ANALYTICAL CALCULATION OF MULTIMASS DYNAMIC SYSTEM

N. A. Dokukova, E. N. Kaftaikina (Minsk, Belarus)

The technique of multibody dynamic systems research is developed which equilibrium equations have occasion to associated inhomogeneous differential equations of the second order with non-separating variables. This paper is described that such mechanical systems have general regularities when their characteristics researches are performed. The developed dependences allows to lower complex mathematical calculations, to replace the system of the second order associated differential equations with the system of the linear inhomogeneous differential constant rate equations independent from each other, to set the regularities of physical parameters influence on dynamic properties of the mechanical system in the form of the simple analytic formula. The suggested method can be used for analysis general regularities of passive uniaxial isolator. Consider oscillating system with the sixth degrees of freedom, as in Fig. 1. (each mass is connected to other five masses).

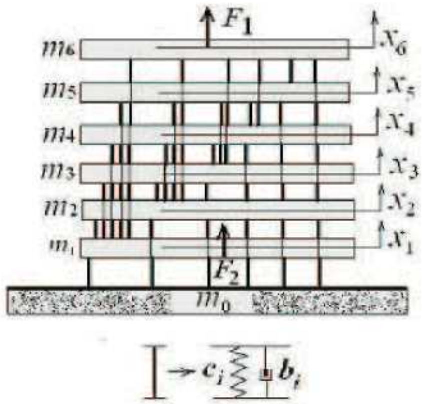

Figure 1: General dynamic scheme of the mechanical system forced oscillation.

Mathematical model of this system is represented by the following equation (see Eq. [1])

$$\ddot{X} = CX + B\dot{X} + F, \tag{1}$$

where $X = (x_1(t), x_2(t), x_3(t), x_4(t), x_5(t), x_6(t))$ – decision vector, $\dot{X} = (\dot{x_1}(t), \dot{x_2}(t), \dot{x_3}(t), \dot{x_4}(t), \dot{x_5}(t), \dot{x_6}(t))$ – speed vector, $\ddot{X} = (\ddot{x_1}(t), \ddot{x_2}(t), \ddot{x_3}(t), \ddot{x_4}(t), \ddot{x_5}(t), \ddot{x_6}(t))$ – acceleration vector, $F = (f_1(t), f_2(t), f_3(t), f_4(t), f_5(t), f_6(t))$ – external loads vector, C – stiffness matrix 6×6, B – damping matrix 6×6. Using operation of repeated differentiation which does not contradict the theorem of existence and uniqueness Peano Ref. [1] allocate from the system Eq. (1) the following differential operators of the second and first orders.

$$L_i = \frac{d^2}{dt^2} + b_{ij}\frac{d}{dt} + c_{ij}, \quad d_{ij} = b_{ij}\frac{d}{dt} + c_{ij}, \tag{2}$$

where $i = \bar{1,6}$, $j = \bar{1,6}$, $i \neq j$. Using the operators Eq. (2). The dynamic system of vibration isolation can be submitted as Eq. (3):

$$L_i(x_i) = d_{ij}(x_j) + d_{ik}(x_k) + d_{il}(x_l) + d_{im}(x_m) + d_{in}(x_n) + F_i, \tag{3}$$

where $i = \overline{1,6}$, $j = \overline{1,6}$, $k = \overline{1,6}$, $l = \overline{1,6}$, $m = \overline{1,6}$, $n = \overline{1,6}$, $i \neq j \neq k \neq l \neq m \neq n$. Consistently applying operators $L_i(i = \bar{1,6})$ to the differential equations system Eq. [3] concerning variables x_i, the new system of six incoherent the twelfth order differential equations is received:

$$\begin{aligned}
D^{12}(x) &= L_1L_2L_3L_4L_5L_6(x)- \\
&- \sum_{i=1,j=i,k=j,l=k,m=1,n=m}^{6} L_iL_jL_kL_ld_{mn}d_{nm}(x) - \sum_{i=1,j=i,k=j,l=1,m=l,n=l}^{6} L_iL_jL_kd_{lm}d_{mn}d_{nl}(x)- \\
&- \sum_{i=1,j=i,k=1,l=k,m=k,n=k}^{6} L_iL_jd_{kl}d_{lm}d_{mn}d_{nk}(x) + \sum_{i=1,j=i,k=1,l=k,m=k,n=m}^{6} L_iL_jd_{kl}d_{lk}d_{mn}d_{nm}(x)- \\
&- \sum_{i=1,j=1,k=j,l=j,m=j,n=j}^{6} L_id_{jk}d_{kl}d_{lm}d_{mn}d_{nj}(x) + \sum_{i=1,j=1,k=j,l=1,m=l,n=l}^{6} L_id_{jk}d_{kj}d_{lm}d_{mn}d_{nl}(x)- \\
&- \sum_{i=1,j=i,k=i,l=i,m=i,n=i}^{6} d_{ij}d_{jk}d_{kl}d_{lm}d_{mn}d_{ni}(x) - \sum_{i=1,j=i,k=i,l=k,m=k,n=m}^{6} d_{ij}d_{ji}d_{kl}d_{lk}d_{mn}d_{nm}(x)+ \\
&+ \sum_{i=1,j=i,k=1,l=k,m=k,n=k}^{6} d_{ij}d_{ji}d_{kl}d_{lm}d_{mn}d_{nk}(x) + \sum_{i=1,j=i,k=i,l=i,m=l,n=l}^{6} d_{ij}d_{jk}d_{ki}d_{lm}d_{mn}d_{nl}(x),
\end{aligned} \tag{4}$$

where $x = \overline{x_1, x_6}$. The heterogeneous parts of the system Eq. (4). are unwieldy and are not gave an abstract. The system enables to determine obviously constant factors of the linear inhomogeneous the twelfth order equations which integrating comes easily. The system decisions are searched on the basis of integral Laplace transform application, see Fig. 2.

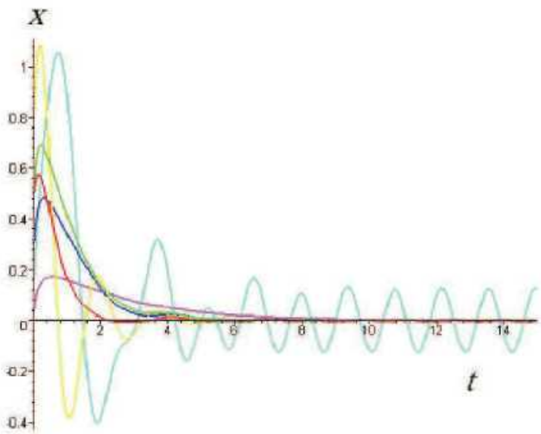

Figure a

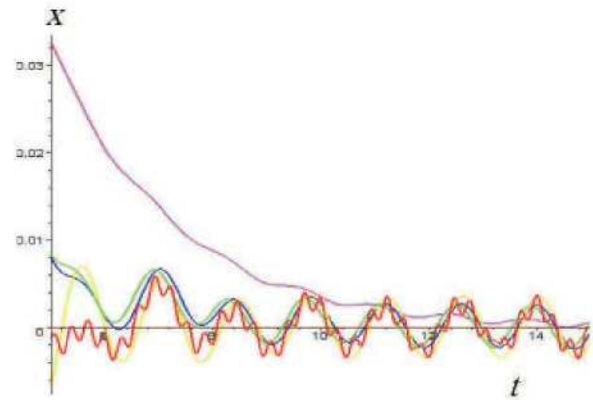

Figure b

Figure 2: Exact analytical solutions $x = \overline{x_1, x_6}$ of coherent heterogeneous differential equations systems on terms $t \in [0, 5]$ – (a) and $t \in [5, 15]$ – (b).

Calculation of the constructive unit with six degree of freedom assigned to lower high vibration level is developed. The inhomogeneous differential equations of the second order with non-separating variables system is analytically integrated Ref. [2]. The accurate theoretical results are found using known initial velocities and displacements. The equivalent replacement of multibody model by unimass oscillating system is concluded. The physical and geometrical parameter selection criterion of vibration damper is developed in the form of inequality between reduced factors.

Refrences

1. Cyril M. Harris, Charles E. Crede *Shock and vibration handbook.[Russian translation]* Leningrad, Sudostroenie, 1980.

2. Dokukova N. A., Konon P. N. *Equivalence of the impedance method and the method of amplitude-frequency characteristics for investigating the vibrations in hydraulically powered supports* // Inzh.-Fiz. Zh., 2003. Vol. 76, no. 6. P. 174–176.