A CLOSED SINGLE-LINE QUEUEING SYSTEM WITH REPEATED CALLS AND LOSSES

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The closed single-server queueing model with repeated attempts is studied in a number of papers. In all these papers it is assumed that a rejected subscriber repeats his attempts until he is satisfied. In the present paper we consider the same model, assuming some restrictions on the number of retrials, which the customer is inclined (or is allowed) to make. The theoretical study of these models being too hard, we investigate them via simulations.

Keywords: Closed queueing systems, repeated calls, simulations

1. INTRODUCTION AND MATHEMATICAL MODEL

The queuing systems with repeated attempts (or retrials) are widely used to model problems in telephone, computer, communication systems, etc. The most obvious example of a retrial queue appears in a telephone call, when a person phones and finds the line engaged. Usually in such a situation the subscriber repeats his attempt after some time. A description of other situations in which retrials arise can be found in [5], and more recent applications of retrial models - in [1]. The standard assumption in analysis of retrial queues is that each customer repeats his attempts until he is satisfied. However, in many real situations this assumption does not hold. It does not always hold even in the classical example of the telephone subscriber who finds engaged line. The objective of the present paper is investigation of retrial queueing models, where the number of repetitions is in some way restricted. The theoretical study of such systems is too difficult, so in the present paper a method of simulations is applied.

We consider retrial queueing system of type $M/G/1//N$ in Kendall's notation. This means that the system has one server, which serves $N, (2 \leq N < \infty)$ customers (subscribers) and each one of the customers produces a Poisson process of demands (calls) with intensity λ. These customers are identified as primary or free customers or sources of primary calls. When an arriving primary customer finds the server busy, he leaves the service area and repeats the request after some random time. Between trials a customer is said to be in orbit (queue), or to be a source of repeated calls, or an active customer. Thus, when a source is free at time moment t (i.e. is not being served and is not waiting for service) it may generate a primary call during the interval $(t,\ t + dt)$ with probability λdt. If the server is free at the instant of arrival of a primary call then the call starts to be served. During the service time the source cannot generate a new primary call. After service the source moves into a free state and can generate a new primary call. If the channel is busy at time of the arrival of a primary call, then the source starts generating a Poisson flow of repeated calls with intensity μ. As in the case of a primary call, after service the source becomes free and can generate a new primary call. We assume that primary calls, repeated attempts and service times are mutually independent and denote the service time distribution function by $G(x)$ both for primary and repeated calls and its first moment by ν^{-1}.

This queueing system and its variants are useful in modeling magnetic disk memory systems [9], a star-like local area networks [6], [8], telephone networks [7], etc and is studied in a number of papers ([3], [4], [7], [9] and others). All these papers deal only with the case, when each secondary customer repeats his retrials until he is satisfied. Now we consider two variants of the described system, regarding some restrictions on the number of retrials, that a secondary subscriber is inclined (or is allowed) to make before he reaches the server free. In the first variant we assume that if the server is busy at the instant of arrival of a secondary subscriber (an active subscriber), this subscriber continues his repeated attempts with probability p and with probability $1-p$, $0 \leq p \leq 1$ he refuses to try entering server any more and becomes again a free one. In the second model we assume, that each secondary customer stops his retrials if his $(k+1)$-th attempt fails, $k = 1, 2, \ldots$ (if not served before). Further in this paper we will call each of these three models as the model without restrictions (unrestricted model or a model without losses), the first restricted model and the second restricted model, respectively. Obviously, if in the first restricted model we take $p = 1$, or in the second one $k = \infty$ we get the model without restrictions. In the same way, if $p = 0$ the first restricted model coincides with the second one with $k = 1$.

2. ANALYSIS OF THE RESTRICTED MODELS

Let $C(t)$ be the number of busy channels at time t (i.e. $C(t)$ is 0 or 1 according to whether the channel is free or busy at time t) and $A(t)$ - the number of active customers at time t (orbit size or queue length). In the first restricted model (where a rejected secondary subscriber continues his attempts with probability p and with probability $1-p$ stops the retrials) the dynamics of the system can be described by the process $(C(t),\ A(t))$. It should be noted that the situation $C(t) = 0$, $A(t) = N$ is impossible and thus the state space of the process $(C(t),\ A(t))$ is the set $\{0, 1\} \times \{0, 1, \ldots, N-1\}$. As $(C(t),\ A(t))$ is a Markov process only when $C(t) = 0$, in order to work with a Markov process also in the case $C(t) = 1$ we introduce a supplementary variable $z(t)$, equal to the elapsed service time. Let

$$p_{1j}(x)dx = \lim_{t\to\infty} P(C(t) = 1,\ A(t) = j,\ x < z(t) \leq x + dx),$$

$$p_{ij} = \lim_{t\to\infty} P(C(t) = i, A(t) = j),\ i = 0, 1,\quad j = 0, 1, \ldots, N-1. \tag{1}$$

In a general way we obtain the equations of statistical equilibrium

$$\frac{d}{dx}p_{1j}(x) = -[(N-j-1)\lambda + \eta(x) + j(1-p)\mu]p_{1j}(x) + (N-j)\lambda p_{1,j-1}(x) \tag{2}$$
$$+(j+1)(1-p)\mu p_{1,j+1}(x),\ p_{1,-1}(x) = p_{1N}(x) = 0,$$
$$[j\mu + (N-j)\lambda]p_{0j} = \int_0^\infty p_{1j}(x)\eta(x)dx, \tag{3}$$
$$p_{1j}(0) = (N-j)\lambda p_{0j} + (j+1)\mu p_{0,j+1}, \quad j = 0, 1, \ldots, N-1, \tag{4}$$

Here
$$\eta(x) = \frac{G'(x)}{1-G(x)}$$
is the service rate at instant x after start of a service.

Recurrent formulas for computing the solutions of the system (2) - (4) in case $p = 1$ are obtained in [9] with the help of discrete transformations and in [3] directly, by means of mathematical induction. For arbitrary p, no formulas are known to us for the solution of this system.

As described before, in the second restricted model we assume that if the $(k+1)$-th attempt of an active subscriber fails, $k = 1, 2, \ldots$, the subscriber leaves the orbit and becomes free again. So, the state of the system at time t can be described by the process $(C(t),\ A_1(t),\ A_2(t), \ldots, A_k(t))$, where $A_i(t)$ is the number of those active subscribers in the orbit which have made exactly k attempts before the moment t, $A(t) = A_1(t) + A_2(t) + \cdots + A_k(t)$. Introducing the steady state probabilities

$$p_{1j_1j_2\ldots j_k}(x)dx = \lim_{t\to\infty} P(C(t) = 1,\ A_1(t) = j_1, \ldots,\ A_k(t) = j_k,\ x < z(t) \le x + dx),$$
$$p_{ij_1j_2\ldots j_k} = \lim_{t\to\infty} P(C(t) = i,\ A_1(t) = j_1,\ A_2(t) = j_2, \ldots, A_k(t) = j_k),$$
$$x \ge 0,\ i = 0, 1,\ \sum_{l=1}^{k} j_l \le N-1,\ j_l \ge 0, 1 \le l \le k,$$

for the joint stationary distribution p_{ij} of the server state and the orbit size (1) it holds

$$p_{ij} = \sum_{j_1+j_2+\cdots+j_k=j}^{N-1} p_{ij_1j_2\ldots j_k}.$$

The equations of statistical equilibrium are very complicated even for small k. For $k = 1$ this is the system (2) - (4) with $p = 0$, for $k = 2$ the system has the form

$$\frac{d}{dx}p_{1j_1j_2}(x) = -[(N-j-1)\lambda + \eta(x) + j\mu]p_{1j_1j_2}(x) + (N-j)\lambda p_{1,j_1-1,j_2}(x)$$
$$+(j_2+1)\mu p_{1,j_1,,j_2+1}(x) + (j_1+1)\mu p_{1,j_1+1,j_2-1}(x),$$
$$[j\mu + (N-j)\lambda]p_{0j_1j_2} = \int_0^\infty p_{1j_1j_2}(x)\eta(x)dx,$$
$$p_{1j_1j_2}(0) = (N-j)\lambda p_{0j} + (j_2+1)\mu p_{0,j_1,j_2+1} + (j_1+1)\mu p_{0,j_1+1,j_2},$$
$$j = j_1 + j_2,\ p_{1j_1j_2}(x) = p_{1j_1j_2} = 0 \text{ if } j_k \notin [0, N-1],\ k = 1, 2.$$

3. SIMULATIONS

In this section, on the basis of obtained via simulations results we study the properties of the orbit size and the server state. The simulations are performed in the case of exponentially distributed service times. Assuming that in the beginning of the process the server is free and there are no active subscribers, the simulation follows the short time changes of the system state as the direction of transformations is chosen according to the probability of each transformation and using generated random numbers. The method is described in detail in [2].

Five examples are presented as illustrations: simulation of the first restricted model with probability $p = 0.5$, of the second restricted model for $k = 1$, $k = 2$ and $k = 3$ and of model without restrictions. The observed mean $E[A]$, standard deviations $std[A]$ and the partition $E[A_1]$, $E[A_2]$, $E[A_3]$ (in case $k = 2$ and $k = 3$) of the orbit size as well as the probability P_1 of a busy channel are given in table 1. The corresponding theoretically obtained values in the system without losses (calculated according to the derived in [3] formulas) are also given. The values of the parameters are the same in each of these examples: $\lambda = 0.012$, $\mu = 0.1$, $\nu = 1$, $N = 50$, $n = 10000$, where n is the duration of the simulation (the number of time steps). The results presented in the table show that the type of the model greatly influences the observed characteristics, especially the mean number of the active subscribers.

Table1

	P_1	$E[A]$ (st.dev.)	$E[A_1]$	$E[A_2]$	$E[A_3]$
I restr. model, $p = 0.5$	0.69	9.22 (2.97)	-	-	-
II restr. model, $k = 1$	0.66	6.35 (2.5)	-	-	-
II restr. model, $k = 2$	0.70	10.11 (3.16)	6.06	4.05	-
II restr. model, $k = 3$	0.72	12.73 (3.16)	5.88	4.07	2.77
Unrestr. model	0.73	18.93 (4.02)	-	-	-
Theor. results (unrestr. m.)	0.72	19.10 (4.15)	-	-	-

In figures 1 - 2 we can see the effect not only of the system's type but also of the system's parameters on the considered characteristics. The graphs on fig.1 represent the dependence of the active subscribers mean percentage $E[A].100/N$ on the system's parameters: λ (the upper-left corner), μ (the upper-right corner), ν (the lower-left corner) and N (the lower-right corner). The dashed lines show the theoretically obtained values of $E[A].100/N$ in the unrestricted model (when active subscribers repeat their attempts until enter service) and the solid lines - the corresponding empirical means for the second restricted model with $k = 2$ (when active subscribers leave the orbit if their third attempt fails), each one obtained via simulations with 10000 steps. Fig.2 has the same structure as Fig.1, but it concerns the busy channel probability P_1. As we can see the behaviour of $E[A].100/N$ and P_1 in both considered models is similar, but the difference between these models is clearly shown as well.

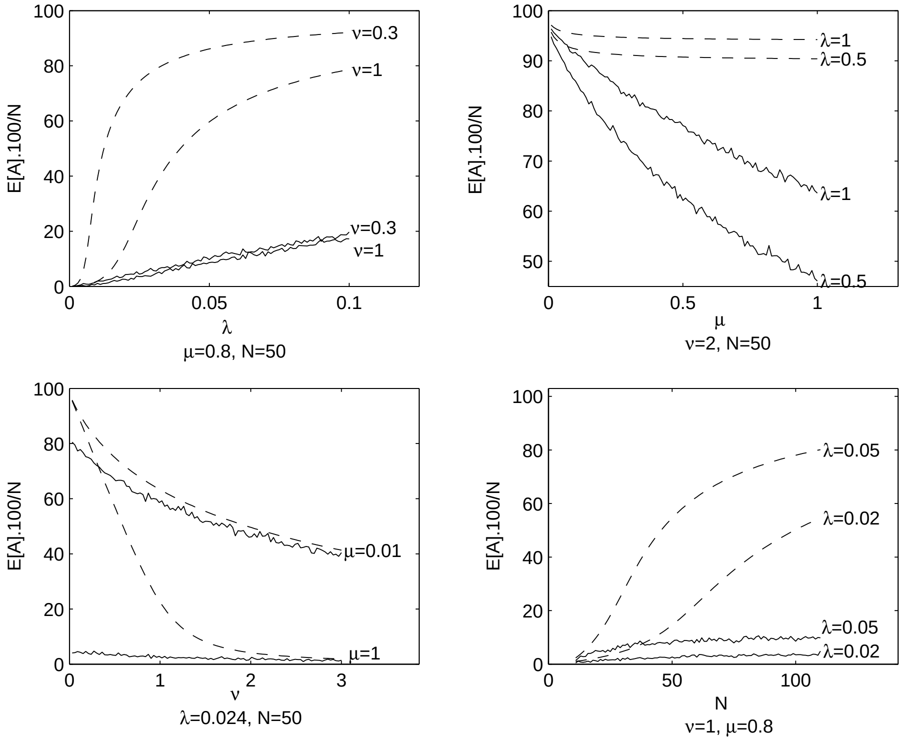
E[A].100/N
λ
μ=0.8, N=50
ν=0.3
ν=1
ν=0.3
ν=1
μ
ν=2, N=50
λ=1
λ=0.5
λ=1
λ=0.5
ν
λ=0.024, N=50
μ=0.01
μ=1
N
ν=1, μ=0.8
λ=0.05
λ=0.02
λ=0.05
λ=0.02

Fig. 1.

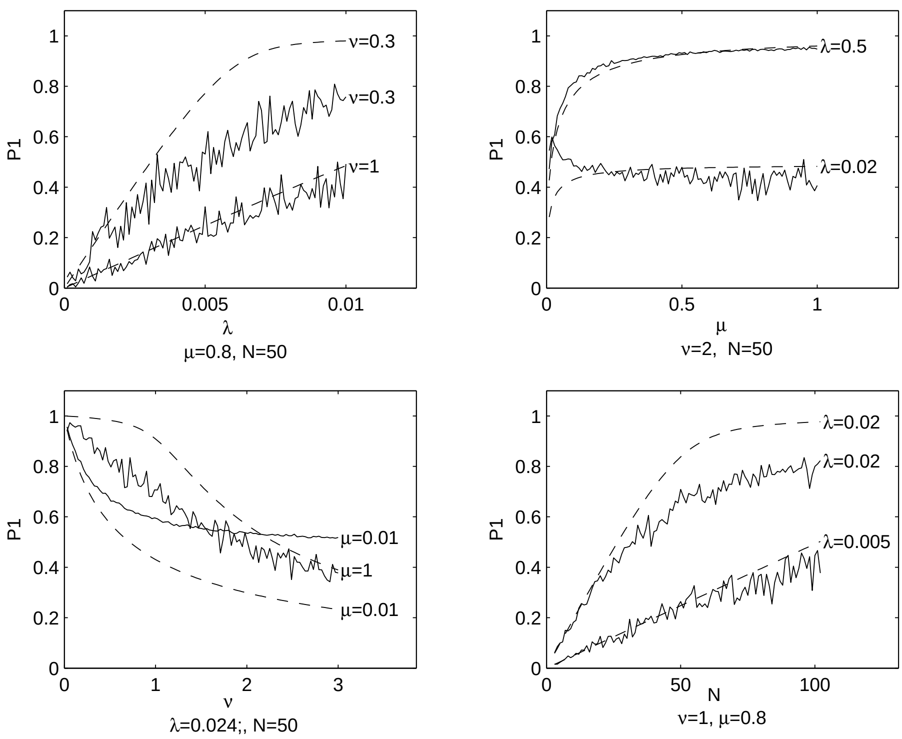
P1
λ
μ=0.8, N=50
ν=0.3
ν=0.3
ν=1
μ
ν=2, N=50
λ=0.5
λ=0.02
ν
λ=0.024;, N=50
μ=0.01
μ=1
μ=0.01
N
ν=1, μ=0.8
λ=0.02
λ=0.02
λ=0.005

Fig. 2.

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