ADAPTIVE CFAR TESTS FOR DETECTION AND RECOGNITION OF TARGET SIGNALS IN RADAR CLUTTER

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Abstract. In this paper, adaptive CFAR tests are described which allow one to classify radar clutter into one of several major categories, including bird, weather, and target classes. These tests do not require the arbitrary selection of priors as in the Bayesian classifier. The decision rule of the recognition techniques is in the form of associating the p-dimensional vector of observations on the object with one of the m specific classes. When there is the possibility that the object does not belong to any of the m classes, then this object is to be classified as belonging to one of the m classes or to class m+1 whose distribution is unspecified. The tests are invariant to intensity changes in the clutter background and achieve a fixed probability of a false alarm.

1. Introduction

Modern air traffic control radar systems rely heavily on automatic target detection and tracking to maximize air traffic safety. Moving target indicator and moving target detector algorithms achieve good target detection performance through the suppression of most or all forms of radar clutter. Unfortunately, real-time information on airborne hazards to aircraft, such as birds and storm systems, is also suppressed. The ability to classify clutter and hence identify these hazards can thus contribute significantly to air traffic safety. In most instances, the prior information about the target observable is limited and the background types vary widely. A possible way to achieve a high probability of detection at a low false-alarm rate for these stressing background clutter-limited situations is through the use of adaptive signal processing techniques that exploit differences between the target signatures and the background in multiple dimensional spaces.

The process of recognition can be formalized as follows. The unprocessed radar data are passed through a feature extractor, which transforms the available data samples into a set of separable features. These features are derived from the reflection coefficients computed using the multisegment version of Burg's formula [2]. The aforementioned coefficients (that contain all spectral information, including the mean Doppler shift) are then transformed and grouped to satisfy the requirements for multivariate Gaussian behavior. Only information that is different from class to class is maintained, and in such a form that a reliable decision, based on a discriminant function derived from the above features, may be made.

Stehwien and Haykin [8] solved the problem of statistical recognition of target in radar clutter in a Bayesian framework. In this paper, the problem is treated in a non-Bayesian setting. A recognition technique is described which allows one to classify target detected in radar clutter into one of several major categories, including bird, weather, and target classes. This technique is based on applying the theory of generalized maximum likelihood ratio testing for composite hypotheses. The results of computer simulations confirm the validity of the theoretical predictions of performance of the suggested technique.

The outline of the paper is as follows. A problem of target signal detection in clutter is considered in Section 2. Section 3 is devoted to a problem of recognition of detected target.

2. Target signal detection

The problem of detecting the unknown deterministic signal $\mathbf{s}$ in the presence of a clutter process, which is incompletely specified, can be viewed as a binary hypothesis-testing problem. The decision is based on a sample of observation vectors $\mathbf{x}_i = (x_{i1}, \dots ,x_{ip})'$, i = 1(1)n, each of which is composed of clutter $\mathbf{w}_i = (w_{i1}, \dots ,w_{ip})'$ under the hypothesis H_0 and a signal $\mathbf{s} = (s_1, \dots ,s_p)'$ added to clutter $\mathbf{w}_i$ under the alternative H_1, where n > p. The two hypotheses that the detector must distinguish are given by

$$H_0\text{: } \mathbf{X} = \mathbf{W} \qquad \text{(clutter alone)}, \tag{1}$$

$$H_1\text{: } \mathbf{X} = \mathbf{W} + \mathbf{sc}' \qquad \text{(signal present)}, \tag{2}$$

where

$$\mathbf{X} = (\mathbf{x}_1, \dots ,\mathbf{x}_n), \quad \mathbf{W} = (\mathbf{w}_1, \dots ,\mathbf{w}_n) \tag{3}$$

are p × n random matrices, and $\mathbf{c} = (1, \dots ,1)'$ is a column vector of n units. It is assumed that $\mathbf{w}_i$, i = 1(1)n, are independent and normally distributed with common mean $\mathbf{0}$ and covariance matrix (positive definite) $\mathbf{Q}$, i.e.

$$\mathbf{w}_i \sim N_p(\mathbf{0},\mathbf{Q}), \qquad \forall i = 1(1)n. \tag{4}$$

Thus, for fixed n, the problem is to construct a test, which consists of testing the null hypothesis

$$H_0: \mathbf{x}_i \sim N_p(\mathbf{0},\mathbf{Q}), \quad \forall i = 1(1)n, \tag{5}$$

versus the alternative

$$H_1: \mathbf{x}_i \sim N_p(\mathbf{s},\mathbf{Q}), \quad \forall i = 1(1)n, \tag{6}$$

where the parameters **Q** and **s** are unknown.

Generalized maximum likelihood ratio. One of the possible statistics for testing H_0 versus H_1 is given by the generalized maximum likelihood ratio (GMLR)

$$\text{GMLR} = \max_{\theta\in\Theta_1} L_{H_1}(\mathbf{X};\theta) \Big/ \max_{\theta\in\Theta_0} L_{H_0}(\mathbf{X};\theta), \tag{7}$$

where $\theta=(\mathbf{s},\mathbf{Q})$, $\Theta_0=\{(\mathbf{s},\mathbf{Q}): \mathbf{s}=\mathbf{0},\ \mathbf{Q}\in Q_p\}$, $\Theta_1=\Theta-\Theta_0$, $\Theta = \{(\mathbf{s},\mathbf{Q}): \mathbf{s}\in R^p,\ \mathbf{Q}\in Q_p\}$, Q_p denotes the set of $p \times p$ positive definite matrices. Under H_0, the joint likelihood for **X** based on (5) is

$$L_{H_0}(\mathbf{X};\theta) = (2\pi)^{-np/2}|\mathbf{Q}|^{-n/2}\exp\left(-\sum_{i=1}^{n}\mathbf{x}_i'\mathbf{Q}^{-1}\mathbf{x}_i/2\right). \tag{8}$$

Under H_1, the joint likelihood for **X** based on (6) is

$$L_{H_1}(\mathbf{X};\theta) = (2\pi)^{-np/2}|\mathbf{Q}|^{-n/2}\exp\left(-\sum_{i=1}^{n}(\mathbf{x}_i-\mathbf{s})'\mathbf{Q}^{-1}(\mathbf{x}_i-\mathbf{s})/2\right). \tag{9}$$

It can be shown that

$$\text{GMLR} = |\hat{\mathbf{Q}}_0|^{n/2}|\hat{\mathbf{Q}}_1|^{-n/2}, \tag{10}$$

where

$$\hat{\mathbf{Q}}_0 = \mathbf{X}\mathbf{X}'/n, \tag{11}$$

$$\hat{\mathbf{Q}}_1 = (\mathbf{X}-\hat{\mathbf{s}}\mathbf{c}')(\mathbf{X}-\hat{\mathbf{s}}\mathbf{c}')'/n, \tag{12}$$

and

$$\hat{\mathbf{s}} = \mathbf{X}\mathbf{c}/n \tag{13}$$

are the well-known maximum likelihood estimators of the unknown parameters **Q** and **s** under the hypotheses H_0 and H_1, respectively. After some algebra, we find that (10) is equivalent finally to the statistic $y = \mathbf{T}_1'\mathbf{T}_2^{-1}\mathbf{T}_1/n$, where $\mathbf{T}_1=\mathbf{X}\mathbf{c}$, $\mathbf{T}_2=\mathbf{X}\mathbf{X}'$. It is known that $(\mathbf{T}_1,\mathbf{T}_2)$ is a complete sufficient statistic for the parameter $\theta=(\mathbf{s},\mathbf{Q})$. Thus, the problem has been reduced to consideration of the sufficient statistic $(\mathbf{T}_1,\mathbf{T}_2)$. It can be shown that under H_0, y is a **Q**-free statistic, which has the property that its distribution does not depend on the actual covariance matrix **Q**. This is given by the following theorem.

Theorem 1 (*PDF of the GMLR statistic y*). Under H_1, the statistic y is subject to a noncentral beta-distribution with the probability density function (PDF)

$$f_{H_1}(y;n,q) = \left[B\left(\frac{p}{2},\frac{n-p}{2}\right)\right]^{-1} y^{\left(\frac{p}{2}\right)-1}(1-y)^{\left(\frac{n-p}{2}\right)-1}\, e^{-q/2}\, {}_1F_1\left(\frac{n}{2};\frac{p}{2};\frac{qy}{2}\right), \quad 0 < y < 1, \tag{14}$$

where ${}_1F_1(a;b;x)$ is the confluent hypergeometric function [1], $q = n(\mathbf{s}'\mathbf{Q}^{-1}\mathbf{s})$ is a noncentrality parameter representing the generalized signal-to-noise ratio (GSNR). Under H_0, when q=0, (14) reduces to a standard beta-function density of the form

$$f_{H_0}(y,n) = \left[B\left(\frac{p}{2},\frac{n-p}{2}\right)\right]^{-1} y^{\left(\frac{p}{2}\right)-1}(1-y)^{\left(\frac{n-p}{2}\right)-1}, \quad 0 < y < 1. \tag{15}$$

Proof. The proof is given by Nechval [4] and so it is omitted here. □

GMLR statistic. It is clear that the statistic y is equivalent to the statistic

$$v = [(n-p)/p]\,y/(1-y) = [n(n-p)/p]\left(\hat{\mathbf{s}}'\left[\hat{\mathbf{G}}_1\right]^{-1}\hat{\mathbf{s}}\right), \tag{16}$$

where

$$\hat{\mathbf{G}}_1 = n\hat{\mathbf{Q}}_1 = (\mathbf{X}-\hat{\mathbf{s}}\mathbf{c}')(\mathbf{X}-\hat{\mathbf{s}}\mathbf{c}')' = \sum_{i=1}^{n}(\mathbf{x}_i-\hat{\mathbf{s}})(\mathbf{x}_i-\hat{\mathbf{s}})'. \tag{17}$$

Here the following theorem clearly holds.

Theorem 2 (*PDF of the GMLR statistic v*). Under H_1, the statistic v is subject to a noncentral F-distribution with p and n-p degrees of freedom, the probability density function of which is

$$f_{H_1}(v;n,q)=\left[B\left(\frac{p}{2},\frac{n-p}{2}\right)\right]^{-1}\frac{\left(\frac{p}{n-p}\right)^{p/2}v^{p/2-1}}{\left(1+\frac{p}{n-p}v\right)^{n/2}}\,e^{-q/2}\,{}_1F_1\left(\frac{n}{2};\frac{p}{2};\frac{q}{2}\left(\frac{p}{n-p}v\left(1+\frac{p}{n-p}v\right)^{-1}\right)\right),\quad 0<v<\infty, \tag{18}$$

where q is a noncentrality parameter. Under H_0, when q=0, (18) reduces to a standard F-distribution with p and n-p degrees of freedom,

$$f_{H_0}(v;n)=\left[B\left(\frac{p}{2},\frac{n-p}{2}\right)\right]^{-1}\frac{\left(\frac{p}{n-p}\right)^{p/2}v^{p/2-1}}{\left(1+\frac{p}{n-p}v\right)^{n/2}},\quad 0<v<\infty. \tag{19}$$

Proof. The proof follows by applying Theorem 1 and being straightforward it is omitted. □

Adaptive test for target signal detection. The test of H_0 versus H_1, based on the GMLR statistic v, is given by

$$v\begin{cases}\ge h, & \text{then} \quad H_1 \quad \text{(signal present)},\\ < h, & \text{then} \quad H_0 \quad \text{(clutter alone)},\end{cases} \tag{20}$$

and can be written in the form of a decision rule u(v) over {v: v∈(0,∞)},

$$u(v)=\begin{cases}1, & v\ge h \quad (H_1),\\ 0, & v<h \quad (H_0),\end{cases} \tag{21}$$

where h > 0 is a threshold of the test which is uniquely determined for a prescribed level of significance α so that

$$\sup_{\theta\in\Theta_0} E_\theta\{u(v)\}=\alpha. \tag{22}$$

For fixed n, in terms of the probability density function (19), tables of the central F-distribution permit one to choose h to achieve the desired test size (false alarm probability P_{FA}),

$$P_{FA}=\alpha=\int_h^\infty f_{H_0}(v;n)dv. \tag{23}$$

Furthermore, once h is chosen, tables of the noncentral F-distribution permit one to evaluate, in terms of the probability density function (18), the power (detection probability P_D) of the test,

$$P_D=\gamma=\int_h^\infty f_{H_1}(v;n,q)dv. \tag{24}$$

The probability of a miss is given by

$$\beta=1-\gamma. \tag{25}$$

It follows from (23) that the GMLR test is invariant to intensity changes in the clutter background and achieves a fixed probability of a false alarm, i.e. the resulting analyses indicate that the test has the property of a constant false alarm rate (CFAR). Also, no learning process is necessary in order to achieve the CFAR. Thus, operating in accordance to the local clutter situation, the test is adaptive.

When the parameter θ=(**s**,**Q**) is unknown, it is well known that no the uniformly most powerful (UMP) test exists for testing H_0 versus H_1 [3]. However, some hypothesis testing problems that do not admit UMP decision rules (tests) nevertheless exhibit certain natural invariance properties [3, 5]. These properties suggest restricting attention to a limited class of decision rules, viz., the invariant decision rules. It is then sometimes possible to derive decision rules that are UMP within this limited class. In this sense, invariance is a concept of fundamental importance in hypothesis testing. The following theorem shows that the test (20) is UMPI for a natural group of transformations on the space of observations.

Theorem 3. (*UMPI test***).** For testing the hypothesis H_0 versus the alternative H_1, the CFAR test given by (20) is uniformly most powerful invariant (UMPI).

Proof. The proof is similar to that of Nechval [6, 7] and so it is omitted here. □

3. Target signal recognition

Suppose that the hypothesis H_0: (clutter alone) is rejected. Then we deal with the target (signal in clutter) recognition problem using target identity information. Let a target detected belong to one of m classes and each class has equal a priori probability. We postulate that this target can be regarded as a "random drawing" from one of the m classes but we do not know from which one. The problem is to classify a detected target as belonging to one of the m specified classes. When there is the possibility that a target does not belong to any of the m above classes, it is desirable to recognize this case.

To adapt to a nonstationary background clutter, consider the situation in which a detected target signal **s** is related to the true target signal, say, of the jth class **s**(j) by

$$\mathbf{s}=\vartheta\mathbf{s}(j)=\vartheta(s_1(j), \ldots, s_p(j))', \quad j\in\{1, \ldots, m\}, \tag{26}$$

where ϑ is a scalar amplitude parameter. It is assumed that the target signal vectors **s**(j), j=1(1)m, are known.

The generalized maximum likelihood ratio statistics for this recognition problem are given by

$$\max_{\vartheta}\{\max_{\mathbf{Q}} L_{H_1(j)}(\mathbf{X};\vartheta,\mathbf{Q})\} \Big/ \max_{\mathbf{Q}} L_{H_0}(\mathbf{X};\mathbf{Q}), \tag{27}$$

where

$$L_{H_0}(\mathbf{X};\mathbf{Q}) = (2\pi)^{-np/2}|\mathbf{Q}|^{-n/2}\exp\left(-\sum_{i=1}^{n}\mathbf{x}_i'\mathbf{Q}^{-1}\mathbf{x}_i/2\right), \tag{28}$$

$$L_{H_1(j)}(\mathbf{X};\vartheta,\mathbf{Q}) = (2\pi)^{-np/2}|\mathbf{Q}|^{-n/2}\exp\left(-\sum_{i=1}^{n}(\mathbf{x}_i-\vartheta\mathbf{s}(j))'\mathbf{Q}^{-1}(\mathbf{x}_i-\vartheta\mathbf{s}(j))/2\right), \tag{29}$$

are the likelihood functions under H_0 and $H_1(j)$, $j\in\{1, \ldots, m\}$, respectively, and

$$\max_{\vartheta}\{\max_{\mathbf{Q}} L_{H_1(j)}(\mathbf{X};\vartheta,\mathbf{Q})\} = \max_{\vartheta}\frac{1}{(2\pi)^{np/2}\left|\widehat{\mathbf{Q}}_1(j)\right|^{n/2}}\exp\left(-\frac{np}{2}\right), \tag{30}$$

$$\max_{\mathbf{Q}} L_{H_0}(\mathbf{X};\mathbf{Q}) = \frac{1}{(2\pi)^{np/2}\left|\widehat{\mathbf{Q}}_0\right|^{n/2}}\exp\left(-\frac{np}{2}\right). \tag{31}$$

The well-known maximum likelihood estimates (MLE's) of the unknown covariance matrix **Q** under the respective hypotheses, H_0 and $H_1(j)$, are given by

$$\widehat{\mathbf{Q}}_0 = \frac{1}{n}\sum_{i=1}^{n}\mathbf{x}_i\mathbf{x}_i' = \frac{1}{n}\mathbf{X}\mathbf{X}', \tag{32}$$

$$\widehat{\mathbf{Q}}_1(j) = \frac{1}{n}\sum_{i=1}^{n}(\mathbf{x}_i-\vartheta\mathbf{s}(j))(\mathbf{x}_i-\vartheta\mathbf{s}(j))' = \frac{1}{n}(\mathbf{X}-\vartheta\mathbf{s}(j)\mathbf{c}')(\mathbf{X}-\vartheta\mathbf{s}(j)\mathbf{c}')'. \tag{33}$$

After several algebraic manipulations, (27) reduces to the following clutter-adaptive test of detection of the jth target signal, $j\in\{1, \ldots, m\}$:

$$v(j)\begin{cases}\geq h(j), & \text{then} \quad H_1(j),\\ < h(j), & \text{then} \quad H_0,\end{cases} \tag{34}$$

where

$$v(j) = \frac{z(j)}{z(j)+1}, \quad z(j) = \frac{[\mathbf{s}'(j)(\mathbf{X}\mathbf{X}')^{-1}\mathbf{X}\mathbf{c}]^2}{[\mathbf{s}'(j)(\mathbf{X}\mathbf{X}')^{-1}\mathbf{s}(j)][1-\mathbf{c}'\mathbf{X}'(\mathbf{X}\mathbf{X}')^{-1}\mathbf{X}\mathbf{c}]}, \tag{35}$$

$h(j)>0$ is a threshold of the test which is uniquely determined for a prescribed level of significance $\alpha(j)$ so that the probability of a false alarm is equal to $\alpha(j)$.

Theorem 4 **(***PDF of the GMLR statistic v(j)***).** The probability density function of v(j) under hypothesis $H_1(j)$ is given as follows:

$$f_{H_1(j)}(v(j);n,q(j)) = \int_0^1 f(v(j);g,n,q(j))f(g;n)dg, \tag{36}$$

where

$$f(g;n) = \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n-p}{2}\right)\Gamma\left(\frac{p-1}{2}\right)}(1-g)^{\frac{p-3}{2}}g^{\frac{n-p-2}{2}}, \tag{37}$$

for $0\leq g\leq 1$, and

$$f(v(j);g,n,q(j)) = \frac{\Gamma\left(\frac{n-p+1}{2}\right)\exp\left(-\frac{q(j)g}{2}\right)}{\Gamma\left(\frac{n-p}{2}\right)\Gamma\left(\frac{1}{2}\right)}[1-v(j)]^{\frac{n-p-2}{2}}[v(j)]^{-\frac{1}{2}}\,{}_1F_1\left(\frac{n-p+1}{2};\frac{1}{2};\frac{q(j)gv(j)}{2}\right) \quad (38)$$

for 0<v(j)<1. In (38) ${}_1F_1(a;b;x)$ is the confluent hypergeometric function, and q(j) is the generalized signal-to-noise ratio (GSNR) defined by

$$q(j) = \mathrm{GSNR} = n\vartheta^2\mathbf{s}'(j)\mathbf{Q}^{-1}\mathbf{s}(j). \quad (39)$$

Under hypothesis H_0, no signal is present. Thus, if one sets q(j)=0 in (38),

$$f_{H_0}(v(j);n) = \frac{\Gamma\left(\frac{n-p+1}{2}\right)}{\Gamma\left(\frac{n-p}{2}\right)\Gamma\left(\frac{1}{2}\right)}[1-v(j)]^{\frac{n-p-2}{2}}[v(j)]^{-\frac{1}{2}}, \quad 0<v(j)<1. \quad (40)$$

Proof. The proof is similar to that of Theorem 2 and so it is omitted here. □

Finally, in terms of the above probability density functions in (36) and (40) the probability of false alarm is given by

$$P_{FA}(j) = \int_{h(j)}^{1} f_{H_0}(v(j);n)dv(j) \quad (41)$$

and the probability of detection of the jth target signal is

$$P_D(j) = \int_{h(j)}^{1} f_{H_{1(j)}}(v(j);n,q(j))dv(j). \quad (42)$$

Thus, if v(j)<h(j) then the jth target class is eliminated from further consideration.

If (m-1) target classes are so eliminated, then the remaining class (say, kth) is the one to which a detected target being classified belongs.

If all the target classes are eliminated from further consideration, we decide that a detected target belongs to the (m+1)th class whose distribution is unspecified.

If the set of target classes not yet eliminated has more than one element, then we declare that a detected target belongs to the class j* if

$$j^* = \arg\max_{j\in D}(v(j)-h(j)), \quad (43)$$

where D is the set of target classes not yet eliminated by the above test.

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