Image Representations in Image Analysis

I. Gurevich, V. Yashina
Dorodnicyn Computing Centre of the Russian Academy of Sciences, Vavilov st. 40, 119333 Moscow, Russian Federation, igourevi@ccas.ru, werayashina@gmail.com

Abstract: *At present, in terms of development of image analysis and recognition, it is critical to understand the nature of the initial information, viz. images, find methods of image representation and description to be used to construct image models designed for recognition problems, establish the mathematical language for the unified description of image models and their transformations that allow constructing image models and solving recognition problems, construct models to solve recognition problems in the form of standard algorithmic schemes that allow, in the general case, moving from the initial image to its model and from the model to the sought solution. The* descriptive approach to image analysis (*DAIA) gives a single conceptual structure that helps develop and implement these models and the mathematical language. A system of concepts characterizing initial data (i.e., images) in image recognition problems is described. This system underlies the formal definition of methods for synthesizing image models and descriptive image models intended for image recognition and analysis.*

***Keywords*:** image mining, image models, descriptive approach to image analysis.

1. INTRODUCTION

This paper describes research results related to the Descriptive Theory for Image Analysis (DTIA), which was proposed and substantiated by I.B. Gurevich and is being developed by members of his scientific school [1, 2]. The main contribution of this paper is that it describes a system of concepts characterizing initial data (images) in recognition problems and uniquely defines a hierarchical system of relations introduced on classes of concepts. This system underlies the formal definition of methods for synthesizing image models and descriptive image models intended for image recognition problems.

At present, the crucial points in the development of image analysis and recognition include the understanding of the nature of initial data (images); methods for image representation and description that provide the construction of image models for image recognition problems; a mathematical language for a unified description of image models and their transformations ensuring the construction of image models and the solution of image recognition problems; and models for solving image recognition problems in the form of standard algorithmic schemes that, in the general case, ensure transitions from the original image to its model and from the latter to a desired solution.

DTIA specifies a unified conceptual structure for the development and implementation of such models and mathematical language. The main goal of DTIA is to represent various methods, operations, and representations used in image analysis and recognition in a structured and standardized form. DTIA provides the conceptual and mathematical foundation for Image Mining. The axiomatics and formal structures of DTIA provide methods and tools for representing and describing images for their subsequent analysis and estimation.

Overall, the following results were obtained by analyzing and adapting descriptive image models (DIMs) for automated image analysis and recognition: (1) a system of concepts was introduced that characterizes initial data (images) in image recognition problems; (2) a system of concepts was introduced that characterizes and defines DIMs for image recognition; (3) a hierarchical scheme was constructed that represents and unifies the conceptual apparatus, definitions, mathematical objects, and transformations defining the construction of an image model in image recognition based on DTIA; (4) a scheme for constructing image models was proposed; and (5) schemes for constructing four classes of DIMs were introduced.

The system of concepts introduced underlies a standard specification of methods for synthesizing image models and descriptive image models for image analysis and recognition.

Section 2 describes the concepts required for a formal description and representation of images in DTIA. Additionally, we consider the structures and mathematical objects used to reproduce an image in the construction of descriptive models. In Section 3, we formalize methods for characterizing images, transformations, and objects used to describe images in a form acceptable for recognition algorithms.

2. IMAGES AS INITIAL DATA IN IMAGE RECOGNITION PROBLEMS

To develop automated image recognition methods, we need techniques for effective image formalization that reflects the image semantics and the information carried by the internal image structure and the structure of external connections in the actual world part (scene) reproduced by the image. No systematic mathematical methods for image formalization and analysis are available at present. The overwhelming majority of image-related methods are heuristic, and their merits are determined by how effectively the pictorial nature of images is overcome by nonpictorial tools.

An image is an object with a complex information structure reproducing information on the original scene using the brightness of discrete image elements (pixels); configurations of image fragments and sets of pixels; and spatial and logical relations between configurations, sets of pixels, and individual pixels. In contrast to other ways of data representation, images are highly informative, illustrative, structured, and naturally perceived by humans. An image is a mixture of original (raw, "actual") data, their representations, and deformations arising in the formation and transformation of digital images. Representations reflect the information and physical nature of objects, events, and processes represented by images, while deformations are caused by the technical characteristics of tools used to record, form, and transform images in the construction of hierarchical representations. Thus, while developing methods for a formal description of images, we have to take into account not only the brightness of image pixels but also additional explicit and implicit information associated with images.

Apparently, while perceiving an image, a human does not create its verbal description but handles it as an integrated pattern or a system of patterns, resorting to a nonverbal internal representation [2-4]. The development of techniques and systems for automated image recognition includes the creation of methods for effective image formalization so that we can deal with representations (descriptions) reflecting the context and semantics of an image and information carried by its internal structure and the structure of external connections in the actual world part (scene) reproduced by the image.

A natural hypothesis underlying the formalization of image descriptions and its conceptual apparatus is that an original image is specified not only by a set of its digital realizations but also by contextual and semantic information associated with the methods used for image recording and formation or with some specific aspects of the image.

To construct a formalized description of an image, transformations admissible for the given type of images have to be applied to the entire information available on the image. Thus, it is a necessity to study (i) the types of information carried by the image (the space of initial data) and (ii) the transformations to be applied to the original image to reduce it to a form acceptable for recognition algorithms (the space of transformations).

The descriptions of procedures for serial and/or parallel applications of transformations form the space of transformations to initial data from the space of initial data constitute a set of schemes for constructing formal descriptions of images (the space of image representations).

To ensure that recognition algorithms can be applied to the resulting formal image descriptions, the schemes constructed (image representations) have to be implemented; i.e., image models have to be constructed by reducing the original image (with allowance for the entire information on it) to a form acceptable for recognition algorithms. The space of image representations is intermediate between the space of initial data and the space of image models.

Thus, the construction of image models involves the synthesis and application of objects from the set of initial data (i.e., images), the set of image transformations for Reducing Images to a Recognizable Form (RIRF), the set of image representations (i.e., schemes for constructing formal image descriptions), and the set of image models.

The diagram illustrates the synthesis of image models by applying transformations whereby images pass from the original to final set. Each image from the set of images is described by two subsets that represent the image itself (image realizations) and the semantic and related context information carried by the image.

DTIA deals with three classes of admissible image transformations: procedural, parametric, and generating transformations (see Definitions 2-4 below). These classes of transformations generate three classes of image representations and three classes of image models, respectively.

The following concepts are used to characterize images in DTIA: initial data (image as a whole with its legend), transformations of initial data, representations of initial data (by a representation, we mean a formal scheme for describing an image and the objects it involves), and models of initial data (by an image model, we mean an image description acceptable for recognition algorithms).

Additional objects are introduced to define the types of representations through which an original image goes in the course of image model construction and to establish the relations between these types. These additional objects include rule-generating structuring elements, semantic and contextual information on an image, digital image realizations, classes of image representations, realizations of image representations, classes of image models, and a correct image model.

The following relations between the objects have been revealed by analyzing the basic concepts related to image description construction:

(1) There are deterministic (obvious) relations between the initial data (image) and

- the transformations applied to it;
- methods for obtaining its digital realizations;
- the results of applying transformations to digital image realizations.

(2) There are relations inherent in DTIA:

- between the classes of image transformations and the classes of admissible image representations;
- between the classes of admissible image representations and the classes of image descriptions in a form acceptable for recognition algorithms (classes of image models).

(3) Special relations were revealed

- between some classes of image models;
- between some class of image models and initial data;
- between the results of applying transformations to digital image realizations and the classes of image models.

The study of these relations has led to the construction of a hierarchy of DTIA concepts. According to the hierarchical scheme shown in Fig. 1, the concepts introduced can be structured so that they can be used to develop algorithmic schemes for image analysis and recognition and to describe images with the help of DTIA. Based on the hierarchy, several axioms of DTIA were formulated in [1].

The scheme constructed reflects several levels of relations between DTIA concepts.

I. An open arrow denotes the relation whereby object 1 is associated with objects 2, 3, …: the original image I is associated with the following three sets: 1) the set of transformations $\{\tilde{O}\}$ (the set of structuring elements $\{\tilde{S}\}$ is auxiliary for $\{\tilde{O}\}$), 2) the set of initial data $\{\tilde{I}_0\}$; 3) the set of image models $\{\tilde{M}\}$.

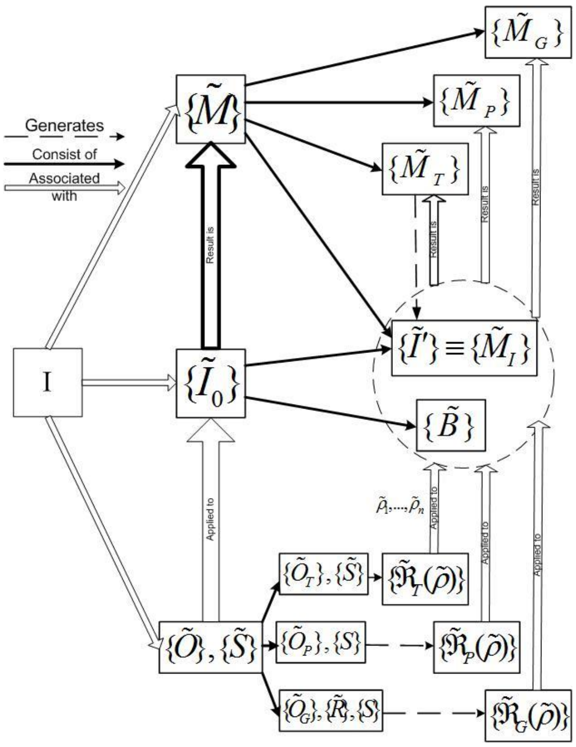

Fig. 3. Hierarchy of DTIA concepts.

II. The application of $\{\tilde{O}\}$ to $\{\tilde{I}_0\}$ generates a set $\{\tilde{M}\}$ of correct image models (this assertion is proved in Theorem 1).

III. A solid heavy arrow denotes the relation "object 1 consists of object 2, object 3…" 1) The set of transformations $\{\tilde{O}\}$ consists of three subsets: procedural transformations $\{\tilde{O}_T\}$, parametric transformations $\{\tilde{O}_P\}$, and generating transformations $\{\tilde{O}_G\}$. Moreover, $\{\tilde{O}\}$ (as well as its subsets $\{\tilde{O}_T\}$, $\{\tilde{O}_P\}$, $\{\tilde{O}_G\}$) is specified together with the set $\{\tilde{S}\}$ of structuring elements, which can be applied to the image together with transformations. The rules for applying generating transformations from $\{\tilde{O}_G\}$ to initial data from $\{\tilde{I}_0\}$ are described by a set of generating rules $\{\tilde{R}\}$. Thus, a subset of $\{\tilde{O}_G\}$ is associated with a fixed subset of $\{\tilde{R}\}$. 2) The set of image models $\{\tilde{M}\}$ consists of four subsets: procedural image models $\{\tilde{M}_T\}$, parametric image models $\{\tilde{M}_P\}$, generating image models $\{\tilde{M}_G\}$, and image I-model $\{\tilde{M}_I\}$. 3) The initial data $\{\tilde{I}_0\}$ contain contextual and semantic information $\{B\}$ on the image and the set of realizations $I' \in \{\tilde{I}'\}$ of I that represent the given object or scene.

IV. A dashed line denotes the relation "an object generates another object". 1) The three classes $\{\tilde{O}_T\}$, $\{\tilde{O}_P\}$, $\{\tilde{O}_G\}$ of transformations generate the following three classes of image representations: procedural representations $\{\Re_T(\tilde{\rho})\}$, parametric representations $\{\Re_P(\tilde{\rho})\}$, and generating representations $\{\Re_G(\tilde{\rho})\}$. 2) It can be proved that any image T-model $M_T \in \{\tilde{M}_T\}$ generates an image realization $I' \in \{\tilde{M}_T\}$ (see Proposition 1).

V. The dashed circle in the scheme stresses that the image realizations $\{\tilde{I}'\}$ are related to the semantic and context information $\{B\}$. This relation means that the various types of initial data are used together in solving problems.

3. DESCRIPTIVE IMAGE MODELS

In DTIA it is assumed [1] that an image is described by a set of initial data $\{\tilde{I}_0\}$. The composition of this set is determined below.

Lemma 1: The set $\{\tilde{I}_0\}$ of initial data consists of two subsets $\{\tilde{I}'\}$ and $\{\tilde{B}\}$: (1) the set of realizations $I' \in \{\tilde{I}'\}$ of *I* representing the given object or scene such that $I' = \{(x, f(x))\}_{x \in D_f}$ is the set of points x lying in the domain D_f of the image realization and the set of values $f(x)$ at each point of D_f; and (2) semantic and contextual information $\{\tilde{B}\}$ on the image.

The domain of an image realization is a subset of the n-dimensional discrete space Z^n. For two-dimensional images, n = 2.

Lemma 1 is illustrated in Fig. 4.

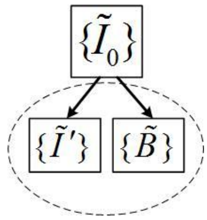

Fig. 4. The set of initial data.

Definition 1: An **I-model** of an image is any element I' of a set $\{\tilde{I}'\}$ of image realizations.

Fig. 5 illustrates the relation between a set of image I-models and a set of image realizations.

$$\boxed{\{\tilde{I}'\} \equiv \{\tilde{M}_I\}}$$

Fig. 5. Identity of image I-models to image realizations.

Consider a set of transformations $\{\tilde{O}\}$ introduced over data given in the form of images.

In this section, we define the basic classes of image transformations (procedural, parametric, and generating) and introduce the related concepts of a structuring element, a generating rule, and a correct generating transformation.

Definition 2: A **procedural transformation** $O_T \in \{\tilde{O}_T\}$ **of arity *r*** over a set of images $\{I_i\}_{1...r}$ is an operation such that its application to $\{I_i\}_{1...r}$ transforms it into another set of images, into an image, or into image fragments.

Accordingly, a procedural transformation $O_T \in \{\tilde{O}_T\}$ of arity *r* over a set of image I-models $\{I'_i\}_{1...r}$ is an operation such that its application to $\{I'_i\}_{1...r}$ transforms it into another set of image I-models, into an image I-model, or into a set of I-models of image fragments. The operands of this operation can be I-models of a single original image or I-models of several different original images.

Definition 3: A **parametric transformation** $O_P \in \{\tilde{O}_P\}$ over an image *I* is an operation such that its application to *I* transforms it into a numerical characteristic *p* that correlates with the properties of geometric objects, brightness characteristics, or configurations formed by regular repetitions of the geometric objects and brightness characteristics of the original image.

To calculate a numerical characteristic *p* of *I,* we can use both the set of image realizations and semantic or contextual information on the image.

Definition 4: A **generating transformation** $O_G \in \{\tilde{O}_G\}$ over an image *I* is an operation generating a particular representation of *I* that reflects some specific properties of *I.*

For the definition of a representation, see Definition 8.

Examples of such transformations are functions describing curves, the conjunction function, the disjunction function, and image coding functions.

Definition 5: A **generating rule** R is a rule for constructing an image model that determines a strict sequence of generating transformations applied to the image in order to construct its model.

For the definition of an image model, see Definition 9.

Definition 6: A **generating transformation** O_G is **correct** for a given image if and only if there are generating rules according to which O_G ensures the construction of a generating image model.

Note that a generating model of an image (image G-model) is constructed using a realization of a generating image representation (G-representation). For the definitions of a generating representation and a realization of a generating representation, see Definitions 17 and 18.

Definition 7: A **structuring element S**$\in \{\tilde{S}\}$ is a two-dimensional spatial object whose convolution with an image yields a partition of the image into a system of fragments suitable for local analysis. A structuring element is specified by parameters defining its form and numerical and geometric characteristics.

Definition 8: An **image representation** $\Re(I)$ is a formal scheme for obtaining a standardized formal description of the surfaces, point configurations, and shapes forming the image and the relations between them.

Definition 9: An **image model** $M(I)$ is a formal image description generated by a realization of an image representation $\Re(I)$.

Definition 10: A realization **of an image representation** is the application of the representation to realizations of the original image with particular parameter values specified for the transformations involved in the representation.

The construction of an image model with the help of the objects and concepts introduced above is shown schematically in Fig. 6.

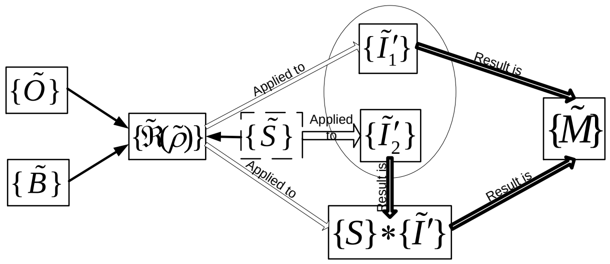

Fig. 6. Construction of an image model.

Definition 11: A **correct representation of an image** *I* is an element of the set of image representations constructed from contextual and semantic information $\{\tilde{B}\}$ by applying transformations $\{\tilde{O}\}$ and structuring elements $\{\tilde{S}\}$, where the sets $\{\tilde{B}\}$, $\{\tilde{O}\}$, and $\{\tilde{S}\}$ are associated with *I* and the set $\{\tilde{S}\}$ may be empty.

Definition 12: A **correct image model** is an element of a set of image models generated by implementing correct image representations on the set of initial data $\{\tilde{I}_0\}$.

Theorem 1: Any element *m* of the set $\{\tilde{M}\}$ generated by applying transformations from $\{\tilde{O}\}$ to the set of initial data $\{\tilde{I}_0\}$ is a **correct image model**.

Corollary. Given an image *I*, the set of correct models of *I* is closed under transformations from $\{\tilde{O}\}$ as applied to the set of initial data $\{\tilde{I}_0\}$ with the use of structuring elements from the set $\{\tilde{S}\}$ associated with *I*.

The proof of this corollary is based on Definitions 11 and 12.

All transformations applied to an image model or an image are introduced to achieve one of the following goals: the construction of a new model; RFSR; or the construction of an aggregate model estimate, i.e., the transition from the space of initial data to a space of estimates on which classifying decision making procedures are implemented in image recognition.

Schemes (1) and (2) illustrate the relations between image representations and image models.

$$\{\tilde{\Re}(I)\} = \{\tilde{O}, \tilde{S}\} : \{I\} \Rightarrow \{\tilde{M}\} \qquad (1)$$

$$\{\Re(I)\}(p) = \{O_1, O_2, ..., O_n, \tilde{S}'\}(p) : \{I'\} \in \{I\} \Rightarrow M_1 \in \{\tilde{M}\} \qquad (2)$$

Definition 13: A **T-representation** $\Re_T(\overline{\eta}, \overline{\mu})$ of an image *I* is a formal scheme for deriving a standardized formal description of *I.* This scheme is constructed from contextual and semantic information $\{B\} \subset \{\tilde{B}\}$ by applying procedural transformations $\{O_T(\bar{\eta})\} \subset \{\tilde{O}_T\}$ and structuring elements $\{S(\bar{\mu})\} \subset \{\tilde{S}\}$ (where $\bar{\eta}, \bar{\mu}$ are parameters of the procedural transformations and the structuring elements, respectively).

The set of all correct T-representations is denoted by $\{\tilde{\Re}_T(\tilde{\eta}, \tilde{\mu})\}$.

Scheme (3) illustrates Definition 13.

$$\left\{\{\tilde{S}\},\{\tilde{O}\}\right\} \xrightarrow{\{B\}} \left\{\{S(\bar{\mu})\},\{O_T(\bar{\eta})\}\right\} \longrightarrow \Re_T(\bar{\eta},\bar{\mu}) \quad (3)$$

Definition 14: A realization of a T-representation $\Re_T(\bar{\eta}, \bar{\mu})$ of an image I is the application of $\Re_T(\bar{\eta}_0, \bar{\mu}_0)$ to realizations $\{I'\} \subset \{\tilde{I}'\}$ of I, where ($\bar{\eta} = \bar{\eta}_0$, $\bar{\mu} = \bar{\mu}_0$) are parameter values chosen for the transformations involved in $\Re_T(\bar{\eta}, \bar{\mu})$.

Scheme (4) illustrates Definition 14.

$$\Re_T(\bar{\eta}_0, \bar{\mu}_0) * \{I'\} = \{S(\bar{\eta}_0), O_T(\bar{\mu}_0)\} * \{I'\} \quad (4)$$

Proposition 1: Any T-representation $\Re_T(\bar{\eta}, \bar{\mu}) \in \{\tilde{\Re}_T(\tilde{\eta}, \tilde{\mu})\}$ of an image generates a set of **image T-models** $\{M_T(\bar{\eta}_0, \bar{\mu}_0\}$ by setting parameter values $\bar{\eta} = \bar{\eta}_0$ and $\bar{\mu} = \bar{\mu}_0$ for the procedural transformations and structuring elements.

The set of all correct image T-models is denoted by $\{\tilde{M}_T\}$. Figure 7 illustrates the generation of image T-models.

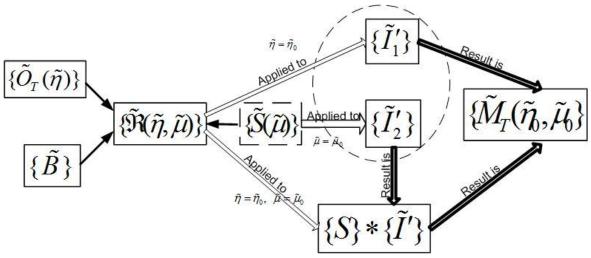

Fig. 7. Generation of image T-models.

Proposition 2: Any image T-model $M_T \in \{\tilde{M}_T\}$ generates an image realization *I'*, i.e., an image I-model $M_I \equiv I' \in \{\tilde{M}_T\} \equiv \{\tilde{I}'\}$.

Proposition 2 is illustrated in Fig. 8.

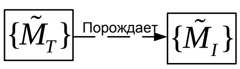

Fig. 8. Generation of an I-model by constructing an image T-model.

Definition 15: A **P-representation** $\Re_P(\bar{\eta}, \bar{\mu})$ of an image *I* is a formal scheme for deriving a standardized formal description of *I*. This scheme is constructed from the contextual and semantic information $\{B\} \subset \{\tilde{B}\}$ by applying parametric transformations $\{O_P(\bar{\eta})\} \subset \{\tilde{O}_P\}$ and structuring elements $\{S(\bar{\mu})\} \subset \{\tilde{S}\}$ (where $\bar{\eta}, \bar{\mu}$ are parameters of the parametric transformations and the structuring elements, respectively).

The set of all correct P-representations is denoted by $\{\tilde{\Re}_P(\tilde{\eta}, \tilde{\mu})\}$. Scheme (5) illustrates Definition 15.

$$\left\{\{\tilde{S}\},\{\tilde{O}\}\right\} \xrightarrow{\{B\}} \left\{\{S(\bar{\mu})\},\{O_P(\bar{\eta})\}\right\} \longrightarrow \Re_P(\bar{\eta},\bar{\mu}) \quad (5)$$

Definition 16: A realization of a P-representation $\Re_P(\bar{\eta}, \bar{\mu})$ of an image I is the application of $\Re_P(\bar{\eta}_0, \bar{\mu}_0)$ to realizations $\{I'\} \subset \{\tilde{I}'\}$ of *I*, where ($\bar{\eta} = \bar{\eta}_0$, $\bar{\mu} = \bar{\mu}_0$) are parameter values chosen for the transformations involved in $\Re_P(\bar{\eta}, \bar{\mu})$.

Scheme (6) illustrates Definition 16.

$$\Re_P(\bar{\eta}_0, \bar{\mu}_0) * \{I'\} = \{S(\bar{\eta}_0), O_P(\bar{\mu}_0)\} * \{I'\} \quad (6)$$

Proposition 3: Any P-representation $\Re_P(\bar{\eta}, \bar{\mu}) \in \{\tilde{\Re}_P(\tilde{\eta}, \tilde{\mu})\}$ generates a set of **image P-models** $\{M_P(\bar{\eta}_0, \bar{\mu}_0\}$ by setting parameter values $\bar{\eta} = \bar{\eta}_0$ and $\bar{\mu} = \bar{\mu}_0$ for the parametric transformations and structuring elements, respectively.

The set of all correct P-models of an image is denoted by $\{\tilde{M}_P\}$. Figure 6 illustrates the generation of image P-models.

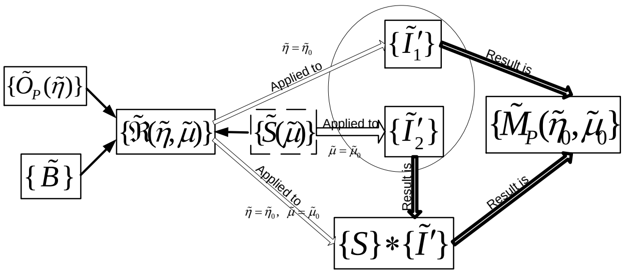

Fig. 9. Generation of image P-models.

Definition 17: A **G-representation** $\Re_G(\bar{\lambda}, \bar{\eta}, \bar{\mu})$ of an image *I* is a formal scheme for deriving a standardized formal description of *I*. This scheme is constructed from the contextual and semantic information $\{B\} \subset \{\tilde{B}\}$ according to generating rules $\{\tilde{R}(\bar{\lambda})\}$ that completely define the sequence of generating transformations $\{O_G(\bar{\eta})\} \subset \{\tilde{O}_G\}$ and structuring elements $\{S(\bar{\mu})\} \subset \{\tilde{S}\}$ applied to *I* (here, $\bar{\lambda}$, $\bar{\eta}$, $\bar{\mu}$ are parameters of the generating rules, generating transformations, and structuring elements, respectively).

The set of all correct G-representations is denoted by $\{\tilde{\Re}_G(\tilde{\lambda}, \tilde{\eta}, \tilde{\mu})\}$. Scheme (7) illustrates Definition 17.

$$\left\{\{\tilde{R}\},\{\tilde{S}\},\{\tilde{O}_G\}\right\} \xrightarrow{\{B\}} \left\{\{R(\bar{\lambda})\},\{S(\bar{\mu})\},\{O_P(\bar{\eta})\}\right\} \longrightarrow \Re_G(\bar{\lambda},\bar{\eta},\bar{\mu})$$

Definition 18: A realization **of a G-representation** $\Re_G(\bar{\lambda}, \bar{\eta}, \bar{\mu})$ of an image *I* is the application of $\Re_P(\bar{\lambda}_0, \bar{\eta}_0, \bar{\mu}_0)$ to realizations $\{I'\} \subset \{\tilde{I}'\}$ of *I*, where

($\bar{\lambda}=\bar{\lambda}_0,\ \bar{\eta}=\bar{\eta}_0,\ \bar{\mu}=\bar{\mu}_0$) are parameter values chosen for the transformations involved in $\Re_G(\bar{\lambda},\bar{\eta},\bar{\mu})$.

Scheme (8) illustrates Definition 18.

$$\Re_G(\bar{\lambda}_0,\bar{\eta}_0,\bar{\mu}_0)*\{I'\}=\{R(\bar{\lambda}_0),S(\bar{\eta}_0),O_G(\bar{\mu}_0)\}*\{I'\} \qquad (8)$$

Proposition 4: Any G-representation $\Re_G(\bar{\lambda},\bar{\eta},\bar{\mu})\in\{\tilde{\Re}_G(\tilde{\lambda},\tilde{\eta},\tilde{\mu})\}$ generates a set of **image G-models** $\{M_G(\bar{\lambda}_0,\bar{\eta}_0,\bar{\mu}_0\}$ by setting parameter values $\bar{\lambda}=\bar{\lambda}_0$, $\bar{\eta}=\bar{\eta}_0$, and $\bar{\mu}=\bar{\mu}_0$ for the generating rules, generating transformations, and structuring elements, respectively.

The set of all correct image G-models is denoted by $\{\tilde{M}_G\}$. Figure 10 illustrates the generation of image G-models.

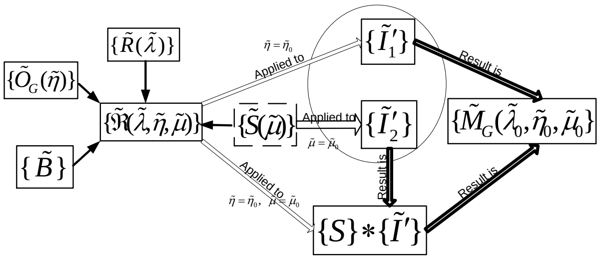

Fig. 10. Generation of image G-models.

The interrelations between the concepts discussed in Section 2 are shown in Fig. 3.

4. CONCLUSION

At present, automated extraction of information from images is a major strategic goal of fundamental research in image analysis, recognition, and understanding. In turn, automation entails the development of a new approach to the analysis and estimation of information represented as images. For this purpose, Yu.I. Zhuravlev's algebraic approach [5] was specialized to initial data represented as images. As a result, DTIA was created.

DTIA was proposed and is being developed as a conceptual and logical foundation for image analysis and recognition. It includes a collection of methods for image analysis and recognition, methods for RIRF, a system of concepts for image analysis and recognition, classes of DIMs, various settings in image analysis and recognition, and a base image recognition model.

The main contribution of this paper is that it describes a system of concepts characterizing initial data (images) in recognition problems and uniquely defines a hierarchical system of relations introduced on classes of concepts. DTIA is being developed as a theoretical foundation for the mathematical theory of image analysis and recognition. Therefore, it is possible to introduce a system of axioms for the desired theory as based on DTIA. Below are examples of such axioms.

Axiom 1: Any image I can be uniquely associated with a collection of sets ($\{\tilde{I}_0\}$, $\{\tilde{O}\}$, $\{\tilde{M}\}$), where $\{\tilde{I}_0\}$ is the set of initial data, $\{\tilde{O}\}$ is the set of transformations applied to $\{\tilde{I}_0\}$, and $\{\tilde{M}\}$ is the set of results produced by applying $\{\tilde{O}\}$ to $\{\tilde{I}_0\}$.

Scheme (9) and Fig. 11 illustrate Axiom 1:

$$\{O\}:(I\cong\{\tilde{I}_o\})\Rightarrow\{\tilde{M}\} \qquad (9)$$

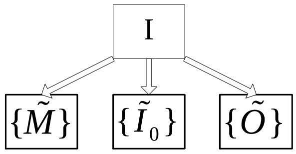

Fig. 11. Illustration of Axiom 1.

Axiom 2: The set of transformations $\{\tilde{O}\}$ is specified by a set $\{\tilde{S}\}$ of structuring elements, by a set $\{\tilde{R}\}$ of generating rules, and by three subsets of transformations $\{\tilde{O}_T\}$, $\{\tilde{O}_P\}$, and $\{\tilde{O}_G\}$, namely, (1) procedural transformations $\{\tilde{O}_T\}$, (2) parametric transformations $\{\tilde{O}_P\}$, and (3) generating transformations $\{\tilde{O}_G\}$.

Figure 12 illustrates Axiom 2.

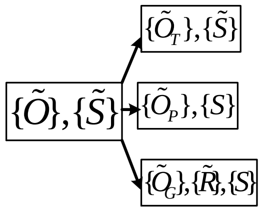

Fig. 12. Illustration of Axiom 2.

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