# A SEMI-PERIODIC BOUNDARY VALUE PROBLEM FOR SYSTEM OF INTEGRO-DIFFERENTIAL EQUATIONS IN PARTIAL DERIVATIVES

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The semi-periodic boundary value problem for system of integro-differential equations in partial derivative from two independent variables is considered on $\bar{\Omega} = \{(t,x) : 0 \leqslant t \leqslant T, 0 \leqslant x \leqslant \omega\}$

$$\frac{\partial^2 u}{\partial t \partial x} = A(t,x)\frac{\partial u}{\partial x} + B(t,x)\frac{\partial u}{\partial t} + C(t,x)u + f(t,x)+$$

$$+ \int_0^T \int_0^x \left[ K_1(t,x,\tau,\xi)\frac{\partial u(\tau,\xi)}{\partial \xi} + K_2(t,x,\tau,\xi)\frac{\partial u(\tau,\xi)}{\partial \tau} + K_3(t,x,\tau,\xi)u(\tau,\xi)\right] d\xi d\tau, \quad (1)$$

$$u(t,0) = \psi(t), \qquad t \in [0,T], \quad (2)$$

$$u(0,x) = u(T,x), \qquad x \in [0,\omega], \tag{3}$$

where $(n \times n)$ - matrices $A(t,x)$, $B(t,x)$, $C(t,x)$, $n$ - vector-function are continuous on $\bar{\Omega}$, $(n \times n)$ - matrices $K_1(t,x,\tau,\xi)$, $K_2(t,x,\tau,\xi)$, $K_3(t,x,\tau,\xi)$ are continuous on $\bar{\Omega} \times \bar{\Omega}$, $n$-vector-function $\psi(t)$ is continuously differentiable on $[0,T]$ and $\psi(0) = \psi(T)$.

The function $u(t,x) \in C(\bar{\Omega}, R^n)$, having the partial derivatives $\frac{\partial u(t,x)}{\partial x} \in C(\bar{\Omega}, R^n)$, $\frac{\partial u(t,x)}{\partial t} \in C(\bar{\Omega}, R^n)$, $\frac{\partial^2 u(t,x)}{\partial t \partial x} \in C(\bar{\Omega}, R^n)$ is called a classical solution to problem (1) - (3), if it satisfies system (1) at any $(t,x) \in \bar{\Omega}$ and meets the boundary conditions (2), (3).

We investigate questions of the existence, uniqueness and finding classical solutions of the problem (1)-(3). Sufficient conditions for existence of solutions periodic in $t$ with period $T$ for systems of partial integro-differential equations with an impulsive effect were established by numerical-analytic method [1]. The non-local boundary value problem with data on characteristics for system of hyperbolic equations without integral summand were considered in [2-4] by the method introduction of functional parameters.

In the present communication the sufficient coefficients conditions of the unique classic solvability of the semi-periodic boundary value problem for system of integro-differential equations of hyperbolic type (1)-(3) are obtained and algorithm finding its solution are proposed.

We introduce new unknown functions $v(t,x) = \frac{\partial u(t,x)}{\partial x}$, $w(t,x) = \frac{\partial u(t,x)}{\partial t}$ and problem (1)-(3) reduce to equivalent problem

$$\frac{\partial v}{\partial t} = A(t,x)v + \int_0^T \int_0^x K_1(t,x,\tau,\xi) v(\tau,\xi) d\xi + f(t,x) +$$

$$+B(t,x)w + C(t,x)u + \int_0^T \int_0^x \Big[K_2(t,x,\tau,\xi) w(\tau,\xi) + K_3(t,x,\tau,\xi) u(\tau,\xi)\Big] d\xi d\tau, \tag{4}$$

$$v(0,x) = v(T,x), \qquad x \in [0,\omega], \tag{5}$$

$$u(t,x) = \psi(t) + \int_0^x v(t,\xi) d\xi, \qquad w(t,x) = \dot{\psi}(t) + \int_0^x \frac{\partial v(t,\xi)}{\partial t} d\xi, \qquad t \in [0,T]. \tag{6}$$

In the problem (4)-(6) condition (6) is realized in relation (6).

A triple $\{v(t,x), u(t,x), w(t,x)\}$ of continuous functions on $\bar{\Omega}$ is called a solution to problem (4)-(6) if the function $v(t,x) \in C(\bar{\Omega}, R^n)$ has a continuous derivative with respect to $t$ on $\bar{\Omega}$ and satisfies one-parameter family of periodic boundary value problem for the system ordinary integro-differential equations (4), (5), in which the functions $u(t,x)$ and $w(t,x)$ are related to $v(t,x)$ and $\frac{\partial v(t,x)}{\partial t}$ by the functional relation (6).

It introduce functional parameter $\lambda(x) = v(0,x)$ and in problem (4)-(6) it realize substitution $v(t,x) = \widetilde{v}(t,x) + \lambda(x)$. Then problem (4)–(6) it reduce to equivalent problem with parameter

$$\frac{\partial \widetilde{v}}{\partial t} = A(t,x)[\widetilde{v} + \lambda(x)] + \int_0^T \int_0^x K_1(t,x,\tau,\xi)[\widetilde{v}(\tau,\xi) + \lambda(\xi)] d\xi + f(t,x) +$$

$$+B(t,x)w + C(t,x)u + \int_0^T \int_0^x \Big[K_2(t,x,\tau,\xi)w(\tau,\xi) + K_3(t,x,\tau,\xi)u(\tau,\xi)\Big] d\xi d\tau, \qquad (7)$$

$$\widetilde{v}(0,x) = 0, \qquad x \in [0,\omega], \qquad (8)$$

$$u(t,x) = \psi(t) + \int_0^x [\widetilde{v}(t,\xi) + \lambda(\xi)]d\xi, \qquad w(t,x) = \dot{\psi}(t) + \int_0^x \frac{\partial \widetilde{v}(t,\xi)}{\partial t} d\xi, \qquad t \in [0,T]. \quad (9)$$

$$\widetilde{v}(T,x) = 0, \qquad x \in [0,\omega]. \qquad (10)$$

For the problem (7)-(10) is propose algorithm of finding solution.

Sufficient conditions for the implementability and convergence of the algorithm and the existence of a unique classical solution to problem (1)-(3) are ensured by the following theorem.

**Theorem.** *Let matrix* $Q(T,x) = \int_0^T A(\tau,x)d\tau$ *it is invertible for all* $x \in [0,\omega]$ *and are hold conditions:* *a)* $||[Q(T,x)]^{-1}|| \leqslant \gamma$, *where* $\gamma$ *- const;*

*b)* $q_1 = e^{\alpha T} \cdot T^2 \cdot \omega \cdot \beta < 1$; *c)* $q_2 = e^{\gamma T^2 \omega \beta} \cdot \gamma \cdot T(\alpha + T\omega\beta) \cdot \frac{e^{\alpha T} - 1 + q_1}{1 - q_1} < 1$,

*where* $\alpha = \max_{(t,x)\in\bar{\Omega}} ||A(t,x)||$, $\beta = \max_{(t,x,\tau,\xi)\in\bar{\Omega}\times\bar{\Omega}} ||K_1(t,x,\tau,\xi)||$.

*Then the problem (1)-(3) has unique classical solution.*

**Sketch of the proof.** At proof of theorem it realize analogously of proof theorem 1 from [2] and it use proposing algorithm.

## References


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