward scattering amplitude follows that

$$
\begin{aligned}
Re(d_1) &= -\frac{2k}{\pi} Im \int \left\{ t_{pN}\left(\vec{\xi}\right) t^C_{pN}\left(\frac{\vec{\xi}+\vec{\eta}}{2}\right) + t_{nN}\left(\vec{\eta}\right) t^C_{pN}\left(\frac{\vec{\xi}+\vec{\eta}}{2}\right) \right\} \left[\left|\varphi_{\pm1}\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 - \left|\varphi_0\left(\vec{\xi}-\vec{\eta},z\right)\right|^2\right] d^2\xi d^2\eta dz \\
&- \frac{2k}{\pi} Im \int t_{pN}\left(\vec{\xi}\right) t_{nN}\left(\vec{\eta}\right) \left[\left|\varphi_{\pm1}\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 - \left|\varphi_0\left(\vec{\xi}-\vec{\eta},z\right)\right|^2\right] d^2\xi d^2\eta dz \\
&- \frac{4k}{\pi} Re \int t_{pN}\left(\vec{\xi}\right) t_{nN}\left(\vec{\eta}\right) t^C_{pN}\left(\frac{\vec{\xi}+\vec{\eta}}{2}\right) \left[\left|\varphi_{\pm1}\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 - \left|\varphi_0\left(\vec{\xi}-\vec{\eta},z\right)\right|^2\right] d^2\xi d^2\eta dz, \\
Im(d_1) &= \frac{2k}{\pi} Re \int \left\{ t_{pN}\left(\vec{\xi}\right) t^C_{pN}\left(\frac{\vec{\xi}+\vec{\eta}}{2}\right) + t_{nN}\left(\vec{\eta}\right) t^C_{pN}\left(\frac{\vec{\xi}+\vec{\eta}}{2}\right) \right\} \left[\left|\varphi_{\pm1}\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 - \left|\varphi_0\left(\vec{\xi}-\vec{\eta},z\right)\right|^2\right] d^2\xi d^2\eta dz \\
&+ \frac{2k}{\pi} Re \int t_{pN}\left(\vec{\xi}\right) t_{nN}\left(\vec{\eta}\right) \left[\left|\varphi_{\pm1}\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 - \left|\varphi_0\left(\vec{\xi}-\vec{\eta},z\right)\right|^2\right] d^2\xi d^2\eta dz \\
&- \frac{4k}{\pi} Im \int t_{pN}\left(\vec{\xi}\right) t_{nN}\left(\vec{\eta}\right) t^C_{pN}\left(\frac{\vec{\xi}+\vec{\eta}}{2}\right) \left[\left|\varphi_{\pm1}\left(\vec{\xi}-\vec{\eta},z\right)\right|^2 - \left|\varphi_0\left(\vec{\xi}-\vec{\eta},z\right)\right|^2\right] d^2\xi d^2\eta dz. \qquad (22)
\end{aligned}
$$

Taking into account (13) and the same assumption as in (11) we are obtain

$$
\begin{aligned}
Re(d_1) &= \frac{3}{\pi} Im \left\{ F_{nN}(0) \int t^C_{pN}\left(\frac{\vec{\xi}}{2}\right) \left\{ \frac{1}{\sqrt{2}} \frac{u(r)W(r)}{r^2} - \frac{1}{4}\frac{W(r)^2}{r^2} \right\} \frac{\xi^2 - 2z^2}{r^2} d^2\xi dz \right\} \\
&- \frac{6}{k} Im \left\{F_{pN}(0)F_{nN}(0)\right\} G - \frac{12k}{\pi^2} G \; Re \int t_{pN}\left(\vec{\xi}\right) t_{nN}\left(\vec{\eta}\right) t^C_{pN}\left(\frac{\vec{\xi}+\vec{\eta}}{2}\right) d^2\xi d^2\eta dz, \\
Im(d_1) &= -\frac{3}{\pi} Re \left\{ F_{nN}(0) \int t^C_{pN}\left(\frac{\vec{\xi}}{2}\right) \left\{ \frac{1}{\sqrt{2}} \frac{u(r)W(r)}{r^2} - \frac{1}{4}\frac{W(r)^2}{r^2} \right\} \frac{\xi^2 - 2z^2}{r^2} d^2\xi dz \right\} \\
&+ \frac{6}{k} Re \left\{F_{pN}(0)F_{nN}(0)\right\} G - \frac{12k}{\pi^2} G \; Im \int t_{pN}\left(\vec{\xi}\right) t_{nN}\left(\vec{\eta}\right) t^C_{pN}\left(\frac{\vec{\xi}+\vec{\eta}}{2}\right) d^2\xi d^2\eta dz. \qquad (23)
\end{aligned}
$$

In (23) the first items for $Re(d_1)$ and $Im(d_1)$ are describe sum of contributions of interference of nuclear n-C interaction with averaged Coulomb p-C interaction and nuclear p-C interaction with averaged Coulomb p-C interaction (lets denote that as 2NC), the second items are describe contribution of interference of nuclear p-C and n-C interactions (NN) and the third items are describe contribution of interference of nuclear p-C, n-C and averaged Coulomb p-C interactions (NNC). Dependencies on energy of contributions of every items to $\sigma_{\pm1} - \sigma_0$ and $Re(d_1)$ are shown on fig.4.

So, according to fig.1 and fig.4 the second model for estimation of deuteron-carbon spin-dependent forward scattering amplitude gives quantitatively almost the same result as the first model but the peaks caused by Coulomb interaction is shifted almost on 1 MeV.

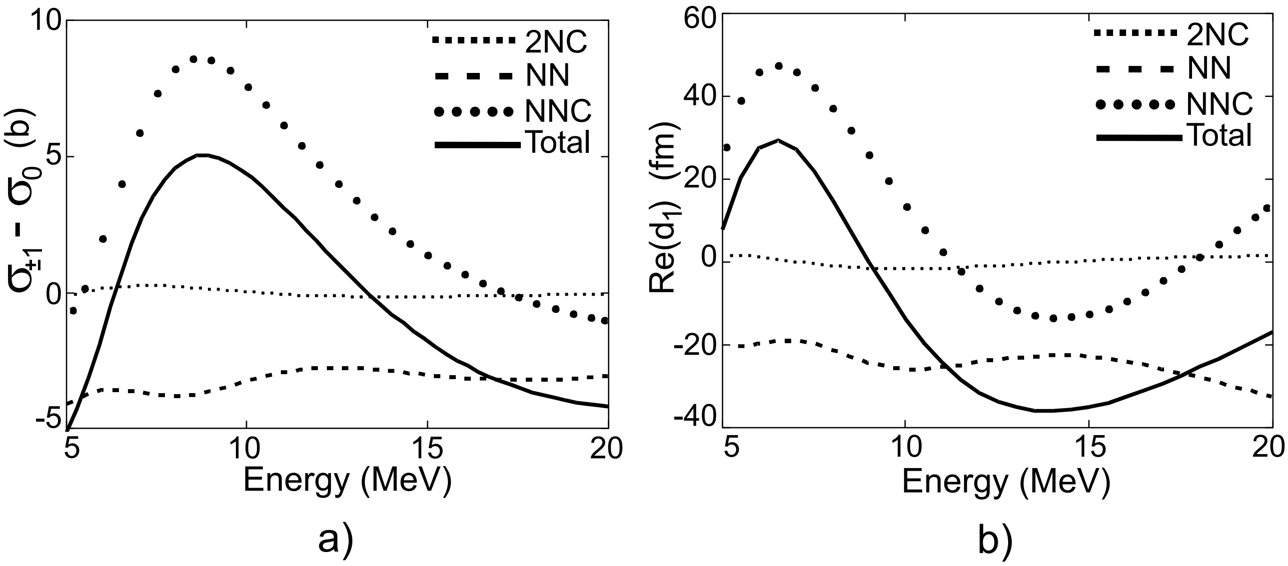

Fig. 4: Dependencies on deuteron energy of contributions of items 2NC, NN, NNC and their sum to a) $\sigma_{\pm1} - \sigma_0$ and b) $Re(d_1)$.

III. SUMMERY

Obtained results show that estimation of spin dichroism is coincide with experimental results for some targets. More over it necessary to note that birefringence (through the parameter G) is very sensitive to deuterons wave functions especially at small distances. Here was used deuterons wave functions from [8], based on CD-Bonn potential of nucleon-nucleon interaction. According to [8] these functions have discrepancies with wave functions based on the another models for $r < 2\,fm$. Especially a big problem arise with description of deuterons wave functions for $r < 0.5\,fm$. So birefringence can be used as additional source of information about deuterons wave functions at small distances.

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