SMALL-AMPLITUDE LIMIT CYCLE BIFURCATIONS IN POLYNOMIAL SYSTEMS

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Consider systems of ordinary differential equations on $\mathbb{R}^2$ of the form

$$\dot{u} = P_n(u, v), \qquad \dot{v} = Q_n(u, v), \tag{1}$$

where P_n and Q_n are polynomials such that the maximal degree of P_n and Q_n is n. The parameter space $\mathcal{E}$ of the system is $\mathbb{R}^{(n+1)(n+2)}$, every point $E \in \mathcal{E}$ corresponds to a system of the form (1). Let $H(n)$ be the maximal possible number of limit cycles in family (1). A question of the second part of 16th Hilbert problem is to find a bound for $H(n)$. The question is not yet answered even in the simplest case of the quadratic system ($n = 2$). Moreover, it is unknown even whether $H(n)$ is finite.

An essential part of the problem, called sometimes the local 16th Hilbert problem, is investigation of small-amplitude limit cycle bifurcations, that is, studying the cyclicity of singular points (a singular point γ of a system $E \in \mathcal{E}$ has *cyclicity* k with respect to $\mathcal{E}$ if and only if any sufficiently small perturbation of E in $\mathcal{E}$ has at most k limit cycles in a sufficiently small neighborhood of γ, and k is the smallest number with this property).

The concept of cyclicity was introduced by Bautin [1]. He showed that the cyclicity problem in the case of an elementary focus or center can be reduced to the problem of finding a basis for the ideal generated by the coefficients of the Poincaré map in the ring of polynomials in the coefficients of the system. Nowadays it is well-known that basing on Bautin's approach the cyclicity problem can be easily resolved using algorithms of modern computational algebra in the case that the ideal generated by the initial string of focus quantities that determine the center variety (the Bautin ideal) is a *radical ideal* [3].

n the talk we describe a method based on algorithms of computational algebra for obtaining an upper bound for the number of small-amplitude limit cycles bifurcating from a center or a focus of polynomial vector field in the case, when the Bautin ideal is non-radical ideal [2]. We apply it to get upper bounds for the number of such limit cycles for some families of cubic system.

Refrences

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3. Romanovski V. G., Shafer D. S. *The Center and Cyclicity Problems: A Computational Algebra Approach.* Boston: Birkhäuser, 2009.