The electron beam instability in a one-dimensional cylindrical photonic crystal

V.G. Baryshevsky* and E.A. Gurnevich†

Research Institute for Nuclear Problems of Belarussian State University,

Bobruiskaya Str. 11, 220050 Minsk, Belarus

The radiative instability of the relativistic electron beam in a periodic dielectric-filled cylindrical waveguide is considered. The dependence of the beam instability increment on the radiated wave frequency near the region of dispersion equation roots degeneracy is studied. It is shown that sharp change in the instability of the beam under the conditions of two-wave diffraction in Compton generation regime, making the increment proportional to the fourth root from the beam density $\rho^{1/4}$ in contrast to conventional law $\rho^{1/3}$, brings radiation generation (amplification) in the considered system to be essentially improved in comparison with conventional devices (BWO, TWT, FEL etc). Numerical calculations of the instability increment for various parameters of the system are performed.

I. INTRODUCTION

Currently, there is a large number of generators and amplifiers of electromagnetic radiation (from microwave to optical wavelengths range) based on electron beams, for example traveling-wave tubes (TWT), backward wave oscillators (BWO), free electron lasers (FEL), ubitrons etc [1, 2]. It is known that any radiating system is characterized by its dispersion equation describing in the case of small perturbations the possible types of waves in the system. A detailed analysis of the properties of this dispersion equation [3] shows that the gain in the Compton regime (increment of electron beam instability) of the most commonly used generators (TWT's, BWO's, FEL's) is proportional to $\rho^{\frac{1}{3}}$, where ρ is the density of the electron beam. However, in the papers [4, 5] it is found that for electron beam moving

in spatially periodic medium under the conditions providing the coincidence of roots of the dispersion equation there is a new physical law: the increment of instability is proportional to $\rho^{1/(3+s)}$, where s is the number of additional waves appearing due to diffraction in the crystal. The analysis shows that this new law leads to reducing of the electron beam current density, needed to reach lasing threshold. This enables development of a new type of Free Electron Lasers called the Volume Free Electron Lasers (VFEL) [5–7]. Due to a significant change in the threshold conditions, VFEL can provide a more efficient radiation process than conventional generators.

Until recently the theoretical studies of the problem of beam instability in photonic crystals were carried out for the case of infinite in the transverse direction crystals. However, in many cases the mode structure of the electromagnetic field in laser cavities can not be neglected (for example, when the lasing is performed in the microwave range). For the first time the process of VFEL lasing in photonic crystal finite in transverse direction is considered in [8] . In [8] it is also pointed out that the four-fold degeneracy of the dispersion equation roots (when increment is $\sim \rho^{1/4}$) is possible in the one-dimensional photonic crystal when the condition $\varepsilon_0 > 1$ (in the conventional generators - BWO, TWT etc. ε_0 is usually equal to 1) is fullfilled. In this paper we consider the simplest example of such one-dimensional photonic crystal – a cylindrical waveguide with a periodic dielectric filling.

II.   BASIC FORMULAS

The system of equations describing the interaction of an electron beam with an electro-magnetic wave in a waveguide can be obtained from the Maxwell and electron movement equations. We rewrite the Maxwell equations as follows

$$\text{rot rot } \vec{E}(\vec{r},\omega) - \frac{\omega^2}{c^2}\varepsilon(\vec{r},\omega)\vec{E}(\vec{r},\omega) = \frac{4\pi i\omega}{c^2}\vec{j}(\vec{r},\omega), \tag{1}$$

$$\text{div }\varepsilon(\vec{r},\omega)\vec{E}(\vec{r},\omega) = 4\pi\rho(\vec{r},\omega), \tag{2}$$

$$i\omega\rho(\vec{r},\omega) - \text{div }\vec{j}(\vec{r},\omega) = 0, \tag{3}$$

where $\vec{E}(\vec{r},\omega) = \int \vec{E}(\vec{r},t)e^{i\omega t}dt$ is the Fourier transformation of the electric field $\vec{E}(\vec{r},t)$; $\varepsilon(\vec{r},\omega)$ is the dielectric permittivity of the medium filling waveguide; $\vec{j}(\vec{r},\omega)$ and $\rho(\vec{r},\omega)$ are Fourier transformations of the current density $\vec{j}(\vec{r},t)$ and electric charge density of the beam

$\rho(\vec{r}, t)$, respectively. The quantities $\vec{j}(\vec{r}, t)$ and $\rho(\vec{r}, t)$ can be expressed in following way

$$\vec{j}(\vec{r}, t) = e \sum_{\alpha} \vec{v}_{\alpha}(t) \delta(\vec{r} - \vec{r}_{\alpha}(t)), \tag{4}$$

$$\rho(\vec{r}, t) = e \sum_{\alpha} \delta(\vec{r} - \vec{r}_{\alpha}(t)), \tag{5}$$

where $\vec{r}_{\alpha}(t)$, $\vec{v}_{\alpha}(t)$ are αth electron radius-vector and velocity; the sum in (4)-(5) is over all electrons in the beam.

The electron movement equations can be written in the form

$$\frac{d\vec{v}_{\alpha}(t)}{dt} = \frac{e}{m\gamma} \left\{ \vec{E}(\vec{r}_{\alpha}(t), t) + \frac{1}{c} \left[\vec{v}_{\alpha}(t) \times \vec{H}(\vec{r}_{\alpha}(t), t) \right] - \frac{\vec{v}_{\alpha}}{c^2} \left(\vec{v}_{\alpha} \vec{E}(\vec{r}_{\alpha}(t), t) \right) \right\}, \tag{6}$$

where $\vec{E}(\vec{r}_{\alpha}(t), t)$ and $\vec{H}(\vec{r}_{\alpha}(t), t)$ are the electric and magnetic field of the electromagnetic wave in the point $\vec{r}_{\alpha}(t)$ at the time moment t, $\gamma = (1 - \frac{v_0^2}{c^2})^{-\frac{1}{2}}$.

Let the z coordinate axis coincide with waveguide axis. We also suppose that electron beam is "cold" (velocity spread of the electrons can be neglected) and initial electrons velocities are directed along z-axis ($\vec{u} = u\vec{e}_z$; ($\vec{e}_x, \vec{e}_y, \vec{e}_z$) are unit vectors of coordinate system). The dielectric permittivity inside the waveguide is $\varepsilon(\vec{r}, \omega) = \varepsilon_0(\vec{\rho}) + \chi(\vec{r}, \omega)$, where χ is periodic function of z: $\chi(\vec{r}, \omega) = \sum_{\tau \neq 0} \chi_{\tau}(\vec{\rho}) e^{i\tau z}$; $\chi_0 \equiv \varepsilon_0 - 1$. With the help of (2,3) and supposing that $|\chi(\vec{r}, \omega)| \ll 1$ we can rewrite (1) in the following way

$$-\vec{\nabla}^2 \vec{E}(\vec{r}, \omega) - \frac{\omega^2}{c^2} \varepsilon(\vec{r}, \omega) \vec{E}(\vec{r}, \omega) \approx \frac{4\pi i}{c^2} \left(\vec{j}(\vec{r}, \omega) + \frac{c^2}{\omega^2 \varepsilon_0} \vec{\nabla} (\vec{\nabla} \vec{j}(\vec{r}, \omega)) \right). \tag{7}$$

The field $\vec{E}(\vec{r}, \omega)$ can be decomposed in terms of the waveguide eigenfunctions

$$\vec{E}(\vec{r}, \omega) = \frac{1}{2\pi} \sum_n \int a_n(k_z) \vec{Y}_n(\vec{\rho}, k_z) e^{ik_z z} dk_z, \tag{8}$$

where $a_n(k_z)$ are the expansion coeffitients (amplitudes), $\vec{Y}_n(\vec{\rho}, k_z)$ and κ_n are the waveguide eigenfunctions and corresponding eigenvalues [9–11].

The beam current appearing on the right-hand side of (7) is a complicated function of the field $\vec{E}$. To study the problem of the system instability, it is sufficient to consider the system in the approximation linear over perturbation, i.e., one can expand the expressions for $\vec{j}(\vec{r}, \omega)$ over the field amplitude $\vec{E}(\vec{r}, \omega)$: $\vec{j} = \vec{j}_0 + \delta\vec{j}$, where $\vec{j}_0$ is the beam current not perturbated by the radiated field, $\delta\vec{j} \sim \vec{E}(\vec{r}, \omega)$ is the beam current induced by the radiated field. With the help of (4) one can find

$$\delta\vec{j}(\vec{k}, \omega) = e \sum_{\alpha} e^{-i\vec{k}\vec{r}_{\alpha 0}} \left\{ \delta\vec{v}_{\alpha}(\omega - \vec{k}\vec{u}) + \frac{1}{\omega - \vec{k}\vec{u}} \vec{u} \left(\vec{k}\delta\vec{v}_{\alpha}(\omega - \vec{k}\vec{u}) \right) \right\}. \tag{9}$$

The quantity $\delta\vec{v}_\alpha(\omega)$ can be obtained from movement equations:

$$\delta\vec{v}_\alpha(\omega) =$$

$$= \frac{ie}{m\omega\gamma} \int \frac{d^3k'}{(2\pi)^3} e^{i\vec{k}'\vec{r}_{\alpha 0}} \left\{ \frac{\omega}{\omega + \vec{k}'\vec{u}} \vec{E}(\vec{k}', \omega + \vec{k}'\vec{u}) + \left(\frac{\vec{k}'}{\omega + \vec{k}'\vec{u}} - \frac{\vec{u}}{c^2} \right) \cdot \left(\vec{u}\vec{E}(\vec{k}', \omega + \vec{k}'\vec{u}) \right) \right\}. \tag{10}$$

After substituting (10) into (9) in the expression for the current density appear the sum $\sum_\alpha e^{i(\vec{k}'-\vec{k})\vec{r}_{\alpha 0}}$. Let us average this sum over distribution of the particles in the beam

$$\sum_\alpha e^{i(\vec{k}-\vec{k})\vec{r}_{\alpha 0}} \simeq \Phi(\vec{k}_\perp - \vec{k}'_\perp) \cdot (2\pi)^3 n_0 \delta(k_z - k'_z), \tag{11}$$

where $\Phi(\vec{k}_\perp - \vec{k}'_\perp) = \frac{1}{(2\pi)^2} \int_S e^{-i(\vec{k}_\perp - \vec{k}'_\perp)\vec{\rho}} \varphi(\vec{\rho}) d^2\vec{\rho}$, $\frac{1}{S}\int_S \varphi(\vec{\rho}) d^2\vec{\rho} = 1$, S is cross-section of the waveguide, n_0 is the electron density of the beam, k_z is the longitudinal component of the wave vector, the function $\varphi(\vec{\rho})$ describes the distribution of the electrons in the beam cross-section.

The expressions (7)-(11) allow us to write the following system of equations for amplitudes $a_m(k_z)$ (this system is similar to that describing the multiwave dynamical diffraction in crystals):

$$\left(k_z^2 - \left(\frac{\omega^2}{c^2}\varepsilon_0 - \kappa_m^2 \right) \right) a_m(k_z) - \frac{\omega^2}{c^2} \sum_m \sum_{\tau \neq 0} \chi_{eff}^{mn}(k_z, k_z - \tau) a_n(k_z - \tau) =$$

$$= -\frac{\omega_l^2}{\gamma} \frac{1}{(\omega - k_z u)^2} \sum_n A_{mn} a_n(k_z), \tag{12}$$

where $\omega_l^2 = \frac{4\pi e^2 n_0}{m}$ is the Langmuir frequency of the beam, effective susceptibility χ_{eff}^{mn} is

$$\chi_{eff}^{mn}(k_z, k_z - \tau) = \int \vec{Y}_m^*(\vec{\rho}, k_z) \chi_\tau \vec{Y}_n(\vec{\rho}, k_z - \tau) d^2\vec{\rho}. \tag{13}$$

Coeffitients A_{mn} in (12) are

$$A_{mn} = \frac{1}{c^4} \int \frac{d^2\vec{k}_\perp}{(2\pi)^2} d^2\vec{\rho} d^2\vec{\rho}' \varphi(\vec{\rho}') e^{i\vec{k}_\perp(\vec{\rho}-\vec{\rho}')} \times$$

$$\times \vec{Y}_m^*(\vec{\rho}, k_z) \left(\vec{u} - \frac{c^2}{\omega\varepsilon_0}\vec{k} \right) \left(-ic^2 \vec{k}_\perp \vec{\nabla}'_\perp + \frac{c^2 - u^2}{u^2}\omega^2 \right) \left(\vec{u}\vec{Y}_n(\vec{\rho}', k_z) \right). \tag{14}$$

III. INCREMENT OF ELECTRON BEAM INSTABILITY

Let us consider the generation on the most commonly used E_{01}-mode of circular waveguide. The vector eigenfunction $\vec{Y}_1(\vec{\rho}, k_z)$ corresponding to E_{01}-mode can be written as [11]

$$\vec{Y}_1 = \frac{1}{\sqrt{\pi} R J_1(\nu_{01})} \left\{ \frac{-ik_z}{\sqrt{k_z^2 + \kappa^2}} J_1(\rho\kappa)\vec{e}_r + \frac{\kappa}{\sqrt{k_z^2 + \kappa^2}} J_0(\rho\kappa)\vec{e}_z \right\}, \qquad (15)$$

where $\nu_{01} \approx 2.4048$, $\kappa = \nu_{01}/R$, R is the waveguide radius, J_0 and J_1 are Bessel functions. The effective susceptibility according to (13) is

$$\chi_{eff}(k_z, \tau) = \chi_\tau \int \vec{Y}_1^*(\vec{\rho}, k_z)\vec{Y}_1(\vec{\rho}, k_z - \tau)d^2\vec{\rho} = \chi_\tau \left(1 - \frac{k_z\tau}{k_z^2 + \kappa^2} \right). \qquad (16)$$

The equations (12) for the E_{01}-mode can be written as

$$(\omega - k_z u)^2 \left((k_z^2 - k_{z0}^2)a(k_z) - k_0^2 \sum_{\tau \neq 0} \chi_{eff}(k_z, \tau)a(k_z - \tau) \right) = \frac{\omega_l^2}{\gamma} A_{11} a(k_z), \qquad (17)$$

where $k_{z0} = \sqrt{\frac{\omega^2}{c^2}\varepsilon_0 - \kappa^2}$ and $k_0 = \omega/c$. We assume that unperturbed beam occupies the entire cross section of the waveguide and $\varphi(\vec{\rho}) = 1$. Then the calculation of the A_{11} in accordance with (14) gives

$$A_{11} = \left(\frac{k_0^2}{\gamma^2} + \beta^2\kappa^2 \right) \frac{\kappa^2}{k_z^2 + \kappa^2}. \qquad (18)$$

Let us consider firstly the case of waveguide with homogeneous filling ($\chi_\tau = 0$). Apparently in this case the diffracted wave is absent and system (17) is reduced to a single equation. Suppose that right-hand side of (17) is small. Since the nonlinearity is insignificant, let us consider as the zero approximation the spectrum of the waves of equation (17) with zero right-hand side. So, the solution of (17) is $k_{z1} = k'_{z1} + \delta k_{z1}$, where δk_{z1} is to be found, and k'_{z1} is the solution of (17) with zero right-hand side $k'_{z1} = k_{z0} = \sqrt{\frac{\omega^2}{c^2}\varepsilon_0 - \kappa^2}$. Repeating exactly the arguments, for example [8], we arrive the following expression for increment

$$\text{Im}\,k_{z1} = \text{Im}\,\delta k_{z1} = \frac{\sqrt{3}}{2} \left(\frac{\omega_l^2}{2k'_{z1}u^2\gamma} A_{11} \right)^{\frac{1}{3}} \sim \rho^{1/3}. \qquad (19)$$

Now let us consider the case $\chi_\tau \neq 0$, $\chi_\tau \ll 1$. Apparently when the diffraction conditions are not fullfilled we get the same expression (19) for increment. We therefore assume that the conditions of two-wave dynamical diffraction in a waveguide are realized, i.e. the wave

amplitude $a(k_z + \tau)$ is comparable with the amplitude $a(k_z)$. In this case the dispersion equation has the next form:

$$(\omega - k_z u)^2 \left\{ (k_z^2 - k_{z0}^2)(k_{z\tau}^2 - k_{z0}^2) - k_0^4 \chi_{eff}(k_{z\tau}, \tau)\chi_{eff}(k_z, -\tau) \right\} = -\frac{\omega_l^2}{\gamma} A_{11}(k_{z\tau}^2 - k_{z0}^2). \quad (20)$$

The solution of this equation is $k_{z2} = k'_{z2} + \delta k_{z2}$, where

$$k_{z2}^{\prime(1,2)} = k_{z0} \left\{ 1 - \frac{1}{4}\alpha_B \beta \pm \sqrt{(\alpha_B \beta)^2 + 4\frac{r}{\gamma_0^4}\beta} \right\}, \quad (21)$$

$r = \chi_{eff}(k_{z0} + \tau, \tau)\chi_{eff}(k_{z0}, -\tau)$, $k_{z0} = \sqrt{\frac{\omega^2}{c^2}\varepsilon_0 - \kappa^2}$, $\gamma_0 = \frac{k_{z0}}{\omega/c}$, $\beta = \frac{k_{z0}}{k_{z0}+\tau}$ is the diffraction asymmetry factor, $\alpha_B = \frac{(2k_{z0}+\tau)\tau}{k_{z0}^2}$ is the off-Bragg parameter ($\alpha_B = 0$ when the Bragg condition of diffraction is exactly fulfilled), $\tau = 2\pi/D$, D is the waveguide period. The calculation gives the following expression for increment $\mathrm{Im}\, k_z$ in the vicinity of the roots degeneracy point

$$\mathrm{Im}\, k_{z2} = \mathrm{Im}\, \delta k_{z2} \approx \left(\frac{\omega_l^2 k_0^2 \sqrt{r}}{\gamma u^2 \tau^2} A_{11} \right)^{1/4} \sim \rho^{1/4}. \quad (22)$$

From (19) and (22) we have

$$\frac{\mathrm{Im}\, k_{z2}}{\mathrm{Im}\, k_{z1}} \approx \left(\frac{\omega_l^2}{\omega^2} \frac{\tau^2}{k_0^2} \frac{A_{11}}{k_0^2} \frac{1}{(\sqrt{r})^3 \beta^2 \gamma} \right)^{-1/12} \gg 1, \quad (23)$$

because $\omega_l^2 \ll \omega^2$, $A_{11} \ll k_0^2$. This means under diffraction conditions the gain increases. As a result, the threshold current density in two-wave diffraction case is lower ($j \sim \frac{1}{(kL)^3 (k\chi_\tau L)^{2s}}$, where L is the interaction length).

IV.   NUMERICAL SOLUTIONS

To be able to experimentally observe this effect it is necessary to know how accurately the diffraction condition must be satisfied. To study this, the equations (20) were solved numerically. Now we demonstrate the results of this calculation by the example of one of the systems studied ($R = 6$ cm, $D = 3.6$ cm, $\varepsilon_0 = 1.23$, $\chi_\tau = 0.05$).

On the fig. 1 one can see the dependence of the increment of instability on the frequency near region of dispersion roots degeneracy (4.06 GHz in our case). Let us explain how the curves in fig. 1 are obtained. Initially, we found the solutions of dispersion equation without beam (21). They are shown on the right-hand side of the graph. Next, for each frequency f_i from the selected range the set of equations (20) was solved and the imaginary part of
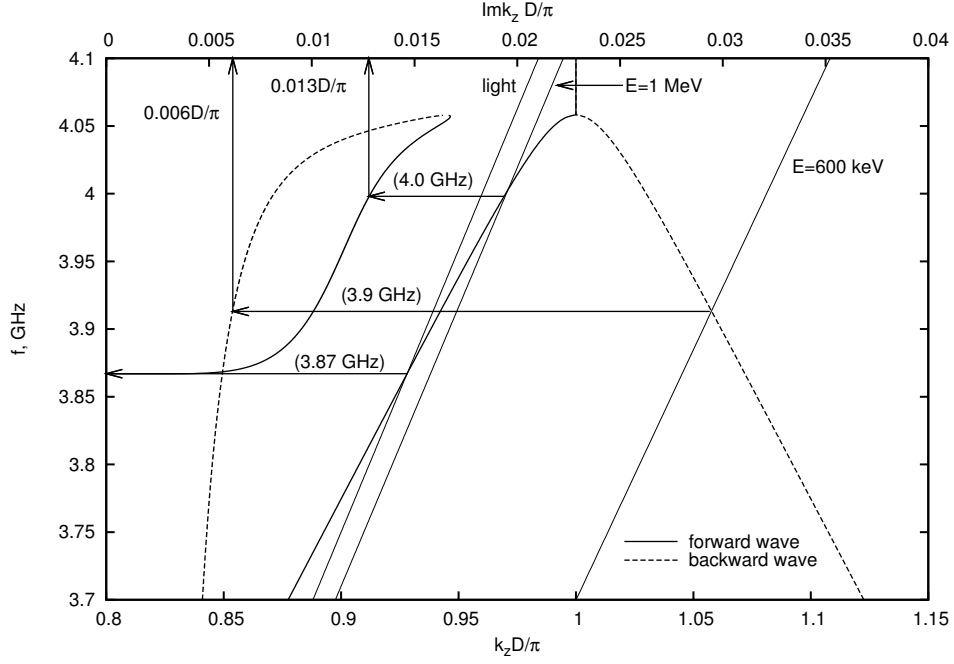
FIG. 1: The dependence of instability increment of electron beam on the frequency near the point of the roots degeneracy of dispersion equation. Beam current is 0.1 kA. Solid and dashed curves on the right-hand side of the graph are dispersion characteristics of the forward and backward waves in the waveguide, respectively; on the left-hand side are plotted corresponding increments. On the graph are also shown the beam lines ($\omega = k_z u$) for the energies 600 keV and 1 MeV and the light line ($\omega = k_z c$).

the solution found $\operatorname{Im} k_z$ was plotted. In the calculations we assumed that the beam current I is given; the beam energy E was chosen so that the Cherenkov synchronism occurred at a frequency f_i. Calculations were performed for the two roots of the dispersion equation.[1] For example, for a backward wave synchronism with the beam with the energy 600 keV occurs at a frequency of $\sim$ 3.9 GHz; if we choose the beam current equal to 0.1 kA, the corresponding increment will be $\operatorname{Im} k_z \approx 0.006\pi/D \approx 0.007$ cm^{-1}, which is shown in Fig. 1.

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[1] Far from the roots degeneracy point they correspond to the so-called forward and backward waves in the waveguide. But near the point of degeneracy such division, generally speaking, is incorrect because in this case "forward" and "backward" waves are coupled. Nevertheless, in the article we use the established terms "forward" and "backward" waves to refer to the roots of the dispersion equation, since we investigate the behavior of the system over a wide frequency range (far from the point of degeneration and in the immediate vicinity from it).
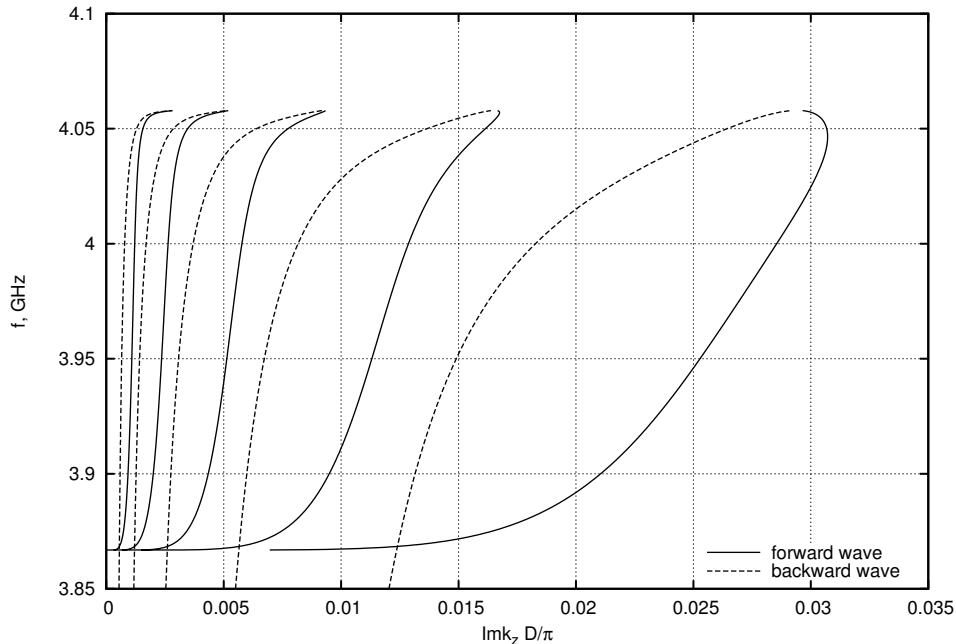
FIG. 2: The growth of the increment with beam current increase. On the graph are shown the increments for cases when the beam current is 0.1 A, 1 A, 10 A, 0.1 kA and 1 kA.

A similar example is given on the fig. 1 for the forward wave.

From Fig. 1 we can trace back the typical dependence of the increment value on the frequency near the point of the roots degeneracy for the systems studied. It is easy to see that the synchronism condition for the forward wave is possible only for frequencies above f_0 (in our case $f_0 \approx 3.87$ GHz), because velocity of the beam can not exceed the speed of light in vacuum c. Therefore, for the forward wave the increment begins to rise from zero at the frequency f_0 to a maximum which can be observed both at the point of the roots degeneracy, and at some distance from it (see fig. 2). It is seen that in the vicinity of the degeneracy point the increment increases with the frequency faster than away from the diffraction conditions. For the backward wave a similar but much more pronounced picture can be seen: relatively slow increment growth away from the diffraction conditions is replaced by the rapid growth near the degeneracy point. In addition, for backward-wave the increment has the highest value exactly in the degeneracy point.

Fig. 2 shows the increment of instability for different beam currents. For the forward wave the increase of the beam current shifts down the frequency at which the increment

reachs maximum. When the current rises, the increment maximum becomes wider and lower. Nevertheless, for the chosen parameters of the waveguide the gain for a backward wave in the vicinity of the degeneracy point can be several times (from 2 to 5) greater than that for the usual no-diffraction case. But to use this in practice the synchronism conditions must be satisfied very precisely. For example, when the beam current is 1 kA as shown in fig. 2 the imaginary part $\mathrm{Im}\, k_z$ is reduced by half when the displacement from the point of degeneracy (4.06 GHz) down in frequency is only ~ 116 MHz (the peak FWHM is ~ 116 MHz). In practice, the control of oscillation frequency with such precision is not always possible due to many factors, such as the presence of an electron velocity spread in the beam. Nevertheless, one can find conditions (such as geometric dimensions of the system, values of ε_0, χ_τ, the beam energy, etc.) under which it will be possible to perform the lasing near degeneracy point with sufficient accuracy. In particular, by use of fig. 1 it is easy to find that for selected geometry the spread in the electron velocities does not play a significant role due to high electron beam energy (for the detection of the effect is enough to have $\delta E \leq 30\%$ at the energy $E \sim 1.6$ MeV).

V. CONCLUSION

We conclude with a remark on the above mentioned model of the one-dimensional crystal. This model is is somewhat idealized since in real systems it is impossible to use the waveguides with a solid dielectric filling. As a rule, in practice are applied waveguides with inner dielectric liner [12] (partially filled waveguides). However in considering of the partially filled waveguide all the above arguments remain valid. In fact, you only need to substitute in the above model other eigenfunctions and derive the expressions for coefficients χ_{eff} and A_{mn}. But the behavior of the instability increment near the degeneracy points remains the same. Moreover to estimate the increment value you can use the model of a waveguide with a solid filling by introducing the average over the cross section of the waveguide dielectric permittivity $\varepsilon(\omega)$ and taking into account the fact that with a wave interacts effectively only part of the beam (located at distance of $\lesssim \lambda\beta\gamma/(4\pi)$ from the surface of the dielectric).

It is shown that for a one-dimensional photonic crystal the gain near region of dispersion equation roots degeneracy rapidly (several times) increases. With a reasonable choice of the waveguide (crystal) and electron beam parameters (in particular, at sufficiently high beam

energy) this effect is experimentally observable and can be used to improve the characteristics of generators and amplifiers of microwave radiation.

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