OPEN NETWORKS WITH MULTIREGIME SERVICE STRATEGIES AND SIGNALS

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Open queueing networks with several types of customers, Poisson incoming flow, exponential service in the nodes and Markov routing are studied. In each of the nodes there is the only device, which can operate in several regimes. Each regime has a residence time, limited by an exponentially distributed random variable. There are signals, which can increase or reduce the regime of service in the node. The problem of stationary distribution of conditions probabilities form is investigated.

Keywords: queueing networks, multiregime service, reversibility, stationary distribution of conditions probabilities.

1. INTRODUCTION

Queueing networks with multiregime service strategies have been investigated relatively recently. The necessity of their study was caused by practical considerations, because such networks allow us to consider models with partially nonreliable devices. With increase the regime number the regime is getting less reliable and the node productivity decreases.

Transitions from one regime into another are considered as "internal" changes. Under such "internal" changes we mean transitions of the serving device to a less reliable regime due to breakdowns and to more reliable regime because of possible recovery due to natural causes. Moreover, such transitions do not depend solely on the number of the device regime, but also on the types of customers that are in the node.

It is assumed that the time of stay in each regime is limited. After this limited time the device transits with corresponding probabilities to either a regime with a larger number or a regime with a lower number. Described transitions are caused by the properties of each of the regimes and do not depend on the customers in the node.

Considered network is modified by the addition of information signals, which can increase or reduce the regime of service in the node. Presence of signals can be interpreted as an external influence on the network.

2. MODEL DESCRIPTION

We consider open queueing network with M types of customers, which contains N nodes. There are three Poisson input flows: the flow of customers with parameter λ

and two flows of signals, which can reduce or increase the number of regime. They have rates ω^- and ω^+ accordingly.

Every customer of input flow passes independently to node i and becomes the customer of type u with probability $p_{0(i,u)}$ $\left(\sum_{i=1}^{N}\sum_{u=1}^{M} p_{0(i,u)} = 1\right)$. Incoming signal of regime increasing and signal of regime reducing pass to node i with probabilities q_{0i}^+ and q_{0i}^- accordingly $\left(\sum_{i=1}^{N} q_{0i}^+ = 1, \sum_{i=1}^{N} q_{0i}^- = 1\right)$. After the service in node i the customer of type u passes to node j immediately with probability $p_{(i,u)(j,v)}$ as the customer of type v and with probabilities $q_{(i,u)j}^+$, $q_{(i,u)j}^-$ as the signal of increasing or reducing regime accordingly. Or it can leave the node with probability $p_{(i,u)0}$ $\left(\sum_{j=1}^{N}\sum_{v=1}^{M}(p_{(i,u)(j,v)} + q_{(i,u)j}^+ + q_{(i,u)j}^-) + p_{(i,u)0} = 1\right)$.

In each of N nodes there is the only device, which can operate in $r_i + 1$ regimes ($i = \overline{1, N}$). The state of the network is characterized by the vector $x(t) = (x_1(t), ..., x_N(t))$, where $x_i(t) = (\bar{x}_i(t), l_i(t)) = (x_{i1}(t), x_{i2}(t), ..., x_{in(i)}(t), l_i(t))$ describes the state of node i at the moment t. Here $x_{i1}(t)$ — the type of customer, which is getting service at the moment t, $x_{i2}(t)$ — the type of customer, which is the first in the queue,..., $x_{in(i)}(t)$ - the type of customer, which is the last in the queue, $n(i)$ — the number of customers in the node i, $l_i(t)$ — the regime of the node i at the moment t. States space for process $x_i(t)$ is $X_i = \{(0, l_i), (x_{i1}, l_i), (x_{i1}, x_{i2}, l_i), (x_{i1}, x_{i2}, x_{i3}, l_i), ... : x_{ik} = \overline{1, M},\ k = 1, 2, ...; l_i = \overline{0, r_i}\}$.

The time of device service of the node i has an exponential distribution with parameter $\mu_i(n(i), l_i)$. Customers are serviced in the order they arrive in the node.

We define the regime 0 as the basic regime. Switching time from some regime to another one has the exponential distribution. The node in the basic regime can pass only to the regime 1 with rate $\nu_i(\bar{x}_i, 0)$. For the states, which have the number of regime $1 \le l_i \le r_i - 1$, the node passes to regime $l_i + 1$ with rate $\nu_i(x_i)$ and to regime $l_i - 1$ with rate $\varphi_i(x_i)$. And the node passes from the regime r_i only to regime $r_i - 1$ with rate $\varphi_i(\bar{x}_i, r_i)$. While the regimes are switching in the node, the number of customers doesn't change. Switch occurs only between the neighborhood regimes.

When the signal of regime reducing incomes to the node with the regime l_i, it turns the node to the regime $l_i - 1$ and doesn't change the number of customers in the node. This signal doesn't produce any action, if the node is in the regime 0. When the signal of regime increasing incomes to the node with the regime l_i, it turns the node to the regime $l_i + 1$ and doesn't change the number of customers in the node. This signal doesn't produce any action, if the node is in the regime r_i. After changing the node regime these signals disappear.

Each regime l_i has a residence time, limited by an exponentially distributed random variable with parameter $\gamma_i(l_i)$ ($l_i = \overline{0, r_i}, i = \overline{1, N}$). After the end of the stay time in the regime l_i the device with probability $p^+(l_i)$ moves to regime $l_i + 1$, and with probability $p^-(l_i)$ moves to regime $l_i - 1$.

Then $x(t)$ is a homogeneous Markov process with states space $X = X_1 \times X_2 \times ... \times X_N$, where $X_i = \{(0, l_i), (x_{i1}, l_i), (x_{i1}, x_{i2}, l_i), ... : x_{ik} = \overline{1, M},\ k = 1, 2, ...; l_i = \overline{0, r_i}\}$.

3. ISOLATED NODE

We consider isolated node i and suppose that three independent Poisson flows come in it: the flow of customers of type u with parameter α_{iu}, the flow of signals, which increase regime of the node, with parameter β_i^+ and the flow of signals, which reduce regime of the node, with parameter β_i^-. Here α_{iu}, β_i^+, β_i^- — average rates of customers, "increasing" signals and "reducing" signals arrivals accordingly to the node i.

Traffic equations for this model are:

$$\alpha_{iu} = \lambda p_{0(i,u)} + \sum_{j=1}^{N}\sum_{v=1}^{M} \alpha_{jv} p_{(j,v)(i,u)},$$

$$\beta_i^+ = \omega^+ q_{0i}^+ + \sum_{j=1}^{N}\sum_{v=1}^{M} \alpha_{jv} q_{(j,v)i}^+,$$

$$\beta_i^- = \omega^- q_{0i}^- + \sum_{j=1}^{N}\sum_{v=1}^{M} \alpha_{jv} q_{(j,v)i}^-.$$

To reduce the calculations we introduce the following operators:

$T_u^+, T^-, S^+, S^- : X_i \to X_i$, setting

$$T_u^+(0, l_i) = (u, l_i),$$

$$T_u^+(x_i) = T_u^+(x_{i1}, \dots, x_{in(i)}, l_i) = (u, x_{i1}, \dots, x_{in(i)}, l_i),$$
$$T^-(x_i) = T^-(x_{i1}, l_i) = (0, l_i), \quad |x_i| = n(i) = 1,$$
$$T^-(x_i) = T^-(x_{i1}, \dots, x_{in(i)}, l_i) = (x_{i1}, \dots, x_{in(i)-1}, l_i), \quad |x_i| = n(i) > 1,$$
$$S^+(x_i) = S^+(x_{i1}, \dots, x_{in(i)}, l_i) = (x_{i1}, \dots, x_{in(i)}, l_i + 1),$$
$$S^-(x_i) = S^-(x_{i1}, \dots, x_{in(i)}, l_i) = (x_{i1}, \dots, x_{in(i)}, l_i - 1),$$

$T^-(x_i)$ is not defined at $x_i = (0, l_i)$, $S^+(x_i)$ — at $x_i = (\bar{x}_i, r_i)$, $S^-(x_i)$ — at $x_i = (\bar{x}_i, 0)$. Consider also the operators describing the change of the network state:

$T_{(i,u)}^+, T_i^-, S_i^+, S_i^- : X \to X$, putting

$$T_{(i,u)}^+(x) = T_{(i,u)}^+(x_1, x_2, \dots, x_N) = (x_1, x_2, \dots, x_{i-1}, \hat{x}_i, x_{i+1}, \dots, x_N), \hat{x}_i = T_u^+(x_i);$$

$$T_i^-(x) = T_i^-(x_1, x_2, \dots, x_N) = (x_1, x_2, \dots, x_{i-1}, \hat{x}_i, x_{i+1}, \dots, x_N), \hat{x}_i = T^-(x_i);$$
$$S_i^+(x) = S_i^+(x_1, x_2, \dots, x_N) = (x_1, x_2, \dots, x_{i-1}, \hat{x}_i, x_{i+1}, \dots, x_N), \hat{x}_i = S^+(x_i);$$
$$S_i^-(x) = S_i^-(x_1, x_2, \dots, x_N) = (x_1, x_2, \dots, x_{i-1}, \hat{x}_i, x_{i+1}, \dots, x_N), \hat{x}_i = S^-(x_i).$$

Lemma. *For the reversibility of the isolated node the following conditions are necessary and sufficient*

$$[\nu_i(\bar{x}_i, l_i-1)+\gamma_i(l_i-1)p^+(l_i-1)+\beta_i^+]\mu_i(n(i),l_i)[\varphi_i(T^-(\bar{x}_i,l_i))+\gamma_i(l_i)p^-(l_i)+\beta_i^-]=$$

$$=[\nu_i(T^-(\bar{x}_i,l_i-1))+\gamma_i(l_i-1)p^+(l_i-1)+\beta_i^+]\mu_i(n(i),l_i-1)[\varphi_i(\bar{x}_i,l_i)+\gamma_i(l_i)p^-(l_i)+\beta_i^-],$$

$$l_i=\overline{1,r_i}.$$

The proof is similar to the corresponding proof in [1].

4. STATIONARY DISTRIBUTION

Let the stationary distribution $\{p(x), x\in X\}$ of $x(t)$ exists, then the stationary state probabilities of the network satisfy the global equilibrium equations:

$$p(x)\sum_{i=1}^{N}\sum_{u=1}^{M}[\lambda p_{0(i,u)}+\mu_i(n(i),l_i)I_{(n(i)\neq 0)}+$$

$$+(\nu_i(x_i)+\gamma_i(l_i)p^+(l_i)+\omega^+q_{0i}^+)I_{(l_i\neq r_i)}+(\varphi_i(x_i)+\gamma_i(l_i)p^-(l_i)+\omega^-q_{0i}^-)I_{(l_i\neq 0)}]=$$

$$=\sum_{i=1}^{N}[p(T_i^-(x))\lambda p_{0(i,x_{in(i)})}I_{(n(i)\neq 0)}+\sum_{u=1}^{M}p(T_{(i,u)}^+(x))\mu_i(n(i)+1,l_i)p_{(i,u)0}+$$

$$+\sum_{j=1}^{N}\sum_{v=1}^{M}p(T_{(j,v)}^+(T_i^-(x)))\mu_j(n(j)+1,l_j)p_{(j,v)(i,x_{in(i)})}I_{(n(i)\neq 0)}+$$

$$+\sum_{j=1}^{N}\sum_{v=1}^{M}p(T_{(j,v)}^+(S_i^-(x)))\mu_j(n(j)+1,l_j)q_{(j,v)i}^+I_{(l_i\neq 0)}+$$

$$+\sum_{j=1}^{N}\sum_{v=1}^{M}p(T_{(j,v)}^+(S_i^+(x)))\mu_j(n(j)+1,l_j)q_{(j,v)i}^-I_{(l_i\neq r_i)}+$$

$$+p(S_i^-(x))[\nu_i(S^-(x_i))+\gamma_i(l_i-1)p^+(l_i-1)+\omega^+q_{0i}^+]I_{(l_i\neq 0)}+$$

$$+p(S_i^+(x))[\varphi_i(S^+(x_i))+\gamma_i(l_i+1)p^-(l_i+1)+\omega^-q_{0i}^-]I_{(l_i\neq r_i)}].$$

Theorem 1. *If for all $i=\overline{1,N}$ the conditions of reversibility are true and the series converges*

$$\sum_{x\in X}q(x)\prod_{i=1}^{N}\prod_{a=1}^{n(i)}\frac{\alpha_{ix_{ia}}}{\mu_i(a,l_i)}\prod_{k=1}^{l_i}\frac{\nu_i(0,k-1)+\gamma_i(k-1)p^+(k-1)+\beta_i^+}{\varphi_i(0,k)+\gamma_i(k)p^-(k)+\beta_i^-},$$

where $(\alpha_{iu}, i=\overline{1,N}, u=\overline{1,M})$ — solution of the traffic equation,

$$q(x)=\lambda+\sum_{i=1}^{N}\mu_i(n(i),l_i)I_{(n(i)\neq 0)}+\sum_{i=1}^{N}[\nu_i(x_i)+\gamma_i(l_i)p^+(l_i)+\omega^+q_{0i}^+]I_{(l_i\neq r_i)}+$$

$$+\sum_{i=1}^{N}[\varphi_i(x_i)+\gamma_i(l_i)p^-(l_i)+\omega^- q_{0i}^-]I_{(l_i\neq 0)},$$

then the Markov process $x(t)$ is ergodic and its stationary distribution of conditions probabilities has the product form

$$p(x)=p_1(x_1)p_2(x_2)...p_N(x_N), x\in X,$$

where $p_i(x_i)$ is determined by the relation

$$p_i(x_i)=\prod_{a=1}^{n(i)}\frac{\alpha_{ix_{ia}}}{\mu_i(a,l_i)}\prod_{k=1}^{l_i}\frac{\nu_i(0,k-1)+\gamma_i(k-1)p^+(k-1)+\beta_i^+}{\varphi_i(0,k)+\gamma_i(k)p^-(k)+\beta_i^-}p_i(0,0),$$

$$p_i(0,0)=\left[\sum_{x_i\in X_i}\prod_{a=1}^{n(i)}\frac{\alpha_{ix_{ia}}}{\mu_i(a,l_i)}\prod_{k=1}^{l_i}\frac{\nu_i(0,k-1)+\gamma_i(k-1)p^+(k-1)+\beta_i^+}{\varphi_i(0,k)+\gamma_i(k)p^-(k)+\beta_i^-}\right]^{-1}.$$

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