SPECTRAL, SEMI-FREDHOLM AND FREDHOLM PROPERTIES OF DIFFERENTIAL OPERATORS

Erovenko V.A. (Belarus, Minsk)

Let T be a closed linear operator densely defined on a complex Banach space. Essential spectra of an operator T could be defined as complements in a complex plane $\mathbb{C}$ of set defined by various Fredholm properties of family of operators $T - \lambda I$:

$$\sigma_{ek}(T) := \mathbb{C} \backslash \Delta_k(T), \quad k = \overline{1,5}, \qquad and \qquad \sigma_{e2}^{\pm}(T) := \mathbb{C} \backslash \Phi^{\pm}(T),$$

where $\Delta_1(T) := \{\lambda \in \mathbb{C} : \overline{R(T-\lambda I)} = R(T-\lambda I)\}$, $\Phi^+(T) := \{\lambda \in \Delta_1(T) : nul(T-\lambda I) < \infty\}$, $\Phi^-(T) := \{\lambda \in \Delta_1(T) : def(T-\lambda I) < \infty\}$, $\Delta_2(T) := \Phi^+(T) \cup \Phi^-(T) = s-\Phi(T)$, $\Delta_3(T) := \Phi^+(T) \cap \Phi^-(T) = \Phi(T)$, $\Delta_4(T) := \{\lambda \in \Delta_3(T) : ind(T-\lambda I) = 0\}$, $\Delta_5(T) := \{\lambda \in \Delta_4(T) :$ a deleted neighborhood of λ lies in the resolvent set $\rho(T)\}$.

Let's consider a formal differential expression for $a \le t < \infty$, $-\infty < a < \infty$

$$\mu := \sum_{k=0}^{n}(a_k + b_k(t))D^k = \tau + \sum_{k=0}^{n} b_k(t)D^k, \qquad \tau := \sum_{k=0}^{n} a_k D^k,$$

where a_k are complex numbers and complex valued functions of real argument $b_k(t) \in C^k[a,\infty)$, $0 \le k \le n$, and $D := d/dt$. Denote by $T(\mu,p,[a,\infty))$ ($T_0(\mu,p,[a,\infty))$) a *maximal* (*minimal*) *operator* corresponding $(\mu,p,[a,\infty))$ which is defined on $L^p(a,\infty)$ for $1 \le p \le \infty$. Let $S(\mu,p,[a,\infty))$, $-\infty < a < \infty$, be a closed linear differential operator in $L^p(a,\infty)$, $1 \le p \le \infty$, which is an extention of minimal operator $T_0(\mu,p,[a,\infty))$ and a restriction of maximal operator $T(\mu,p,[a,\infty))$ generated by differential operation μ, $T_0 \subseteq S \subseteq T$. Let coefficients b_n, $1/(a_n + b_n) \in L^\infty(a,\infty)$, and coefficients $b_k(t)$, $0 \le k \le n$, satisfy conditions

$$\sup_{m \le s < \infty} \int_s^{s+1} |b_k(t)|^p dt \to 0 \text{ as } m \to \infty.$$

Theorem. *For the minimal $T_0(\mu,p,[a,\infty))$, maximal $T(\mu,p,[a,\infty))$ and intermediate $S(\mu,p,[a,\infty))$ differential operators in $L^p(a,\infty)$, $1 \le p < \infty$, the following generalizations of the classic Weyl invariance essential spectrum theorem hold:*

$$\sigma_{ek}[S(\mu,p,[a,\infty))] = \sigma_{ek}[S(\tau,p,[a,\infty))], \quad k = 1,2,2^\pm,3,$$
$$\sigma_{ek}[T_0(\mu,p,[a,\infty))] = \sigma_{ek}[T_0(\tau,p,[a,\infty))], \quad k = 4,5,$$
$$\sigma_{ek}[T(\mu,p,[a,\infty))] = \sigma_{ek}[T(\tau,p,[a,\infty))], \quad k = 4,5,$$

Using this base theorem it is possible to receive the exact formulas for a finding of essential spectra of perturbed differential operators with constant coefficients and Fuchsian differential operators.

References. **1.** Erovenko V.A. // Math. Model. and Anal. 1998. V. 3. P. 86-92. **2.** Erovenko V.A. // Differential equations. 1999. V. 35, № 1. P. 58-64. **3.** Erovenko V.A. // Differential equations. 2000. V. 36, № 8. P. 1029-1036.