IDENTIFICATION OF STATIONARY AND UNSTATIONARY REGIMES OF RESPIRATORY NEURON NETWORK WITH COMPUTER USING

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Abstract. The problem of respiratory neuron networks (RNN) stationary regime's identification was discussed. The method of minimal realization according to behaviorist approach was presented. It is based on compartmental-cluster theory of RNN when the order m of model presents the number of compartments. The software provides the identification of optimum interval of stimulus time – length according to calculation of the Perron roots of matrices A. The biological interpretation of the method is presented for one class of biological dynamic system (BDS) - respiratory neuron network.

1. Compartmental-cluster's modeling of RNN and their stationary regime

Modern theory of natural neuron networks (NN) needs of effective method for parameter's and structural identification of its mathematical model. It is clear that neuron networks are various and difficult objects for investigation of their structure. More then the direct investigation of its behaviour is very difficult problem for investigator. More convenient method for modeling of NN and its identification is based on behavioristic approaching (BA). It is evident that the method of response analyses of NN (the relation between output and input) signal of NN is very effective for the investigation of neuron networks [1-4]. The BA can be realized by different method but we use one of the famous – the method of minimal realization (MMR) [3-5].

The usefulness of MMR is based on modern compartmental-cluster theory (CCT) of the RNN [1, 3]. According to such theory the RNN have a compartmental (pool) structure and the mathematical model of such NN have a vector-matrix form [1, 6]:

$$\frac{dx}{dt} = AP(y)x - bx + ud \ , \tag{1}$$

where vector $x \in R^m$, b is a damping coefficient, ud present the input (chemoreceptor drive CD of RNN for example) drive shaping the trajectory of central inspiratory activity, vector $d(d \in R^m)$ is the weighting coefficient of CD and $u \geq 0$ is a scaling factor. Matrix $P = P(y)$ presents the inhibition between pools, where y is the integral activity of RNN. Value y is defined by the formula

$$y = c^T x \ , \tag{2}$$

where vector c presents the weighting coefficient c_i. The components $p_i(y)$ of matrix $P(y)$ reduce the conductivity of excitation from j-th to i-th pool in $p_i(y)$ times. Matrix $A = \{a_{ij}\}_{i,j=1}^{m}$ presents the conductivity of excitation between pools.

So the compartmental approach [1-4, 7] is especially suitable for the modeling of the RNN's rhythm generation and the stationary regimes (SR). The SR of neural network is a special regime when RNN can be presented as linear system and all parameters of RNN does not changing. In this case the mathematical model of RNN has the form of linear differential equation and matrix $P(y) = I$.

For this case we find the coordinates of vector x (here $x = x_0$ present the stationary point of RNN's model) for different configuration of RNN's connectedness. For example if RNN is presented by oriented graph with an irreducible cycle form $(a_{im} \neq 0)$ the coordinates of SR (according to our calculation for model (1)) are [6]:

$$x_0^1 = \frac{u_0}{b^m - 1}\left(b^{m-1}, b^{m-2}, \ldots, 1\right)^T \ \text{for} \ d = (1, 0, \ldots 0)^T \tag{3}$$

$$x_0^2 = \frac{u_0}{b^m - 1}\left(b^{m-2}, b^{m-3}, \ldots, b^{m-1}\right)^T \ \text{for} \ d = (0, 0, \ldots, 1)^T \tag{4}$$

$$x_0^3 = \frac{u_0 \sum_{i=1}^{m} b^{i-1}}{b^m - 1} (1,1,\ldots,1)^T \text{ for } d = (1,1,\ldots 1)^T . \quad (5)$$

Other case for cyclic RNN with subcycle we can obtain SR with coordinates of stationary point x_0 :

$$x_0^1 = \frac{-(1-b)u_0}{b^{m+1} + b^m - b^{m-1} + 1} B_1^T \text{ for } d = (1,0,\ldots 0)^T \quad (6)$$

$$x_0^2 = \frac{-(1-b)u_0}{b^{m+1} + b^m - b^{m-1} + 1} B_2^T \text{ for } d = (0,0,\ldots,1)^T \quad (7)$$

$$x_0^3 = \frac{-(1-b)u_0}{b^{m+1} + b^m - b^{m-1} + 1} B_3^T \text{ for } d = (1,1,\ldots 1)^T , \quad (8)$$

where B_1 , B_2 , B_3 are some special vectors [4].

So for construction of mathematical model of RNN we must have a SR of respiratory neuron networks and the method of minimal realization is useful for its identification. Let's consider this problem more detail.

2. Stationary regimes and usefulness of MMR for its identification.

The problem of identification of stationary regime of RNN is one of the main problems in the theory of respiratory rhythm generation. It is evident that the SR is presented by stationary point on the mathematical model of RNN (see (1)) if such model we can identify. For our case we assume that the stationary regime of RNN is characterized by stable of value of the integral activity y . The value y is presented by output of RNN. For example it is the activity of the phrenic nerve or an intercostals muscles (recording with special gold electrodes in our experiments).

In some special cases the value y may be equal zero. In our experiments we observed the stationary regime of RNN after hyperventilation of experimental animal when $y = 0$. In other cases we obtain the SR after application of solution of fenibutum or karfedonum (GABA generatives) to the ventral surface of the medulla oblongata and to structures located ventrolaterally in rostral medulla [3]. After such application and microinjection the frequency of breathing was not changed, but the output of RNN (phrenic nerve activity) slowly approaching to zero $(y \to 0)$.

Other case we have $y > 0\,(y = const)$ but the SR takes place because output signal y does not change. Such situation we observed after micro-injection of karfedonum or fenibutum (derivator of GABA) into the parabrachial nucleus [3-4, 6]. So there are two cases presenting the biological conception of the RNN's SR such as: $y = 0$ (a) or $y = const$ (b). In our experiments we obtain these two types of SR with special biological manipulation and such results are new results in physiology. For biology it is one of the typical examples of principle NN dynamic. We must say that SR is not so simple regime. Our investigation was illustrated the radical changes of the structure of RNN according to our procedure of identification. In our experiments we obtain the different order m of mathematical models of RNN if we change the dose of drugs (for anesthesia) when $y = 0$ after hyperventilation or application of GABA and its generatives. In this case the necessary condition ($y = 0$ or $y = const$) takes place but the order m of linear model is variable. So such experiments we can present as a special control manipulation on RNN.

It is evident for us that the identification procedure needs not only on determination of matrix A and the order m of mathematical model of RNN but it needs the calculation of matrices spectrum. We must compare the characteristic polynomials of the obtained matrices for different sets of neurophysiology experiments. Now it is clear that the equality of characteristic polynomials for matrices with a simple spectrum means, that one model can be transformed into the other by means of a change of variables. So if we identity the SR of RNN we must be sure that the characteristic polynomials of matrices A and the spectrum of the matrices does not change after some biological manipulation. Other way we do not have a stationary regime of RNN and some structural change of RNN may be. Such procedure we construct for creation of our software for RNN SR identification.

There are some special methods providing structural and parameter identification of mathematical models of RNN in a SR. One of them is the method of minimal realization [5, 8]. The MMR provides not only structural and parameter identification of RNN's model but also determines the lowest order of the model and optimum length t and amplitude u for the stimulus (it perturbs the RNN in SR). An important aspect of the problem of linear approximations and utilization of the MMR in neurophysiology and especial for RNN's investigation is the problem of selecting the optimal duration of stimulus [5].

It is clear that we can't use the method of minimal realization if the RNN has not linear property. So we must prove a special linear regime of RNN before its identification. For this purpose we must analyze the empirical relationships between the effect at the input of the RNN and the deviation of the outgoing variable obtained at the output. If we increase the amplitude u of input impulse in k time and then the output of RNN increase in same k time the RNN has the linear property.

The second question in the problem of RNN identification is connected with selection of optimum stimulus parameter. The investigator must know the optimum length and optimum amplitude of stimulus when the RNN has a linear property. The method of minimal realization can provides the selection of optimal parameters of input stimulus. How it must be? The answer of the question we are presenting now.

3. Experimental results of MMR using for RNN's SR identification.

As an example we represent the data of identification of compartmental structure of expiratory neuron network according to fig.1. Here we present the various responses of internal intercostals nerve, according to various duration of stimulus $(t = 5, 10, 15, 20\, m\sec)$ (see fig. 1). The procedure of reduction the matrix A of all connectedness between compartments to an eventually non-negative form for a case $t = 5\, m\sec$ was obtained by us in our calculation with MMR using.

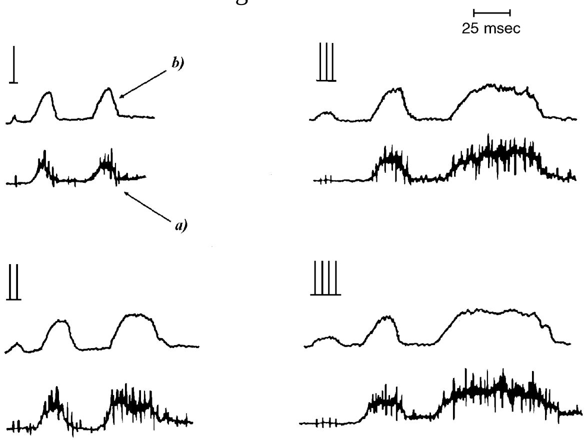

Fig. 1. *The dependence of spinal-bulbo response (SBR) activity of 10-th internal intercostal nerve on duration of stimulation of 11-th internal intercostal nerve of cat in conditions of a hyperventilation and chloralos anesthetized. The duration of one impulse 1* $m\sec$, *frequency in a series 200 Hz duration of a series varies from 5* $m\sec$ *up to 20* $m\sec$*; a) – natural nerves activity; b) – the integral nerves activity after stimulation.*

It is necessary to note, that the duration of testing stimulus in our experiments was selected according to two procedures. For first, we required that the invariants of a matrix A must be equal to the length of duration of stimulus. The next condition equal to requirement minimal order of RNN model ($m \leq 7$). If the order $m > 7$ it may be the point of catastrophe. Particularly in considered experiment border of an admissible interval (when the system remained quasilinear property) were the following $t \in [t_{\min}, t_{\max}] = [1\, m\sec, 60\, m\sec]$. If $t > t_{\min}$ the linear properties of RNN was observed, and if $t > t_{\max}$ it may be the oscillatory regime for SBR-responses and we have the point of catastrophy when $m >> 7$. Outside of borders of the indicated interval of stimulus duration we considered that the procedure of MMR is non-effective.

Table 1 are indicated (in conventional units) values of Markov parameters for each case of duration of testing stimulus (see fig.1).

Table 1

y_i	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9

$t = 5\, m\sec$	0,8	5,89	8,2	8,6	5,8	3,4	1,2	1,1	1,0
$t = 10\, m\sec$	1,0	4,36	6,0	10,3	7,5	5,0	2,0	1,0	0,5
$t = 15\, m\sec$	0,9	5,0	9,1	9,2	9,0	8,8	4,5	0,8	0,2
$t = 20\, m\sec$	1,0	5,0	9,2	9,3	9,1	9,0	4,1	0,5	0,1

We use the MMR for identification of RNN for case when the duration of stimulus is $t = 5\, m\sec$. After computer identification of outputs (see Table 1) we obtain the numerical values of triple of matrices A, B, C according to MMR procedure [7]. The eigenvalues of a matrix A, for the indicated case satisfied the condition of the Perron-Frobenius theorem: $\lambda_1 = 1.55$, $\lambda_2 = 0.97$, $\lambda_{3,4} = 0.57 \pm 0.63i$, $\lambda_5 = -0.91$.

Really the eigenvalue equal 1,55 will be Perron root, because it is positive and it's modulo exceeds all stayed eigenvalues. Because the Perron root was calculated for our model we can propose that the condition of the Perron-Frobenius theorem may be realized and the non-negative form of matrix may be.

For a case $t_2 = 10\, m\sec$, $t_3 = 15\, m\sec$ and $t_4 = 20\, m\sec$ a triple of matrixes A, B, C were calculated too and all of them have a Perron roots ($2.44, 4.89$ *and* 5.35). Thus according to the Perron-Frobenius theorem the existence of Perron root for the considered cases was proved with experimental data by us. So the existence of an eventually non-negative matrix A may be as a result of the interaction between compartments and a synergetic character. Such relation in RNN provides the increasing of the Perron roots according to special condition between the increasing of stimulus duration and eigenvalues of matrices A [5, 6].

There is a special property of eigenvalues L_i of the matrix A (when the duration of stimulus equal t) and eigenvalues $\overline{L_i}$ of matrix $\overline{A}$ (when the duration of stimulus equal qt, $q = 2,3,...$) when the linear property of RNN does not changes. According to famous definition it must be $L_i^q = \overline{L_i}$. It is easy to see that all Perron roots satisfy of this condition. So we can say: the linear property of our RNN on interval [5, 20] msec may be. Such mathematical procedure for experimental investigation can be used for identification of RNN's properties and MMR can be usefulness for RNN investigation.

So we present the possibility of software using for identification of optimal input signal (time interval) when RNN are in linear regime and its synergetic property may be. The procedure provides the identification of such regimes according to compartmental-cluster's theory and MMR using. It is principle new method for RNN investigation in special stationary condition. Before that the investigators have not mathematical criteria for comparison of SR for different animal in different experiments.

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