ON AN APPLICATION OF ONE DIFFERENCE-DIFFERENTIAL EQUATION IN ANALYTIC NUMBER THEORY

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Let $F(z)$ be a normalized Hecke eigenform of weight k for the full modular group having the Fourier series expansion

$$F(z) = \sum_{m=1}^{\infty} c(m) e^{2\pi i m z}, \quad c(1) = 1.$$

The zeta-function $\varphi(s, F)$, $s = \sigma + it$, attached to the form $F(z)$ is defined, for $\sigma > \frac{k+1}{2}$, by

$$\varphi(s, F) = \sum_{m=1}^{\infty} \frac{c(m)}{m^s},$$

and can be analytically continued to an entire function.

Suppose that $w \neq 0$ is an arbitrary complex number, and, for $\sigma > \frac{k+1}{2}$,

$$\varphi^w(s, F) = \sum_{m=1}^{\infty} \frac{g_w(m)}{m^s}.$$

Then the function $g_w(m)$ is multiplicative, and, in many applications, the asymptotics for the mean value

$$M(x) = \sum_{m \leqslant x} h_w(m), \quad x \to \infty,$$

where $h_w(m) = g_w^2(m) m^{1-k}$, is used. It turns out that the above asymptotics can be obtained by using the difference-differential equation

$$u\varrho'(u) = w^2 \varrho(u-1). \tag{1}$$

In [1], [2], the case $|w| \leqslant \frac{1}{2}$ has been considered. This report is devoted to the case $|w| > \frac{1}{2}$.

We note that the equation (1) is applied to obtain the asymptotics, as $x \to \infty$, for the sum

$$S_1(x, y) = \sum_{m_1 \leqslant x} h_w(m_1),$$

where all prime divisors of m_1 are greater than y, $y \to \infty$, and $\frac{\log x}{\log y} \to \infty$ as $x \to \infty$.

Refrences

1. Ivanauskaitė R., Laurinčikas A., Macaitienė R. *One application of differential equations* // First Intern. Conf. Mathematical Modeling and Differential Equations, Abstracts, Inst. Math., Minsk, 2007. P. 75–76.

2. Laurinčikas A., Steuding J. *On zeta-functions associated to certain cusp forms.*I. // Central European J. Math. 2004. Vol. 2, no 1. P. 1–18.