ON A CONVERGENCE'S PROPERTY OF CONTROL SYSTEMS PROGRAM MANIFOLD

S. S. Zhumatov (Almaty, Kazakhstan)

The problem of construction of all the set of differential equations possessing by given integral manifold have been formulated and a method of solving this problem is given in the work [1]. Later on Erugin's method was developed for construction of stable system of differential equations and nonlinear automatic control systems under given program manifold in [2] - [5]. It is known that realization of stability and other qualitative property of program manifold is required for construction program motion's systems. Establishment of convergence's properties of nonlinear systems and nonlinear automatic control's systems were studied in [6, 7]. The problem of finding of convergence conditions of control systems' program manifold is investigated with respect to the given vector-function in this paper.

Let us consider the material system subjected to external action, possessing by $(n-s)$ -dimensional integral manifold $\Omega(t) \equiv \omega(t,x) = 0$, where the motion of which described by equations

$$\dot{x} = f(t,x) - B\xi - g(t),\ \xi = \varphi(\sigma),\ \sigma = P^T\omega, \tag{1}$$

where $B \in R^{n\times r}$, $P \in R^{s\times r}$ are matrices, $x \in R^n$ is vector of objects state,$f \in R^n$ is vector-function satisfying of unique existence conditions of solution, $g \in R^s$ is vector-function of external perturbations , $\omega \in R^s$ is vector-function $s \leqslant n$, $\xi \in R^r$ is differentiable on σ vector of control on deflection from given program, for which are valid the following local quadratic connection's conditions

$$\begin{cases} \varphi^T\theta(\sigma - K^{-1}\varphi) > 0, \quad \theta = diag\|\theta_1,\dots,\theta_r\|, \quad K = K^T > 0, \\ K_1 \leqslant \dfrac{\partial\varphi}{\partial\sigma} \leqslant K_2, \quad K_i = diag\|k_1,\dots,k_r\| \quad (i=1,2), \quad K_2 >> 0. \end{cases} \tag{2}$$

As a result of the fact that $\Omega(t)$ is integral manifold for the system (1) hold $\dot{\omega} = F(t,x,\omega)$. Here $F(t,x,\omega)$ - is some Erugin's s -vector-function satisfying condition $F(t,x,0) \equiv 0$ [1]. Assuming $F = -A\omega, -A \in R^{s\times s}$ is Hurwitz matrix and differentiating the manifold with respect to time t in view of (1), we derive that

$$\dot{\omega} = -A\omega - HB\xi - Hg(t),\ \xi = \varphi(\sigma),\ \sigma = P^T\omega,\ H = \frac{\partial\omega}{\partial x}. \tag{3}$$

Theorem 1. *Let $(-A)$ is Hurwitz matrix and nonlinearity $\varphi(\sigma)$ satisfies (2), exist diagonal matrices K,θ,β, moreover $\beta_i > 0$ $(i = 1,\dots,r)$, such that*

$$\pi(\varpi) = \theta K^{-1} + Re[(\theta + i\varpi)W(i\varpi)] > 0$$

and for arbitrary $t \geqslant t_0$ is valid

$$\|\omega_1(t) - \omega_2(t)\| \leqslant N\exp[-\alpha(t-t_0)]\|\omega_1(t_0) - \omega_2(t_0)\|. \tag{4}$$

Then program manifold $\Omega(t)$ possess by property exponential convergence with respect to vector-function ω.

Here $W(i\varpi) = P^T(A + i\varpi E)^{-1}HB$ is transfer matrix of linear part of the system (3).

Theorem 2. *Let linearizing system (3) is asymptotical stable under nonlinearity $\varphi(\sigma)$ satisfying (2) and $\varphi(\sigma) = \mu\sigma, \mu \leqslant K$, i.e.*

$$l_s^{-1} V_0 exp[\alpha_1(t - t_0)] \leqslant \omega^2 \leqslant l_1^{-1} V_0 exp[\alpha_1(t - t_0)]$$

and for arbitrary $t \geqslant t_0$ is valid (4).
Then program manifold $\Omega(t)$ possess by property exponential convergence with respect to vector-function ω.

Here l_1, l_s are the least and the greatest roots of characteristic matrix, α_1, α_2 are combinations of corresponding characteristic matrices' roots.

Ideas of proof. In the first frequently and algebraical conditions of asymptotical stability of the system (3) are established by the method of construction Lyapunov's function in the form of "quadratic form plus integral from of non-linearity"and Popov's frequently criteria. In the second we consider two arbitrary solutions of (3) $\omega_1, \omega_2, \xi_1, \xi_2, \sigma_1, \sigma_2$. Introducing $z = \omega_1 - \omega_2, \zeta = \xi_1 - \xi_2, \eta = \sigma_1 - \sigma_2$. we obtain

$$\dot{z} = -Az - HB\zeta, \quad \eta = P^T z. \tag{5}$$

Hence the system (5) will be consider as linear part of some nonlinear system which entrance ζ and exist η satisfy the following condition

$$K_1 \leqslant \frac{\zeta(t)}{\eta(t)} \leqslant K_2 \quad for \quad \eta(t) \neq 0.$$

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