SYSTEMS OF CONTROL WITH AFTER-EFFECT AND OPEN PROBLEMS

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Abstract. The paper deals with the qualitative control and observation theory for linear systems with after-effect. The main problems under consideration are finite and infinite dimensional controllability and observability problems, and modal control. Some principle results in such a field are presented and open problems are formulated.

1. History and available results

The beginning of the research in the qualitative time-delay system control theory dates back to N.N.Krasovskii's (1963)-report at the Second IFAC Congress [1] where the problem of total quieting (controllability to zero function, complete controllability) was formulated for the simplest system with delay. As a whole, headed by academician N.N.Krasovskii Sverdlovsk's School of Control Theory, prepared such leading experts in the area as academicians A.B.Kurzhansky and Yu.S.Osipov, professors R.Gabasov and F.M.Kirillova etc.. This school possesses a world priority in statement of such basic qualitative control theory problems as complete controlability (N.N.Krasovskii, A.B.Kurzhansky (1963, 1966)), stabilization under action of integral feedback (N.N.Krasovskii, Yu.S.Osipov (1963, 1965)) etc. Simultaneously, F.M.Kirillova and S.V.Churakova (1967) and independently L.Weiss investigated the problem of relative(Euclidean, R^n-) controllability. Since then different aspects of controllability and observability problems of systems with after-effect became the subject of investigations of many authors (see, for example, survey papers [2,3] and its references).

Let us take a look at some basic stages of development of the qualitative control and observation theory for systems with after-effect on an example of the simplest time-delay system (SDS):

$$\dot{x}(t) = Ax(t) + A_1x(t-h) + Bu(t),\ y(t) = Cx(t),\ t > 0\,, \tag{1}$$

The first question under consideration is the form of initial-value problem:
Join continuous form (N.N. Krasovskii, A.D. Myshkis, J. Hale)

$$x(\tau) = \varphi(\tau),\ \tau \in C([-h,\ 0],\ R^n)\ ; \tag{2}$$

Initial jump form (N.V. Azbelev, A. Manitius and others)

$$x(0) = x(+0) = \varphi_0 \in R^n,\ \ x(\tau) = \varphi(\tau),\ \ \tau \in [-h, 0)\ ;$$

Our form of initial conditions

$$x(0) = x(+0) = \varphi_0 \in R^n,\ \ A_1x(\tau) = A_1\varphi(\tau),\ \ \tau \in [-h, 0) \tag{3}$$

where φ is a piecewise continuous in $[-h,\ 0]$ vector function.

System (1) is said to be:

i) relatively t_1-controllable (R^n-controllable at time t_1) where $t_1 > t_0 = 0$ if for any $n-$vector $x_1 \in R^n$ and initial data φ, φ_0 from (3) there exists a piecewise continuous $r-$vector function control $u = u(t), t \in [t_0,\ t_1]$ such that the corresponding solution $x(t) = x(t, t_0, \varphi_0, \varphi, u)$, $t > t_0$, of System (1) satisfies the condition $x(t) = x_1$;

ii) relatively zero-t_1-controllable (R^n-zero-controllable at time t_1) if $x_1 = 0 \in R^n$ in the previous definition;

iii) pointwisely controllable in points $\beta_0, \beta_1, \ldots, \beta_\gamma$ where $0 = \beta_0 < \beta_1 < \ldots < \beta_\gamma$ if there exists a time moment $t_1 > t_0 + \beta_\gamma$ such that for any initial data φ_0, φ from (3) and for any vectors $c_j \in R^n, j = 0, 1, \ldots, \gamma$, there exists a piecewise continuous control $u = u(\cdot)$ such that the corresponding solution of the system satisfies the condition $x(t_1 - \beta_j,\ t_0,\ \varphi_0,\ \varphi,\ u) = c_j;\ \ j = 0, 1, \ldots, \gamma$;

iv) α-pointwisely controllable for $\alpha \ge 0$ if it is pointwise controllable for arbitrary points $\beta_0,\ \beta_1,\ \ldots,\ \beta_\gamma$ such that $0 = \beta_0 < \beta_1 < \ldots < \beta_\gamma \le \alpha$;

v) poinwisely (multipoint) controllable if it is $\alpha-$ pointwisely controllable for all non-negative real number α.

In contrast to Kalman's systems ($A_1=0$), examples show that the first two controllability problems are not equivalent and it did not completely take into account in the first papers on controllability of time-delay systems. For the problem of relative controllability an effective rank condition was obtained [see, for example [2,3]] in terms of the determining equation:

System (1) is relatively t_1 -controllable if and only if the following condition holds

$$rank\left[X_k\left(jh\right);k=0,1,\ldots,n-1;\,j=0,1,\ldots,\alpha\right]=n, \tag{4}$$

where $\alpha=\lim_{\varepsilon\to+0}\left[\frac{t_1-\varepsilon}{h}\right]$ and symbol $[d]$ denotes the entire part of number d , and $X_k\left(t\right),\ t\geq 0,\ k=0,1,\ldots$ is a solution of the corresponding determining equation

$$X_{k+1}\left(t\right)=AX_k\left(t\right)+A_1X_k\left(t-h\right),\ t\geq 0\,,k=0,1,2,\ldots, \tag{5}$$

with initial conditions of the form $X_0\left(0\right)=B\,,X_0\left(t\right)=0$ for $t\neq 0$. Later, the condition (4) was extended (see [2,3]) to systems with several delays and neutral type concentrated delay systems. But, it seems that the role of the determining equation in investigating properties of time-delay systems has not been completely studied yet and does not restrict to the role in investigating the relative controllability problem. In our opinion, it deals with some structure properties of system under consideration, for example, the fundamental matrix $F(t)$ of System (1) solutions can be expressed in the form (B.Sh.Shklyar)

$$F\left(t\right)=\sum_{i=0}^{+\infty}\sum_{j=0}^{p}X_i^1\left(jh\right)\frac{\left(t-jh\right)^i}{i!},t\in\left[ph,\left(p+1\right)h\right],p=0,1,2,\ldots,$$

where $X_i^1\left(t\right),\ t\geq 0,\ i=0,1,\ldots$ is a solution of the equation (4) with $B=I_n$ that allows to obtain a determining equation series representation (V.M.Marchenko) for solutions of System (1)

$$x(t)=v(t,\,\varphi_0,\,\varphi)+\sum_{k=0}^{+\infty}\sum_{t-jh>0}X_k(jh)\int_0^{t-jh}\frac{(t-\tau-jh)^k}{k!}u(\tau)d\tau,\ t>0\,. \tag{6}$$

where function $v(\cdot,\,\cdot,\,\cdot)$ depends on initial data only. A direct way for obtaining such a representation, which makes more precise of the well-known representation of Bellman and Cook, is based on algebraic properties of the determining equation solutions and delay operator [2,3], in particular, we have

Lemma 1. For the solution of the determining equation the following identity is valid

$$(A+mA_1)^kB=\sum_{j=0}^{k}m^jX_k(jh)\ \text{ for }\ m\in R\ \text{ and }\ k=0,1,\ldots \tag{7}$$

Lemma 2 (generalized Hamilton-Cayley theorem). The solution $X_k(t)$ of the determining equation satisfies to the system characteristic equation

$$X_{n+k}(\gamma h)=-\sum_{i=1}^{n}\sum_{j=0}^{i}r_{ij}X_{n-i+k}((\gamma-j)h)\qquad \text{for }\ \gamma=0,1,\ldots\ \text{ and }\ k=0,1,\ldots \tag{8}$$

where r_{ij} for $j=0,1,\ldots,i$ and $i=0,1,\ldots,n$ are the coefficients of the characteristic equation

$$\det(\lambda I_n-A-e^{-\lambda h}A_1)=\sum_{i=0}^{n}\sum_{j=0}^{i}r_{ij}\lambda^{n-i}e^{-\lambda jh}=0. \tag{9}$$

where $\lambda\in C$, C is the field of complex numbers.

Lemma 3. For any natural numbers p and q the following *rank* condition holds

$rank[X_k(jh)$ for $j=0,1,\ldots,q$ and $k=0,1,\ldots,p]==rank[X_k(jh)$ for $j=0,1,\ldots,\min\{q,n-1\}$ and $k=0,1,\ldots,\min\{p,n-1\}]$. (10)

Results formulated allow to find a finite number of generators in linear span of the determining equation columns and, as a result, gives a way for describing the set of relatively controllable (or, precisely, relatively attainable) states of System (1). They also give a techniques, along with the regional frequency formal operator proof (R.Gabasov, F.M.Kirillova and S.V.Churakova) for obtaining several proofs of relative controllability criterion (4) that are very similar to the well-known ones for Kalman's controllability criterion.

Investigation of the problem of relative zero-controllability is essentially complicated because of possible degeneration (congruence) of solutions. It means that all the trajectories of system under consideration may not completely fill out the space R^n. Systems are said to be complete if they are not degenerate. In case of complete systems criteria of relative and zero-relative controllability are coincide. Such a result is true for general neutral type distributed delay systems of the second order ($n=2$) but it is not true, in general, for retarded type distributed delay systems as well as for neutral type concentrated delays systems for $n=3$. It is shown [1,2] that System (1) is complete for $n \le 5$ and may be degenerate for $n=10$. The question about the greatest dimension n, for which the properties of relative and zero-relative controllability are equivalent, is open. It has remained open the problem of finding effective zero-controllability criteria in such a completed form as criterion (4).

Parametric criteria for the kinds iii) and iv) of pointwise controllability can be obtained (S.A. Minyuk) in a similar way as (4) with techniques and in terms of the determining equation solution. The problem of pointwise controllability was stated and solved by V.M. Marchenko. He proved that the property of α-pontwise controllability is completed, i.e. if System (1) is α-pointwisely controllable for $\alpha = \alpha_0 = \frac{(n-1)(n-2)h}{2}$ then it is α-pointwisely controllable for $\alpha \ge \alpha_0$ and, as a result, is pointwisely controllable. And what is more, it was proved that *System* (1) *is poinwisely controllable if and only if the following one-parameter system without delay*

$$\dot{x}(t) = (A + mA_1)x(t) + Bu(t),\ t > 0, \tag{11}$$

is controllable in Kalman's sense of least for one real number m. This result gives the base for generalizing the majority of statements of Kalman's mathematical system theory to time-delay systems. The results can be generalised to the neutral and retarded type systems with several delays.

One of the most difficult problem of the qualitative control theory of systems with delay is the problem of complete controllability in sense of Krasovskii. A general scheme for testing every concrete time-delay system from the point of view of its complete controllability is proposed by R.Gabasov and F.M.Kirillova, by A. Olbrot , and by G.P.Razmyslovich for numerous attempts. A parametric criterion for the complete controllability (Problem of Krasovskii) was obtained by V.M.Marchenko [2]:

System (1) *is completely controllable (at time* $t_1 = t_0 + s$ *where* $s > nh$*) if and only if the following condition holds*

$$rank\left[\lambda I_n - A - A_1 \exp(-\lambda h), B\right] = n, \forall \lambda \in C. \tag{12}$$

An important number of questions are realised in the field of feedback control. The main attention is paid here to the problem of modal control. Observe that the traditional statement of the problem of modal control given by W.M. Wonham in 1967 can not be used for time-delay systems because of infinite dimensionality of such systems. The first problem of modal control of time-delay systems was considered in 1974 by V.I. Bulatov, R.F. Naumovich and T.S. Kaluzhnaya as a partial modal control problem of control by arbitrary given finite part of the system spectrum. The method applied by the authors was a modified method of Krasovskii and Osipov in investigating the problem of stabilization. The problem of modal control, in general statement, as a problem of control by the coefficients of the system characteristic equation was given in 1976 by I.K.Asmykovich and V.M.Marchenko. Then infinite dimensional, according to the statement, problem of control by eigenvalues was reduced to a finite dimensional problem of control by the coefficients. Various kinds of regulators have been used intensively by several authors to solve the problem stabilization and modal control for different types of systems with after-effect. Krasovskii and Osipov considered an integral type in $[-h,\ 0]$ of feedback to solve the problem of stabilization.. More general feedback $u(t) = \int_{-\theta}^{0} dQ(s)x(t+s),\ t > 0,$ where $\theta \ge 0$ is to be defined, was used by Marchenko for solving the general problem of modal control. In case where the kernel Q is constant, except of a finite number of finite jumps, the feedback is reduced to the difference regulators

$$u(t) = \sum_{j=0}^{\theta} Q_j x(t - jh), t > t_0, Q_j \in R^{r \times n} \ (j = 0,1,\ldots,\theta)\ . \tag{13}$$

It seems that regulator (13) is more convenient in practical realizations. From this point of view, the following type of regulator (13)

$$u(t) = Q_0 x(t) + Q_1 x(t-h)$$

is of great interest to study. Difference regulators (13) were introduced in 1976 by I.K. Asmykovich and V.M. Marchenko and independently A.S. Morse in their investigations of the problem of modal control.

System (1) is said to be modally controllable by action of feedback (13) if for any real numbers $r^1_{ij},\ j=0,1,\dots,i$ and $i=1,\dots,n$ there exists a regulator (13) such that the characteristic equation of System (1) closed by such a regulator has form

$$\lambda^n+\sum_{i=1}^{n}\sum_{j=0}^{i} r^1_{ij}\lambda^{n-i}e^{-\lambda jh}=0\ .$$

In a similar way the definitions of modal controllability of System (1) by action other types of regulators can be given. We state [2,3] that *System* (1) *with single input* ($r=1$) *is modally controllable by regulator* (13) *if and only if the following condition holds*

$$\det W(m)=\det\left[B,(A+mA_1)B,\dots,(A+mA_1)^{n-1}B\right]\equiv const\neq 0,\qquad \forall m\in R\ . \tag{14}$$

The problem of modal control by action of integral regulators is investigated (V.M.Marchenko) by using the integer function theory methods, in particular, by using the Wiener-Paley theorem, and it is proved that *System* (1) *is modally controllable by integral regulators if and only if it is completely controllable in Krasovskii's sense*. It is a generalization to systems with delay of the well-known theorem of W.M. Wonham.

Several generalizations of the modal control problem to systems with several delays, neutral type systems and distributed delay systems are given in [2,3] for both complete and incomplete information case where an analogy of dynamic regulator of Pearson is proposed.

Under the influence of abstract state approach in the general mathematical system theory there appear such approaches in the qualitative time delay control and observation theory. Actually, in the approach, a set $\Omega\ni\varphi$ of initial data for join continuous form $\varphi(0)=\varphi_0$ or $(\varphi_0,\varphi)\in R^n\times\Omega$ for initial jump form (3) is interpreted as a set □ of initial states of the system. Then Â -controllability (functional) is considered as the existence of a control function for which the corresponding trajectory joins two arbitrary given points from Â . Similarly, complete observability is interpreted as the possibility to differ initial data by the output measurements. It proves to be that even Â is isomorphic to Sobolev space $W_2^{(1)}\left([-h,0],R^n\right)$ or $M_2=R^n\times L_2\left([-h,\ 0],\ R^n\right)$, the problem of Â -controllability can be solved in exceptional cases only. That is why the property of controllability became the consideration in more weak sense, for example, as a problem of approximate controllability. Let us formulate some results [1,2]:

i) System (1) is M_2 -approximate controllable (S.A. Minyuk, S.N. Luakhovets) if, along with (12), the condition $\det A_1\neq 0$ holds;

ii) System (1) is completely observable (A.V. Metelskii, S.A. Minyuk, B.Sh. Shklyar) if the conditions $\det A_1\neq 0$ and $rank\left[\lambda I_n-A^T-A_1^Te^{-\lambda h},C^T\right]=n$ for all $\forall\lambda\in C$ are valid.

It is not difficult to see that such notions of controllability and observability are not generalizations of Kalman's ones for Systems (1) with $A_1=0$. It takes place because of state in the above sense is not minimal, in general. Historically, one of the first problems on functional controllability was the problem (E.A.Barbashin) of realization of motion along to a given trajectory. Another approach to the investigation of functional controllability and observability properties was proposed by V.M. Marchenko on the base of "minimal state" notion. According to such an approach, at the investigation of concrete control and observation processes (studying the established processes) it is often necessary to consider a control problem after a certain time s of "idle" system work. In this situation from the point of view of control it makes no difference how the system has behaved before. Thus, it is not necessary to distinguish those solutions which coincide for $t\geq t_0+s$. Let Ω be an arbitrary set of initial data. Introduce the relation of equivalence L_s on Ω, having put $(\varphi_0,\ \varphi)L_s(\psi_0,\ \psi)$ if and only if for any admissible control $u=u(\cdot)$ the corresponding solutions of the system satisfy the condition $x(t,\ t_0,\ \varphi_0,\varphi,u)=x(t,\ t_0,\ \psi_0,\ \psi,v)$ for $t\geq t_0+s$. Then the set Ω/L_s of equivalent solution classes (factor-set) is interpreted as a set of admissible initial s -states of the system and its image with respect to the system "input-output" mapping gives the set of s -solution of the system. At last, the restriction ${}_sX_t$ of the set of s -solutions on the interval $[t_0+t-h,\ t_0+t]$ makes sense of the set of admissible s -states at the moment t . Hence by analogy with Kalman's definition, the problem of s -observability is considered [2,3] as the possibility of restoration (distinguishing), according to the output measurement, of the corresponding initial s-states. Analogously, we define the property of s-controllability as the possibility to transfer the system trajectory from arbitrary initial state to arbitrary final one by choice of a control function. The case where s is not fixed, we say weak state case and put $s=\infty$, is of special interest because of the sets ${}_{\infty}X_t$ are isomorphic and the weak system solutions corresponding to different weak initial data are also different, i.e. the system operator semi-group turns into its quotient group. Such a minimal state approach can be applied to the general non-stationary neutral type distributed delay systems and to more general concepts of $(s,\ t)-$ and $R^n-(s,\ t)$ -controllability and observability. But the best way to understanding the $R^n-(s,\ t)$- and $(s,\ p)-$ controllability problems is its game interpretation as a problem of pursuit of objects of the same dynamics type when the start t_*-s of motion for pursuivant and t_*-p for evasive are different, and the control v of evasive is known in the whole interval $[t_*-p,\ t_*]$ of pursuit (we say with discrimination of evasive).

For given $p\geq 0,\quad s>p$, and $s>0$ SDS (1) is said to be:

(i) $R^n-(s,\ p)$-controllable at time $t_*=t_0+s$ if for any initial data $\varphi_0,\ \varphi$ and $\psi_0,\ \psi$ and for any piecewise continuous r-vector function v there exists a piecewise continuous control function $u=u(\cdot)$ such that for the corresponding solutions $x(t)=\ x(t,\ t_*-s,\ \psi_0,\ \psi,\ u),\ \ t\geq t_*-s$, and $x(t)=x(t,\ t_*-p,\ \psi_0,\ \psi,\ v),$, $t\geq t_*-p$ the following condition holds: $x(t_*,\ t_*-s,\ \varphi_0,\ \varphi,\ u)=x(t_*,\ t_*-p,\ \psi_0,\ \psi,\ v)$;

(ii) $(s,\ p)-$ controllable at time $t_*=t_0+s$ if for any initial data $\varphi_0,\ \varphi$ and $\psi_0,\ \psi$ and for any piecewise continuous r-vector function v the exists a piecewise continuous control function $u=u(\cdot)$ such that the corresponding solutions satisfy the condition $x(t_*+t,\ t_*-s,\ \varphi_0,\ \varphi,\ u)=x(t_*+t,\ t_*-p,\ \psi_0,\psi,\ v)$ for $t\geq 0$;

(iii) superstrongly controllable if it is $R^n-(s,\ p)$ - controllable for all real s and p such that $s\geq p\geq 0$ and $s>0$. Dual observability concepts can be found in [2].

The following duality principle is true [2]:

(i) $R^n-(s,\ t)-$ *controllability is dual to* $R^n-(s,\ t)-$ *observability;*

(ii) $(s,\ t)-$ *controllability is dual to linear* $(s,\ t)-$ *observability;*

(iii) *approximate controllability is dual to* $(s,\ t)-$ *observability;*

This principle essentially places the role of Fredholm alternative in the corresponding problem of control. Notice that the introduced concept of controllability are direct generalizations of Kalman's ones and the following statements are valid:

(i) $R^n-(s,\ p_1)$-controllability implies $R^n-(s,\ p_2)$ - controllability for $s>0$ and $s\geq p_2\geq p_1\geq 0$;

(ii) SDS (1) is superstrongly controllable if and only if $rank\left[B,\ AB,\ \ldots,\ A^{n-1}B\right]=n$ and $rank\left[A_1,\ B\right]=rankB$;

(iii) SDS (1) is $R^n-(s,\ p)$ - controllable for $0\leq p\leq s\leq h,\quad s>0$ if and only if it is superstrongly controllable.

Similar results can be formulated for the problem of $(s,\ t)-$ controllability.

Notice that property of $R^n-(s,\ p)$ - controllability at time s of SDS (1) is equivalent for $p=0$ to the relative controllability one. It is also equivalent for $p=s$ to the relative zero-controllability. The problem of $(s,\ s)-$ controllability at time t_0+s is equivalent to the complete controllability in Krasovskii's sense. Then we can state that SDS (1) is $R^n-(s,\ 0)$- controllable at time t_0+s if and only if condition (4) holds. Similarly, SDS (1) is $(s,\ s)-$ controllable at time t_0+s with $s>nh$ if and only if condition (12) is valid. Concluding, observe that $(s,\ p)-$ and $R^n-(s,\ p)$-controllability are equivalent for $0\leq p\leq s\leq h,\ s>0$.

One of the basic problems of the qualitative control theory is controllability in special classes of control functions [1,2]. Let us consider the following two classes.

Dynamic Control (DC):

$$\dot{u}(t)=A^cu(t)+A^c{}_1u(t-h^c)+B^cv(t),\quad t>0,$$

where $A^c \in R^{r\times r}$, $A^c{}_1 \in R^{r\times r}$, $B^c \in R^{r\times q}$ and $h^c > 0$ with initial conditions like (3) for $u(\cdot)$ and with "new" control $v(\cdot)$. The case of $B^c = 0$ is of special interest.

Simplex Control (SC): the function $u(\cdot)$ is a piecewise continuous function which satisfies the condition $A^c u(t) \le b^c$ for $A^c \in R^{m\times r}$ and $b^c \in R^r$, and the inequality is for the corresponding entries.

2. Controllability concept comparison

Consider SDS (1) with $B = b \in \square^n$ and introduce abbreviations:
"ssc" for superstrongly controllability;
"mcdf" for modal controllability by action of difference feedback (13);
"cc" for complete controllability in Krasovskii's sense (at $t_* = t_0 + s,\ s > nh$);
"ccad" for complete controllability for all delay $h \ge 0$;
"mcif" for modal controllability by action of integral feedback;
"pcp" for pointwise controllability in points $0 = \beta_0 < \ldots < \beta_\gamma \le \alpha$;
" α -pc" for α -pointwise controllability;
"pc" for pointwise controllability
"rc" for relative controllability (for non-fixed time).

The following implications are valid:

"ssc" $\Rightarrow$ "mcdsf" $\Rightarrow$ "ccad" $\Rightarrow$ "cc" $\Leftrightarrow$ "mcif" $\Rightarrow$ "pc" $\Leftrightarrow$ " $\frac{(n-1)(n-2)}{2} h -$ pc"

$\Downarrow$ $\Downarrow$

" α -pc" $\Rightarrow$ pcp"($0 = \beta_0 < \ldots < \beta_\gamma \le \alpha$) $\Rightarrow$

$\Rightarrow$ "rc"($t_* = t_0 + s,\ s > (n-1)h$). Similarly, we can compare the dual observability concepts. Examples show that the inverse implications are not true in general.

3. Open problems

1. Criteria for $R^n - (s,\ p) -$ controllability.
2. Criteria for $(s,\ p) -$ controllability.
3. Find the minimal dimensional n of the system for which $(s,\ s) -$ and $(s,\ 0) -$ controllability are not equivalent (and the same for the $R^n - (s,\ s) -$ controllability).
4. Criteria for modal control of System (1) in the simplest linear time delay feedback class $u(t) = Q_0 x(t) + Q_1 x(t-h)$ (and the same for stabilization).
5. Complete controllability criteria for all delays (for small delays, at least for one delay, for fixed t_1).
6. DC and SC controllability criteria.

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