# Dynamic Reliability Indices in Reliability Analysis of Multi-State System

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***Abstract:*** *The reliability of Multi-State System (MSS) is analysed in this paper. In a MSS, both the system and its components may experience more than two reliability states. Many practical and theoretical problems needs still to be solved in this area. One of the crucial ones is to identify how a change in a state of an individual component or changes in states of several ones affect(s) the system reliability. The Multiple-Valued Logic (MVL) tools are employed for handling this problem in this paper. In the paper the structure function and Logical Differential Calculus of MVL function are combined to evaluate the dynamic behaviour of a MSS. The Logical Differential Calculus extends potentialities of structure function tool to analyse also the MSS dynamical properties. The evaluation of MSS components changes is considered in this paper.*

***Keywords*:** Reliability Analysis, Multi-State System, Importance measure, Dynamic Reliability Indices (DRI)

## 1. INTRODUCTION

As a rule reliability is considered as one of important measures for technical systems. There are many methods for reliability analysis of these systems. Evolution of these methods allows to estimate different types of systems such as economical, financial, social and etc [1 – 3]. But in some tasks new type system caused implementation of new methods of reliability analysis. Therefore two problems arise: firstly, it is adaptation methods of reliability analysis of technical system to any type system; secondly, it is interpretation of initial real-world systems as one of mathematical model in reliability analysis. From the point of view of reliability analysis a system is interpreted as one of four basic mathematical models (Fig.1).

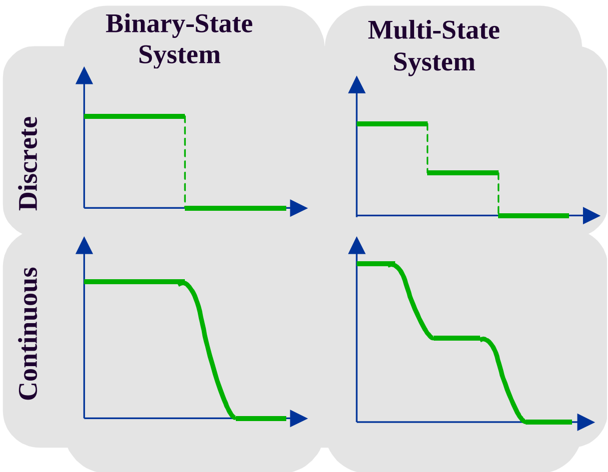


**Fig. 1. – Classification of mathematical models in reliability analysis**

These models are alternative and are used according to initial conditions. They are classified by level of availability (reliability) and type of depicted function. The function for description system model can be discrete or continuous. Function type causes mathematical approach for system reliability analysis [4, 5]. There are two types of these models dependent of level availability and reliability: Binary System and *Multi-State System* (MSS). In the most studied model to which are investigated, system and system component can take on one of two states: failure or functioning. This model is named Binary-State System. Many problems for these systems have been settled. But this approach fails to describe many situations where the system can have more than two distinct states [4, 6, 7]. MSS has proposed alternative decision system for reliability analysis. In a MSS, both the system and its components may experience more than two states, for example, completely failed, functioning and perfect functioning. The MSS is frequently required for applied problem. This model has been exploited in Reliability Analysis more 30 years [8].

There are different tools for a MSS reliability estimation [4, 6 - 8]. For example, Markov processes are used to analyze the system state transition process [6] or the structure function approach is used to investigate the system topology [4, 9, 10]. We have been developed structure function tool for computation system reliability measures. One principal problem in MSS reliability analysis is how to infer the effects of state changes of each individual component upon the system reliability [4, 6 – 9].

We propose method on the basis of structure function to estimate the dynamic properties of the MSS reliability evolving results proposed by Boedigheimer and Kapur [9]. In paper [11] basic and theoretical concepts of this approach have been determined and *Dynamic Reliability Indices* (DRIs) have been proposed as measures of MSS reliability. These indices are used to estimate changes of system reliability caused by changes in the states of its components [11 - 13]. DRIs are computed based on MSS structure function and Logical Differential Calculus of *Multiple-Valued Logic* (MVL). These indices are one of importance measures for MSS [].

Proposed method a MSS estimation by DRIs is independent of initial type system and can be applied for reliability analysis different system that primary interpreted as discrete MSS. In this paper basic conception of the MSS reliability analysis based on DRIs and some examples of this method application are considered.

## 2. MSS STRUCTURE FUNCTION

The structure function declares a system reliability depending on its components states [4, 7, 11 - 13]:

$$\phi(x_1, \ldots, x_n) = \phi(x): \{0, \ldots, m\text{-}1\}^n \rightarrow \{0, \ldots, m\text{-}1\} \quad (1)$$

where $\phi(x)$ – system reliability (system state), $x_i$ – components state ($i$ = 1, …, $n$), $n$ – number of system components, $m$ – discrete levels of reliability for system and its components ($m$ = 0, …, $m$-1): zero correspond to complete failure of system or its components and ($m$-1) is perfect functioning of MSS or its components.

The component probability characterizes every system component state $x_i$ form zero to ($m$-1):

$$p_{i,s_i} = \Pr\{x_i = s_i\} \quad (2)$$

where $i$ = 1, …, $n$ and $s_i$ = 0, …, $m$-1.

These assumptions for structure function in reliability analysis of MSS are used [4, 6, 7, 11]: ($a$) the structure function is monotone and $\phi(\boldsymbol{s})=s$ ($s\in\{0, \ldots, m\text{-}1\}$); ($b$) all components are $s$-independent and are relevant to the system; ($c$) the structure function $\phi(\boldsymbol{x})$ (1) is interpreted as a MVL function. Last allows applying MVL mathematical tools for reliability analysis of the MSS.

Consider approach of Direct Partial Logic Derivatives of MVL function for reliability analysis of MSS. This approach is part of Logical Differential Calculation. There are two types of these derivatives: with respect to one variable and with respect to variables vector [12]. The first type permits to examine the influence of one variable change to modification of MVL function value. The second type of derivative reveals MVL function value changes depending on changes of some function variables. So the second type of derivatives is generalization of the first type Direct Partial Logic Derivatives with respect to one variable.

A Direct Partial Logic Derivatives of a structure function $\phi(\boldsymbol{x})$ of $n$ variables with respect to variables vector $\boldsymbol{x}^{(p)} = (x_{i_1}, x_{i_2}, \ldots, x_{i_p})$ reflects the fact of changing of function from $j$ to $h$ when the value of every variable of vector $\boldsymbol{x}^{(p)}$ is changing from $a$ to $b$ [5]:

$$\partial\phi(j\rightarrow h)/\partial\boldsymbol{x}^{(p)}(\boldsymbol{a}^{(p)}\rightarrow\boldsymbol{b}^{(p)}) = \\ = \begin{cases} m-1, \text{if } \phi(a_{i_1},\ldots,a_{i_p},\boldsymbol{x}) = j \;\&\; \phi(b_{i_1},\ldots,b_{i_p},\boldsymbol{x}) = h \\ 0, \quad \text{in the other case} \end{cases} \quad (3)$$

where $\phi(a_{i_1},\ldots,a_{i_p},\boldsymbol{x}) = \phi(x_1,\ldots,a_{i_1},\ldots,a_{i_p},\ldots,x_n)$ and $\phi(b_{i_1},\ldots,b_{i_p},\boldsymbol{x}) = \phi(x_1,\ldots,b_{i_1},\ldots,b_{i_p},\ldots,x_n)$, $a_{i_j}, b_{i_j}$ ∈ {0, …, $m$-1}.

These derivatives for structure function of MSS in more details are considered in papers [11, 12].

In (3) a change of value of $i_j$-th variable $x_{i_j}$ form $a$ to $b$ agrees with a change of MSS component efficiency form $a$ to $b$. So, changes of some components states correspond with change of a variables vector $\boldsymbol{x}^{(p)} = (x_{i_1}, x_{i_2}, \ldots, x_{i_p})$. Every variable values of this vector changes form $a$ to $b$. So, vector $\boldsymbol{x}^{(p)}$ can be interpreted as components states vector or components efficiencies vector.

For example, consider MSS 2-out-of-3 with three states of reliability ($m$=3). This system consists of three components ($n$=3) and works if two or more system components are function. The structure function of this MSS is:

$$\phi(x) = (x_1\, x_2) \vee (x_2\, x_3) \vee (x_1\, x_3). \quad (4)$$

The structure function of MSS 2-out-of-3 and its Direct Partial Logic Derivative $\partial\phi(0\rightarrow1)/\partial x_1(0\rightarrow2)$ are in Table 1.

**Table 1. The example of the structure function of MSS 2-out-of-3 and the Direct Partial Logic Derivative**

| $x_1x_2x_3$ | $\phi(\boldsymbol{x})$ | $\partial\phi(0\rightarrow1)/\partial x_1(0\rightarrow2)$ | $x_1x_2x_3$ | $\phi(\boldsymbol{x})$ | $\partial\phi(0\rightarrow1)/\partial x_1(0\rightarrow2)$ | $x_1x_2x_3$ | $\phi(\boldsymbol{x})$ | $\partial\phi(0\rightarrow1)/\partial x_1(0\rightarrow2)$ |
|---|---|---|---|---|---|---|---|---|
| 0 0 0 | **0** | **0** | 1 0 0 | **0** | **-** | 2 0 0 | **0** | **-** |
| 0 0 1 | **0** | **2** | 1 0 1 | **1** | - | 2 0 1 | **1** | - |
| 0 0 2 | **0** | **0** | 1 0 2 | **1** | - | 2 0 2 | **2** | - |
| 0 1 0 | **0** | **2** | 1 1 0 | **1** | - | 2 1 0 | **1** | - |
| 0 1 1 | **1** | **0** | 1 1 1 | **1** | - | 2 1 1 | **1** | - |
| 0 1 2 | **1** | **0** | 1 1 2 | **1** | - | 2 1 2 | **2** | - |
| 0 2 0 | **0** | **0** | 1 2 0 | **1** | - | 2 2 0 | **2** | - |
| 0 2 1 | **1** | **0** | 1 2 1 | **1** | - | 2 2 1 | **2** | - |
| 0 2 2 | **2** | **0** | 1 2 2 | **2** | - | 2 2 2 | **2** | - |

Direct Partial Logic Derivatives permit describe every change of MSS depending of component efficiencies changes. But system failure and repair is more important for real-word system.

*The MSS failure* is represented as the changing of the function value $\phi(\boldsymbol{x})$ from $j$ into zero and as decrease of a system components efficiencies vector $\boldsymbol{x}^{(p)}$ from $\boldsymbol{a}^{(p)}$ to $\boldsymbol{b}^{(p)}$: $\partial\phi(j\rightarrow0)/\partial\boldsymbol{x}^{(p)}(\boldsymbol{a}^{(p)}\rightarrow\boldsymbol{b}^{(p)})$, $a_{i_j}, b_{i_j} \in \{0,\ldots,m-1\}$, $a_{i_j} > b_{i_j}$, $j$ = 1,…, $p$. Because the structure function is monotone (assumption ($a$)) the MSS failure is declared by a change function $\phi(\boldsymbol{x})$ from "1" into zero and decreases of every of $p$ system components availability from $a_{i_j}$ to $(a_{i_j} - 1)$:

$$\partial\phi(1\rightarrow0)/\partial\boldsymbol{x}^{(p)}(\boldsymbol{a}^{(p)}\rightarrow\tilde{\boldsymbol{a}}^{(p)}), \quad (5)$$

where $\tilde{\boldsymbol{a}}^{(p)} = (\tilde{a}_{i_1},\ldots,\tilde{a}_{i_p}) = ((a_{i_1}-1),\ldots,(a_{i_p}-1))$ and $a_{i_j} \in \{1,\ldots,(m-1)\}$.

*The MSS repair* for replacements of failed system components is defined in papers:

$$\partial\phi(0\rightarrow h)/\partial\boldsymbol{x}^{(p)}(\boldsymbol{0}\rightarrow(\boldsymbol{m}\text{-}\boldsymbol{1})), \quad (6)$$

where $h\in\{1, \ldots, m\text{-}1\}$, $\boldsymbol{0} = (\underbrace{0,\ldots,0}_{p})$ and $(\boldsymbol{m}-\boldsymbol{1}) = (\underbrace{(m-1),\ldots,(m-1)}_{p})$.

In this case the efficiency of $p$ failed system components changes from zero into ($m$-1). Therefore the elements of variable vector $\boldsymbol{x}^{(p)}$ change from zero into ($m$-1) and the structure function value changes from zero into $h$ ($\phi(\boldsymbol{x})$: $0 \rightarrow h$) in (6).

There is another variant of MSS repair, where failed components don't replacement and change state from zero into levels $b_{i_j} \in \{1,\ldots,m-2\}$ ($j$ = 1, …, $p$) by renewal of failed components: $\partial\phi(0\rightarrow h)/\partial\boldsymbol{x}^{(p)}(\boldsymbol{0}\rightarrow\boldsymbol{b}^{(p)})$. But in this paper we examine the first variant of MSS repair that is caused by failed components replacement.

In this paper we investigate the influence of state changes of more than one component to the system reliability by Direct Partial Logic Derivatives. MSS failure and MSS repair caused by changes of some components efficiency are defined in Direct Partial Logic Derivative terminology.

## 3. DYNAMIC RELIABILITY INDICES

DRIs are probabilistic indices and include two groups: *Component Dynamic Reliability Indices* (CDRIs) and *Dynamic Integrated Reliability Indices* (DIRIs). CDRIs allow measuring an influence of each individual component or a fixed group of components to the system reliability. A point of view of system reliability the unstable components are determined by these indices. DIRIs characterize a probability of impact of one or some of system components to the system reliability.

***Definition 1***. CDRIs for a MSS failure are probability of this MSS failure that is caused by $\boldsymbol{p}$ system components efficiencies decrease [5]. These indices are calculated as:

$$P_f(\boldsymbol{x^{(p)}}) = (\rho_f / \rho_1) \cdot \prod_{j=1}^{p} p_{i_j, a_j}\ , \qquad (7)$$

where $\rho_f$ – number of system states when the value of variables vector $\boldsymbol{x^{(p)}}$ of MSS structure function changes from $\boldsymbol{a^{(p)}}$ to $\tilde{\boldsymbol{a}}^{(\boldsymbol{p})}$ and forces the system failure (5):

$$\rho_f \equiv \partial\phi(1\to 0)/\partial \boldsymbol{x^{(p)}}(\boldsymbol{a^{(p)}} \to \tilde{\boldsymbol{a}}^{(\boldsymbol{p})}) \neq 0, \qquad (8)$$

$\rho_1$ – number of system states for which $\phi(\boldsymbol{x})$=1 and $x_{i_j} = a_{i_j}$ ($j$ =1, …, $p$); $p_{i_j, a_j}$ – component state probability in (2).

***Definition 2***. CDRIs for a MSS repair are probability of this MSS repair that is caused by replacements of $p$ failure system components [5]. An equation for computation of these indices is:

$$P_r(\boldsymbol{x^{(p)}}) = \left( \sum_{h=1}^{m-1} \rho_r^{(h)} \Big/ \rho_0 \right) \cdot \prod_{j=1}^{p} p_{i_j, 0}\ , \qquad (9)$$

where $\rho_r^{(h)}$ – number of system states when the vector $\boldsymbol{x^{(p)}}$ value change from zero to ($m$-1) forces the system repair and is calculated by Direct Partial Logic Derivative (6):

$$\rho_r^{(h)} \equiv \partial\phi(0\to h)/\partial \boldsymbol{x^{(p)}}(\boldsymbol{0} \to (\boldsymbol{m}\text{-}\boldsymbol{1})) \neq 0, \qquad (10)$$

$\rho_0$ – number of zero system states ($\phi(\boldsymbol{x})$ = 0 and $x_{i_j} = 0$ for $j$ =1, …, $p$), $p_{i_j,0}$ = probability of component state that are declared in (2).

***Definition 3.*** DIRIs is probability of the system failure or repairing if one of system component fails or restores:

$$P_f = \sum_z P_f(\boldsymbol{x^{(p)}}) \prod_{z-1} \left(1 - P_f(\bar{\boldsymbol{x}}^{(\boldsymbol{p})})\right), \qquad (11)$$

$$P_r = \sum_z P_r(\boldsymbol{x^{(p)}}) \prod_{z-1} \left(1 - P_r(\bar{\boldsymbol{x}}^{(\boldsymbol{p})})\right), \qquad (12)$$

where $P_f(\boldsymbol{x^{(p)}})$ and $P_r(\boldsymbol{x^{(p)}})$ – probabilities that are determined in (7) and (9) accordantly; $\bar{\boldsymbol{x}}^{(\boldsymbol{p})}$ – the variable vector of $p$ variables for which $\bar{\boldsymbol{x}}^{(\boldsymbol{p})} \neq \boldsymbol{x^{(p)}}$; $z$ – number of combinations of $n$ things taken $p$, that determines number of variables vector $\boldsymbol{x^{(p)}}$ for the structure function of MSS which consist of $n$ components.

Let's calculate CDRIs and DIRIs for the 2-out-of-3 MSS for $m$=3 (Table 1). The probabilities of the component state (2) are declared in Table 2.

Derivatives $\partial\phi(1\to 0)/\partial x_i(1\to 0)$ for the system failure and $\partial\phi(0\to 1)/\partial x_i(0\to 2)$, $\partial\phi(0\to 2)/\partial x_i(0\to 2)$ for the system repair ($i$ = 1, 2) are calculated firstly. Then numbers $\rho_f$ and numbers $\rho_r^{(h)}$ for $h$ = {1, 2} are determined in Table 3. The numbers $\rho_1$ and $\rho_0$ are calculated by structure function: $\rho_0$ = 5, $\rho_1$ = 7. The CDRIs for this MSS 2-out-of-3 is calculated by (7) and (9) and are presented in Table 3.

**Table 2. – Component state probability**

| Component | State 0 | 1 | 2 |
|---|---|---|---|
| $x_1$ | 0.1 | 0.6 | 0.3 |
| $x_2$ | 0.4 | 0.5 | 0.1 |
| $x_3$ | 0.2 | 0.2 | 0.6 |

**Table 3.** – CDRIs calculation for the MSS 2-out-of-3

| | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| $\rho_f$ | 4 | 4 | 4 |
| $\rho_r^{(1)}$ | 2 | 2 | 2 |
| $\rho_r^{(2)}$ | 2 | 2 | 2 |
| $P_f(i)$ | 0.185 | 0.154 | 0.062 |
| $P_r(i)$ | 0.058 | 0.230 | 0.116 |

The analysis of Table 3 is shown. The 1-st component is cause the system failure at most because the CDRIs for this component $P_f(1)$=0.185 is the largest. The probability of the system failure after the failure of the 3-rd component is minimum ($P_f(3)$=0.062). The 2-nd component permits to repair the system by highest possible probability $P_r(2)$=0.230.

DIRIs for this MSS (11) and (12) are: the probability of the system failure when a change in the state of one of the component that is $P_f$=0.308 and the probability of the MSS repairing above by replacement of the failure component that is $P_r$=0.315.

## 4. APPLICATION OF DRIs

Consider some examples of a MSS analysis by CDRIs and DIRIs. The first example presented application of DRIs in Human Reliability Analysis. In paper [15] the examination of human errors by these indices is in more detail.

*Example 1*. In paper [15] considered system 2-out-of-3:G (Fig.2) with hardware failure events $C_i$ and consecutive human errors events $H_i$ (I = 1, 2, 3). The system fails if two or more trains (events $C_i$ and $H_i$) fail.

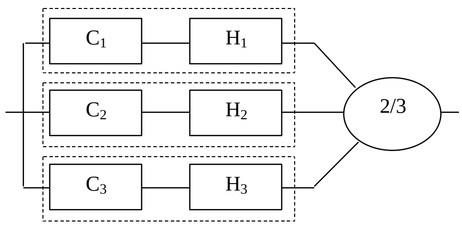

**Fig. 2. – A system with hardware failure events $C_i$ and consecutive human errors events**

The system includes 6 components with 3 level of availability (the probabilities of components states are in Table 4) and declared by structure function:

$$\phi(\mathbf{x}) = x_1x_4\, x_2x_5 \vee x_1x_4\, x_3x_6 \vee x_2x_5\, x_3x_6. \qquad (13)$$

**Table 4. Component state probability for system in Fig.2 (*m*=3)**

| State | Component | | | | | |
|---|---|---|---|---|---|---|
| | $x_1$ ($C_1$) | $x_2$ ($C_2$) | $x_3$ ($C_3$) | $x_4$ ($H_1$) | $x_5$ ($H_2$) | $x_6$ ($H_3$) |
| 0 | 0.03 | 0.03 | 0.03 | 0.12 | 0.26 | 0.17 |
| 1 | 0.28 | 0.28 | 0.28 | 0.26 | 0.26 | 0.20 |
| 2 | 0.69 | 0.69 | 0.69 | 0.62 | 0.48 | 0.63 |

Consider this system failure depending on human error events. These events agree with variables $x_4$, $x_5$ and $x_6$ in structure function (13). CDRIs for these variables (events) are:

$$P_f(x_4) = 0.131024,\ P_f(x_5) = 0.179310,\ P_f(x_6) = 0.068376.$$

Therefore the maximal probability of the system failure correspond to event $H_2$ and it is $P_f(x_5) = 0.179310$.

Consider this system failure depending on breakdown or availability decrease of two system components if one of events is human error ($H_i$). These CDRIs for the system are in Table 5.

**Table 5. CDRIs of the system in Fig.2 failure**

| Components | | Components changes | | CDRI $P_f(\mathbf{x}^{(p)})$ |
|---|---|---|---|---|
| $x_i$ | $x_j$ | $x_i$ | $x_j$ | |
| $x_1$ | $x_4$ | $1 \to 0$ | $1 \to 0$ | 0.043955 |
| $x_1$ | $x_4$ | $1 \to 0$ | $2 \to 1$ | 0.104815 |
| $x_2$ | $x_4$ | $1 \to 0$ | $1 \to 0$ | 0.072800 |
| $x_2$ | $x_4$ | $1 \to 0$ | $2 \to 1$ | 0.104815 |
| $x_3$ | $x_4$ | $1 \to 0$ | $1 \to 0$ | 0.072800 |
| $x_3$ | $x_4$ | $1 \to 0$ | $2 \to 1$ | 0.104815 |
| $x_1$ | $x_5$ | $1 \to 0$ | $1 \to 0$ | 0.072800 |
| $x_1$ | $x_5$ | $1 \to 0$ | $2 \to 1$ | 0.057191 |
| $x_2$ | $x_5$ | $1 \to 0$ | $1 \to 0$ | 0.056000 |
| $x_2$ | $x_5$ | $1 \to 0$ | $2 \to 1$ | 0.103385 |
| $x_3$ | $x_5$ | $1 \to 0$ | $1 \to 0$ | 0.072800 |
| $x_3$ | $x_5$ | $1 \to 0$ | $2 \to 1$ | 0.086563 |
| $x_4$ | $x_5$ | $1 \to 0$ | $1 \to 0$ | 0.067600 |
| $x_4$ | $x_5$ | $1 \to 0$ | $2 \to 1$ | 0.053106 |
| $x_4$ | $x_5$ | $2 \to 1$ | $1 \to 0$ | 0.097328 |
| $x_4$ | $x_5$ | $2 \to 1$ | $2 \to 1$ | 0.000000 |
| $x_1$ | $x_6$ | $1 \to 0$ | $1 \to 0$ | 0.032000 |
| $x_1$ | $x_6$ | $1 \to 0$ | $2 \to 1$ | 0.100800 |
| $x_2$ | $x_6$ | $1 \to 0$ | $1 \to 0$ | 0.056000 |
| $x_2$ | $x_6$ | $1 \to 0$ | $2 \to 1$ | 0.100800 |
| $x_3$ | $x_6$ | $1 \to 0$ | $1 \to 0$ | 0.040727 |
| $x_3$ | $x_6$ | $1 \to 0$ | $2 \to 1$ | 0.128291 |
| $x_4$ | $x_6$ | $1 \to 0$ | $1 \to 0$ | 0.029714 |
| $x_4$ | $x_6$ | $1 \to 0$ | $2 \to 1$ | 0.093600 |
| $x_4$ | $x_6$ | $2 \to 1$ | $1 \to 0$ | 0.052766 |
| $x_4$ | $x_6$ | $2 \to 1$ | $2 \to 1$ | 0.000000 |
| $x_5$ | $x_6$ | $1 \to 0$ | $1 \to 0$ | 0.052000 |
| $x_5$ | $x_6$ | $1 \to 0$ | $2 \to 1$ | 0.093600 |
| $x_5$ | $x_6$ | $2 \to 1$ | $1 \to 0$ | 0.018732 |
| $x_5$ | $x_6$ | $2 \to 1$ | $2 \to 1$ | 0.000000 |

The main area of applications in mind for the results has been the unavailability analysis of redundant standby safety systems that are periodically tested, calibrated or maintained. The probabilities obtained here are relevant input to system models. The current formalism applies also for calculating the probability of a plant transient (initiating event) if such is caused by repeated operator errors.

In paper [16] the reliability analysis of logical networks and logical gates by DRIs is presented. Next example illustrates this application of DRIs.

*Example 2*. Consider the logical network ($n$ = 3) in Fig. 3 that is declared by logical function:

$$F(y_1, y_1, y_2,) = F(\mathbf{y}) = y_1y_2, \vee y_1y_2 \qquad (14)$$

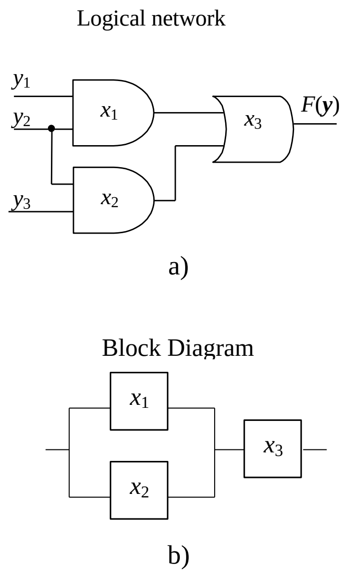


**Fig.3. – Logical network (*a*) and graphical interpretation of its structure function by Block Diagram**

The structure function is different form logical function $F(y)$ that is realized by logical network. The structure function describes the topology of a logical network and can be equal to logical function $F(y)$ in some time [16, 17].

In this example the structure function of the logical network is:

$$\phi(\mathbf{x}) = (x_1 \vee x_2)\, x_{3.} \qquad (15)$$

Consider this network as MSS of three components ($n$ = 3) with three levels of components availability ($m$ = 3). The probabilities of components states are in Table 6.

**Table 6. Component state probability for bridge MSS (*m*=3)**

| State | Component | | |
|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ |
| 0 | 0.03 | 0.12 | 0.03 |

| | | | |
|---|---|---|---|
| 1 | 0.28 | 0.26 | 0.26 |
| 2 | 0.69 | 0.62 | 0.71 |

CDRIs for repair estimation of this logical network are in Table 7.

**Table 7. CDRIs for the logical network repair that is caused by replacement of failed components**

| Components | | CDRI $P_r(\mathbf{x}^{(p)})$ |
|---|---|---|
| $x_1$ | | 0.460000 |
| $x_2$ | | 0.413333 |
| $x_3$ | | 0.710000 |
| $x_1$ | $x_2$ | 0.427800 |
| $x_1$ | $x_3$ | 0.489900 |
| $x_2$ | $x_3$ | 0.440200 |

Therefore probabilities of the logical network restoration are in Table 7 for replacement of failed gate or two failed gates.

The last example illustrates analysis of social system reliability.

*Example 3*. Consider the system (Fig.4, a) with two working groups: ($x_1$, $x_4$) and ($x_2$, $x_5$). Every of these groups has one communication language. It is English for the first group ($x_1$, $x_4$) and it is Russian for the second group ($x_2$, $x_5$). There is the interpreter in this system ($x_3$). Every working group includes programmer ($x_1$) and ($x_2$) and economist ($x_4$) and ($x_5$). In paper [2] the similar system is interpreted as "bridge" system (Fig.4, b).

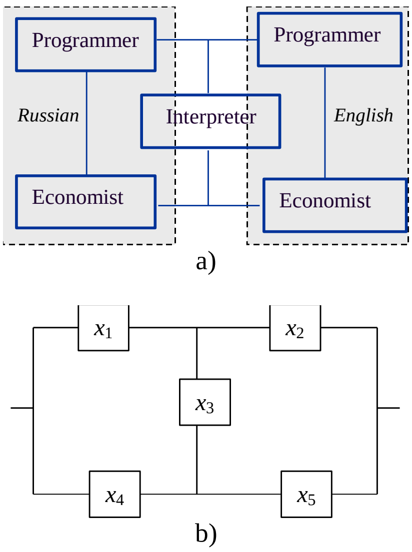


**Fig. 4. – Example of the social system (a) and its interpretation in terms of MSS reliability analysis (b)**

Consider this system as the MSS ($m$ = 4) with 5 components ($n$ = 5). The structure function of this MSS is:

$$\phi(\mathbf{x}) = x_1\, x_2 \vee x_1\, x_3\, x_5 \vee x_4\, x_5 \vee x_4\, x_3\, x_2, \qquad (16)$$

The probabilities of components states are in Table 8.

DIRIs for this system failure are calculated by (11) and CDRIs are determined according to (12). CDRIs for this system are:

$$P_f(x_1) = 0.100892,\ P_f(x_2) = 0.079873,\ P_f(x_3) = 0.030265,$$
$$P_f(x_4) = 0.050446,\ P_f(x_5) = 0.088280$$

and are in Table 9 for two and three components break.

**Table 8. Component state probability for bridge MSS ($m$=4)**

| State | Component | | | | |
|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ |
| 0 | 0.03 | 0.12 | 0.03 | 0.02 | 0.02 |
| 1 | 0.24 | 0.19 | 0.19 | 0.12 | 0.21 |
| 2 | 0.29 | 0.28 | 0.31 | 0.37 | 0.3 |
| 3 | 0.44 | 0.41 | 0.47 | 0.49 | 0.47 |

**Table 9. CDRIs for the bridge MSS failure that is caused by breakdowns of two and three system components for MSS with *m*=4**

| ***p*** = 2 (two system components breakdowns) | | | ***p*** = 3 (three system components breakdowns) | | | |
|---|---|---|---|---|---|---|
| Components | | CDRI $P_f(\mathbf{x}^{(p)})$ | Components | | | CDRI $P_f(\mathbf{x}^{(p)})$ |
| $x_i$ | $x_j$ | | $x_i$ | $x_j$ | $x_s$ | |
| $x_1$ | $x_2$ | 0.026600 | $x_1$ | $x_2$ | $x_3$ | 0.005054 |
| $x_1$ | $x_3$ | 0.024873 | $x_1$ | $x_2$ | $x_4$ | 0.005472 |
| $x_1$ | $x_4$ | 0.032989 | $x_1$ | $x_2$ | $x_5$ | 0.009576 |
| $x_1$ | $x_5$ | 0.028800 | $x_1$ | $x_3$ | $x_4$ | 0.005472 |
| $x_2$ | $x_3$ | 0.019691 | $x_1$ | $x_3$ | $x_5$ | 0.009576 |
| $x_2$ | $x_4$ | 0.014924 | $x_1$ | $x_4$ | $x_5$ | 0.006048 |
| $x_2$ | $x_5$ | 0.039900 | $x_2$ | $x_3$ | $x_4$ | 0.004332 |
| $x_3$ | $x_4$ | 0.012436 | $x_2$ | $x_3$ | $x_5$ | 0.007581 |
| $x_3$ | $x_5$ | 0.021764 | $x_2$ | $x_4$ | $x_5$ | 0.004788 |
| $x_4$ | $x_5$ | 0.014700 | $x_3$ | $x_4$ | $x_5$ | 0.002793 |

The MSS failure will be most possible if the first and the fourth components or the second and fifth components break down, because in this case CDRIs have maximum values $P_f(\mathbf{x}^{(2)}) = P_f(x_1, x_4) = 0.032989$ or $P_f(\mathbf{x}^{(2)}) = P_f(x_2, x_5) = 0.039900$.

The first, the second and the fifth components fail or the first, the third and the fifth components break have maximal influence to the failure of investigation system: $P_f(\mathbf{x}^{(3)}) = P_f(x_1, x_2, x_5) = 0.009576$ and $P_f(\mathbf{x}^{(3)}) = P_f(x_1, x_3, x_5) = 0.009576$.

Therefore, DIRIs for MSS failure are $P_f = 0.191297$ if two system components fail and $P_f = 0.057497$ if three system components fail.

## 5. CONCLUSION

In this paper a method for MSS reliability analysis and its application is considered. This method allows to estimate MSS reliability based on DRIs. The DRIs reveal changes of some efficiency of system components and the influence of these changes on the system reliability. These indices are similar to importance measure [14]. Comparison the DRIs and others importance measures are considered in paper [18]. But mathematical approach for estimation of MSS reliability by DRIs is universal and can be used for different real-world system. It illustrates examples for application of MSS reliability analysis by the DRIs in this paper. Note the CDRIs and DIRIs can be used for reliability (availability) estimation of different types systems, for example, technical or social.

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