SCHRÖDINGER EQUATION IN POTENTIAL REPRESENTATION FOR WOODS-SAXON POTENTIAL

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Method of potential representation for solution of Schrödinger equation was proposed in [1], [2] for nucleus scattering calculations using optical model. Main features of that method is expression of wave functions or solutions of Schrödinger equation like functions of potential [1].The theorem for new multiplicative perturbation theory was presented [2]. According this theorem, when a new potential is added, the new solution can be obtained multiplying previous solution by the function depending on the new potential. This theorem was improved [3] for Coulomb and short range potentials, using modified method of undetermined coefficients of Lagrange. Presented theory can be presented for broad class of potentials but we most of our investigations applied for Woods-Saxon potential, useful for nuclear reactions rates calculations in reactors and astrophysical phenomenon [4].

Hear we consider analytical solution $u(r)$ of the radial Schrödinger equation

$$\frac{d^2u}{dr^2} - \left[k^2 + cV(r)\right]u = 0, \quad -k^2 = cE, \quad c = \frac{2m}{\hbar^2}, \quad V(r) = V_0/\left(1 + \exp\left(\frac{r-R}{a}\right)\right), \tag{1}$$

where m – mass, bound states energies $E < 0$, V_0, a, R – parameters of Woods-Saxon potential $V(r)$. We consider case of large radii $R \gg a$ of nuclei. Substituting $u(r) = \varphi(V(r))e^{-kr}$ into (1) and introducing the new variable $z = 1 - V(r)/V_0$, which depends on potential $0 < z \leqslant 1$ we obtain differential equation

$$(z^2 + z^3)\frac{d^2\varphi(z)}{dz^2} - (2z^2 - (1-2ka)z)\frac{d\varphi(z)}{dz} - ca^2V_0\varphi(z) = 0. \tag{2}$$

Requiring that solution of (2) must satisfy standard boundary condition $\lim_{r\to 0}\varphi(r) \to 0(r^1)$, we obtained asymptotical solution φ_0 for equation

$$z^2\frac{d^2\varphi_0(z)}{dz^2} + (1-2ka)z\frac{d\varphi_0(z)}{dz} = 0, \quad \varphi_0 = z^\beta, \quad \beta = 2ka. \tag{3}$$

Here we used approximation $\exp(-R/a) \approx 0$ (for large nucleus [1] Pb^{208} $R/a = 14$) where terms in (3) $\lim_{r\to 0} z^2 = e^{-2R/a}$, $\lim_{r\to 0} z^3 = e^{-3R/a}$ was not included. Now solution $\varphi(z)$ can be expressed $\varphi_0\varphi_g(z)$. Substituting last expression in (2) we obtained equation for $\varphi_g(z)$

$$(z^2+z^3)\frac{d^2\varphi_g(z)}{dz^2} - ((2+4ka)z^2 - (1-2ka)z)\frac{d\varphi_g(z)}{dz} - ((4k^2a^2 - 2ka)z + ca^2V_0)\varphi_g(z) = 0. \tag{4}$$

The solution $\varphi_g(z) = \sum_n z^n b_n$ was presented in the power series and recurrent connections between b_{n-1} and b_n were obtained. Physical solutions or wave functions for bound states must be polynomials of finite order and satisfy boundary conditions $\lim_{r\to 0} u(r) = 0$, $\lim_{r\to\infty} u(r) = 0$. Requiring that coefficient at b_{n-1} must be equal zero we obtain discrete eigenvalues - k_n and solutions present like polynomials. Physical eigenvalues must satisfy requirement $k_n > 0$.

For positive energies in (1) we must change $-k^2$ to the k^2. Just [1] solutions f can be expressed multiplying solutions $f_0(\pm k, r) = \exp(\pm ikx)$ for $V(r) = 0$

$$f(\pm k, r) = \varphi(\pm k, V(r))f_0(\pm k, r) \tag{5}$$

on function φ which depends on the potential $V(r)$. After substitution f in to (1) and introduction of the new variable $y = \ln(1 - V/V_0)$, for large nucleus case $\lim_{r\to 0}\exp(y) \approx 0$, following simplified equation [1] was obtained

$$\frac{d^2\varphi^2(\pm k, y)}{dy^2} \mp 2ika\frac{d\varphi(\pm k, y)}{dy} - ca^2V_0\varphi(y) = 0. \tag{6}$$

This equation was solved analytically and analytical expression of scattering matrix $S(k) = \lim_{r\to 0}(\varphi(+k, V)/\varphi(-k, V))$ was got. Analytical solutions are important for investigations of nuclei scattering and interactions. For this aim we used potential representation method [1].

Refrences

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