SOME SPECIFICATION ASPECTS FOR THREE-FACTOR MODELS OF A COMPANY’S PRODUCTION POTENTIAL TAKING INTO ACCOUNT INTELLECTUAL CAPITAL

S. Aivazian¹, M. Afanasiev², V. Rudenko
Central Economics and Mathematics Institute of RAS
Moscow, RUSSIA

e-mail: ¹ aivazian@cemi.rssi.ru; ² afanasiev@cemi.rssi.ru

Abstract

We propose a general algorithm that gives a solution of some problems related to specification of 3-factor stochastic models of a company’s production potential that take into account main production factors. The proposed formal scheme is based on procedures of statistical hypothesis testing and provides the necessary means to choose a reasonable alternative within the analyzed class of models.

According to [1], a model of production function is a deterministic component of a production potential model. We consider the class of production potential models that is given by relation

$$R = \beta_0 K^{\beta_1} L^{\beta_2} I^{\beta_3} e^{V-U},$$

where $R$ is the overall production of a company, $K$ is the physical and financial capital input, $L$ is the labor input, $I$ is the intellectual capital input; the random variable (r.v.) $V$ is normally distributed with parameters $(0; \sigma_V^2)$ (i.e. $V \in N(0; \sigma_V^2)$), the r.v. $U$ is distributed according to a truncated at zero normal law with a mean value $\mu$ and a variance $\sigma_U^2$ (i.e. $U \in N^+(\mu; \sigma_U^2)$). The r.v.’s $V$ and $U$ are considered to be stochastically independent. The parameters $\beta_0, \ldots, \beta_3$ are subject to statistical estimation.

The main goal of the research is to describe a general algorithm of statistical hypothesis testing that provides answers to the following questions related to the model’s specification.

(a) Is it reasonable to consider the 3-factor model (1) under given methods of intellectual capital measurement (alternative is the standard 2-factor model)?

(b) In case the answer to the question (a) is positive: is there any inefficiency in use of production factors (alternative: $\sigma_U^2 = 0$)?

(c) If the answer to the question (a) is positive and there’s inefficiency in use of inputs ($\sigma_U^2 > 0$): is it reasonable to apply the model (1) with $\mu = 0$ (alternative: $\mu \neq 0$)?

(d) Finally, in case the answers to the questions (a) and (b) are positive and there’s possibility to identify indicators $z^{(1)}, z^{(2)}, \ldots, z^{(p)}$ that can influence efficiency in use of the main production factors: how one can proceed with statistical hypothesis testing related to the dependence character of the parameters $\mu$ or $\sigma_U^2$ of those indicators?

In order to estimate intellectual capital (IC) one can use any known method that meets the requirements of the research. We use the following approach to find out whether it is reasonable to apply one or another way of IC measurement. If the
influence of IC on the overall production of a company is positive and statistically significant (under positive and statistically significant estimated coefficients for capital input $K$ and labor input $L$) we say that the corresponding measurement method is acceptable for practical purposes. Otherwise the considered way of IC measurement is said to be inapplicable for the analysis.

Proceeding with econometric analysis of the model (1) we generally have the data array $E^2 = \left\{ R_i, K_i, L_i, I_i, z_i^{(1)}, z_i^{(2)}, \ldots, z_i^{(p)} \right\}_{i=1}^n$, where $R_i$ is the total production of the $i$th company, $K_i, L_i, I_i$ are the values of the main production factors for the $i$th company, $z_i^{(1)}, \ldots, z_i^{(p)}$ are the values of the measurable indicators that characterize efficiency in use of the main production factors for the $i$th company; $n$ is the number of companies in the sample. If there’s no information about any efficiency indicators the specification of the model (1) is carried out on the basis of the reduced data array $E^1 = \{ R_i, K_i, L_i, I_i \}_{i=1}^n$.

In order to formalize the problems given by items (a) - (d) we should consider the following models:

$M_0 : \ R = \beta_0 K^{\beta_1} L^{\beta_2} I^{\beta_3} e^V, \ \text{where} \ V \in N(0; \sigma^2_V), \ \text{and} \ R^2 \text{ cannot be inapplicable for the analysis.}$

$M_1 : \ R = \beta_0 K^{\beta_1} L^{\beta_2} I^{\beta_3} e^{V-U}, \ \text{where} \ V \in N(0; \sigma^2_V), \ U \in N^+(0; \sigma^2_U).$

$M_2 : \ R = \beta_0 K^{\beta_1} L^{\beta_2} I^{\beta_3} e^{V-U}, \ \text{where} \ V \in N(0; \sigma^2_V), \ U \in N^+(\mu; \sigma^2_U).$

$M_3 : \ R = \beta_0 K^{\beta_1} L^{\beta_2} I^{\beta_3} e^{V-U}, \ \text{where} \ V \in N(0; \sigma^2_V), \ U \in N^+(0; \sigma^2_U(z)), \ \ln \sigma^2_U(z) = \theta_0 + \theta_1 z^{(1)} + \ldots + \theta_p z^{(p)}.$

$M_4 : \ R = \beta_0 K^{\beta_1} L^{\beta_2} I^{\beta_3} e^{V-U}, \ \text{where} \ V \in N(0; \sigma^2_V), \ U \in N^+(\mu(z); \sigma^2_U), \ \mu(z) = \delta_0 + \delta_1 z^{(1)} + \ldots + \delta_p z^{(p)}.$

To find out whether a method of IC measurement is acceptable for practical use one should test the following hypothesis:

$H_0 : \ \text{there exists } i(i \in \{1, 2, 3\}) \text{ such that: } \beta_i \leq 0 \text{ i.e. there exists a production factor that is not statistically significant or that provides negative influence on the total production; }$

$H_{0i} : \beta_i > 0, \ i = 1, 2, 3, \text{ i.e. all the considered production factors are statistically significant and provide positive impact on the overall production.}$

Testing procedure of the hypothesis $H_0$ against the alternative $H_{0i}$ is based on the fact that the statistics $t_i = \hat{\beta}_i / s_{\hat{\beta}_i}$ (where $\hat{\beta}_i$ is an estimate of the coefficient $\beta_i$ and $s_{\hat{\beta}_i}$ is an estimate of the standard deviation in $\beta_i$ estimation) is distributed according to $t(n - k)$ - law.

To get an answer to the question (b) one should test the following statistical hypothesis within the frame of the model $M_1$:

$H_1 : \sigma^2_{U^1} = 0 \text{ (no inefficiency in use of production factors) with the alternative } H^1_{U^1} : \sigma^2_{U^1} > 0 \text{ (inefficiency is observed).}$

Testing of the hypothesis $H_1$ (against the alternative $H^1_{U^1}$) is done on the basis of asymptotic characteristics of the likelihood ratio statistics (see corresponding results in [3, 4, 6]).

The choice of a proper model between $M_1$ and $M_2$ is formalized be the hypothesis:

$H_{1,2} : \mu = 0 \text{ for the model } M_2 \text{ (inefficiency in the models } M_1 \text{ and } M_2 \text{ cannot be distinguished),}$
H_{4,1}^3 : \quad \mu \neq 0 \text{ for the model } M_2 \text{ (inefficiency in the models } M_1 \text{ and } M_2 \text{ can be distinguished).}

The corrected Akaike criterion is used in testing of the hypothesis H_{1,2} against the alternative H_{4,2}^1 (see [5], [2]).

To find out whether the indicators \( z^{(1)}, \ldots, z^{(p)} \) have a real effect on the variance \( \sigma_U^2 \) in the model \( M_3 \) we test the hypotheses:

\( H_{3,1}^4 : \quad \forall j = 1, \ldots, p : \quad \theta_j = 0 \) (influence of all the efficiency indicators in the model \( M_3 \) are not statistically significant),

\( H_{3,1}^3 : \quad \exists j = 1, \ldots, p : \quad \theta_j \neq 0 \) (there exists at least one statistically significant efficiency factor in the model).

The problem is solved by means of general theory of linear hypotheses testing using the corresponding F statistics (see [2]).

To form the a posteriori set of efficiency indicators for the model \( M_3 \) one should test the hypotheses:

\( H_{4,2} : \quad \exists j = 1, \ldots, p : \quad \theta_j = 0 \) (there are some (at least one) non-significant efficiency factors in the model \( M_3 \)),

\( H_{4,2}^3 : \quad \forall j = 1, \ldots, p : \quad \theta_j \neq 0 \) (all the efficiency indicators in the model \( M_3 \) are statistically significant).

Testing procedure of the hypothesis \( H_{3,2} \) (against the alternative \( H_{3,2}^A \)) is based on the fact that the statistics \( \hat{z}_j = \hat{\theta}_j^2 / s^2_{\hat{\theta}_j} \) is distributed according to \( \chi^2(1) \)-law.

Analysis of the model \( M_4 \) should clarify whether the efficiency indicators \( z^{(1)}, \ldots, z^{(p)} \) really influence the value \( \mu \) in the distribution \( N^+(\mu; \sigma_U^2) \). The following hypothesis should be tested:

\( H_{4,1} : \quad \forall j = 1, \ldots, p : \quad \delta_j = 0 \) (all the efficiency factors in the model \( M_4 \) are not statistically significant) against the alternative

\( H_{4,1}^3 : \quad \exists j = 1, \ldots, p : \quad \delta_j \neq 0 \) (there’s at least one statistically significant efficiency indicator in the model \( M_4 \)).

To form the a posteriori set of efficiency indicators for the model \( M_3 \) one is advised to test the hypotheses:

\( H_{4,2} : \quad \exists j = 1, \ldots, p : \quad \delta_j = 0 \) (there are some (at least one) non-significant efficiency factors in the model \( M_4 \)),

\( H_{4,2}^3 : \quad \forall j = 1, \ldots, p : \quad \delta_j \neq 0 \) (all the efficiency indicators in the model \( M_4 \) are statistically significant).

The dependence character between efficiency in use of the main production factors and the indicators \( z^{(1)}, \ldots, z^{(p)} \) can be clarified by testing the following hypotheses

\( H_{2,3} : \quad \mu \neq 0, \quad \sigma_U^2 = \text{const} \) (the variance of the component \( U \) should not be expressed via the efficiency indicators),

\( H_{2,3}^4 : \quad \mu = 0, \quad \sigma_U^2 = e^{\theta_0 + \theta_1 z^{(1)} + \ldots + \theta_p z^{(p)}} \) (the variance of the component \( U \) should be decomposed by the efficiency indicators under assumption that the mathematical expectation \( \mu \) is equal to 0).

Finally, if one assumes that both parameters of the r.v. \( U \) (\( \mu \) and \( \sigma_U^2 \)) might depend on the indicators \( z^{(1)}, \ldots, z^{(p)} \) we recommend to test the hypothesis:

\( H_{3,4}^4 : \quad \mu = 0, \quad \sigma_U^2 = e^{\theta_0 + \theta_1 z^{(1)} + \ldots + \theta_p z^{(p)}} \) (the variance \( \sigma_U^2 \) (but not the mathemat-
ical expectation $\mu$) should be decomposed by the efficiency indicators), against the alternative

$$H^A_{3,4}: \quad \mu = \delta_0 + \delta_1 z^{(1)} + \ldots + \delta_p z^{(p)}; \quad \sigma^2_U = \text{const}$$

(the mathematical expectation $\mu$ (but not the variance $\sigma^2_U$) should be decomposed by the efficiency indicators).

To describe the expanded general methodological algorithm that helps to choose a proper model for the class (1) and takes into account availability of the information regarding efficiency indicators we use the following notation:

- **Input 1** — start of the analysis with the data array $E^1 = \{R_i, K_i, L_i, I_i\}_{i=1}^n$,
- **Mi** — calculation of the estimates in the model $M_i$,
- **Mi+** — the estimates are successfully obtained,
- **Mi-** — the estimates are not obtained (due to the sample specifics, problems of non-identifiability etc.),
- **Hi** — application of the hypothesis testing procedure,
- **Hi+** — the hypothesis is accepted,
- **Hi-** — the hypothesis is discarded in favor of the alternative $H^A_i$,
- **ICNS** — the model $M_i$ is finally chosen,
- **Check** — conclusion that the used estimate of intellectual capital (IC) is not statistically significant,
- **Input 2** — start of the analysis with the data array $E^2 = \{R_i, K_i, L_i, I_i, z^{(1)}_i, z^{(2)}_i, \ldots, z^{(p)}_i\}_{i=1}^n$,
- **ENSF(z_i)** — exclusion of the $i$th non-significant efficiency indicator that has the maximum $p$-value in the testing of the hypotheses $H_{3,2}$ and $H_{4,2}$,
- **ENF(z_i)** — exclusion of the $i$th efficiency indicator that has the maximum absolute correlation coefficient with intellectual capital indicator,
- **Check** — check whether there still exist non-excluded efficiency indicators,
- **Check+** — in the analyzed model there still exist non-excluded efficiency indicators,
- **Check-** — in the analyzed model all the efficiency indicators are excluded.

As shown at figure 1, the algorithm starts with the model that has the biggest number of variables and provides the widest opportunities for analysis, i.e. with the
model $M_4$. In case there’s at least one statistically significant efficiency indicator in the models $M_4$ or $M_3$ one should choose one of these models as the final result (given that the basic principles of the provided methodology are not violated). If in both models $M_3$ and $M_4$ all the efficiency indicators are not statistically significant or at least one of production factors is not statistically significant the analysis is reduced to the models of production potential that do not take into account the dependence of efficiency in use of production factors of any additional indicators.

In [2] we consider the modeling of production potential for a sample of US companies that operate in the sector “Biotechnology and Drugs”. The sequence of procedures given below leads to a conclusion that one should use the 3-factor model $M_3$ to estimate the production potential:

$$
\begin{align*}
&\left\{E^2; M_4; M_4^-; ECF(z^1); \text{Check}; \text{Check}^+; M_4; M_4^-; ECF(z^2); \right. \\
&\left. \text{Check}; \text{Check}^+; E^2; M_3; M_3^+; H_0; H_0^-; H_{3,1}; H_{3,2}; H_{3,2}^+; \\
&\text{ENSF}(z^1); \text{Check}; \text{Check}^+; M_3; M_3^+; H_0; H_0^-; H_{3,1}; H_{3,1}^-; H_{3,2}; \\
&H_{3,2}^-; E^1; M_2; M_2^+; \hat{M}_3
\end{align*}
$$

The detailed description of initial data and the results of calculations done according to the described scheme is given in [2].

References


Figure 1: General methodological algorithm of specification problem solution