OPTIMIZATION OF EMERGENCY INVENTORY REPLENISHMENT OF A PERISHABLE PRODUCT

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Profitability of an additional replenishment cycle in an inventory system with a perishable product with deterministic shelf life, Poisson demand process, and fixed replenishment lead time is studied. Formulae for calculating income and expenses in the system are obtained. Behaviour of the net income as a function of ordering moment and order quantity is analyzed and an approach for determining parameter values that maximize net income is proposed.

Keywords: Inventory system, ordering moment optimization, order quantity optimization, stochastic demand, deterministic shelf life.

1. INTRODUCTION

Classical inventory systems, e.g., studied in [5] offer flexibility in determining optimal duration of the replenishment cycle based on particular values of storage and replenishment costs. In contrast to them the systems with perishable products have to deal with some time constraints stipulated by the assumptions about product deterioration, as in [6, 7, 8, 9].

Stochastic demand creates uncertainty about inventory level at some moment in the future, and a number of papers focus on optimization of system performance, binding ordering decisions to the moments when inventory reaches some level, see [1, 3, 4].

This paper studies a system with fixed-life product which can be sold up to some predefined time. Time intervals between depletion moments are assumed to be exponentially distributed. Incurred costs include item price, delivery and storage costs. Revenue is generated by sales. Unsold items can be utilized at some profit or some loss. Stochastic nature of demand implies that initial inventory may be insufficient to last till the end of the sales cycle, therefore a possibility of placement of an additional order with a fixed lead time is considered. Ordering moment and quantity are the parameters to be determined in the task for maximization of the net income in the system.

The proposed model is useful in describing midday production cycle of a highly perishable product in a grocery store or replenishment of a seasonal product or fashion apparel in the middle of season. In many situations precise estimation of demand during the future sale cycle is not available, what translates to the high probability to discard a lot of unsold products. A practical alternative is to start with some inventory level corresponding to the pessimistic estimation of demand and to place a additional order later, depending on the actual intensity of sales. Particular benefit of determining the optimal order placement moment from the standpoint of grocery store production practice is that it facilitates in finding the optimal production schedule in the store.

2. MODEL

It is assumed that inventory level in the system at moment 0 is r. Items are consumed in accordance with Poisson process with intensity μ, generating income p per each item sold before moment T, after which all unsold items are utilized with income q. Holding cost is h per each item per unit time. There is a possibility to place an order for delivery of some l additional items with purchase cost d per each item and c cost for delivery. The order is delivered b time units after the ordering moment x. The purpose is to determine the optimal values of x and l that maximize total income in the system.

Storage cost during time t when starting inventory level was j can be calculated as

$$s_1(t) = \frac{h\left(1 - e^{-\mu x}\right)}{\mu},$$

$$s_j(t) = hjte^{-\mu t} + \int_0^t \left(hj\tau + s_{j-1}(t-\tau)\right) d\left(1 - e^{-\mu\tau}\right), j > 1.$$

Using Laplace transform, as in [2], it can be expressed explicitly as

$$s_j(t) = \frac{h}{\mu}\sum_{k=1}^{j} k\left(1 - \sum_{n=0}^{j-k}\frac{(\mu t)^n}{n!}e^{-\mu t}\right).$$

Applying this formula for intervals $[0, x+b]$ and $[x+b, T]$ the expression for total storage cost in the system can be written as

$$S_l(x) = \frac{h}{\mu}\sum_{k=1}^{r} k\left(1 - \sum_{n=0}^{r-k}\frac{(\mu(x+b))^n}{n!}e^{-\mu(x+b)}\right) +$$
$$+\frac{h}{\mu}\sum_{m=0}^{r}\psi_m^{(r)}(x+b)\sum_{k=1}^{l+m} k\left(1 - \sum_{n=0}^{l+m-k}\frac{(\mu(T-x-b))^n}{n!}e^{-\mu(T-x-b)}\right)$$

where $\psi_i^{(j)}(t)$ are the probabilities to have i items in the system after time t if the starting inventory was j, calculated as

$$\psi_i^{(j)}(t) = \begin{cases} \frac{(\mu t)^{j-i}}{(j-i)!}e^{-\mu t}, i > 0 \\ 1 - \sum_{n=0}^{j-1}\frac{(\mu t)^n}{n!}e^{-\mu t}, i = 0 \end{cases}$$

Replenishment expenses are $c + ld$.

The revenue is formed by the income from sales:

$$p\sum_{m=0}^{r}\psi_m^{(r)}(x+b)\sum_{i=0}^{m+l}(m+l-i)\psi_i^{(m+l)}(T-x-b)+p\sum_{m=0}^{r}(r-m)\psi_m^{(r)}(x+b)$$

and utilization income:

$$q\sum_{m=0}^{r}\psi_m^{(r)}(x+b)\sum_{i=0}^{m+l}i\psi_i^{(m+l)}(T-x-b)$$

Explicitly total revenue can be expressed as:

$$R_l(x)=p(r+l)-(p-q)e^{-\mu T}\sum_{n=0}^{r-1}\frac{(\mu(x+b))^n}{n!}\sum_{j=0}^{r-n+l-1}\frac{(\mu(T-x-b))^j}{j!}(r-n+l-j)-$$

$$-(p-q)\left(1-\sum_{n=0}^{r-1}\frac{(\mu(x+b))^n}{n!}e^{-\mu(x+b)}\right)\sum_{j=0}^{l-1}\frac{(\mu(T-x-b))^j}{j!}e^{-\mu(T-x-b)}(l-j)$$

3. OPTIMIZATION OF NET INCOME

Net income in the system accrued on time interval $[0,T]$ can be expressed as:

$$I_l(x)=R_l(x)-S_l(x)-c-dl$$

Search for the optimal parameters l and x that maximize $I_l(x)$ can be limited to the area $l <= \mu(T-x-b)$ because net income in the system with $l > \mu(T-x-b)$ is less then net income in the system with parameter $l=\mu(T-x-b)$.

For each fixed value of l there exist single value of x that maximizes $I_l(x)$ because of its convexity in the aforementioned area, as $I_l''(x)<0$:

$$I_l''(x)=-(p-q)\mu^2e^{-\mu T}\left(\frac{(\mu(x+b))^{r-1}}{(r-1)!}\sum_{j=0}^{l-2}\frac{(\mu(T-x-b))^j}{j!}+\right.$$

$$\left.+\frac{(\mu(T-x-b))^{(l-1)}}{(l-1)!}\left(e^{\mu(x+b)}-\sum_{n=0}^{r-2}\frac{(\mu(x+b))^n}{n!}\right)\right)-$$

$$-\mu h\left[le^{-\mu(x+b)}\frac{(\mu(x+b))^{r-1}}{(r-1)!}-e^{-\mu T}\frac{(\mu(x+b))^{r-1}}{(r-1)!}\sum_{k=1}^{l-1}k\frac{(\mu(T-x-b))^{l-k-1}}{(l-k-1)!}+\right.$$

$$\left.+e^{-\mu T}\sum_{n=0}^{r-2}\frac{(\mu(x+b))^m}{m!}\sum_{k=1}^{l}\frac{(\mu(T-x-b))^{l-k}}{(l-k)!}-e^{-\mu(T-x-b)}\sum_{k=1}^{l}\frac{(\mu(T-x-b))^{l-k}}{(l-k)!}\right]$$

So that, optimal value of x can be found by iterations with the help of $I_l'(x)$:

$$I_l'(x)=(q-p)\mu\left[e^{-\mu(T-x-b)}\sum_{j=0}^{l-1}\frac{(\mu(T-x-b))^j}{j!}+\right.$$

$$+\, e^{-\mu T}\sum_{j=0}^{l-1}\frac{(\mu(T-x-b))^j}{j!}\sum_{n=0}^{r-2}\frac{(\mu(x+b))^n}{n!}\Bigg]-$$

$$-h\left[e^{-\mu T}\sum_{m=0}^{r-1}\frac{(\mu(x+b))^m}{m!}\sum_{k=1}^{l}k\frac{(\mu(T-x-b))^{l-k}}{(l-k)!}-\right.$$

$$\left.-e^{-\mu(T-x-b)}\sum_{k=1}^{l}k\frac{(\mu(T-x-b))^{l-k}}{(l-k)!}-e^{-\mu(x+b)}\sum_{m=0}^{r-1}\frac{(\mu(x+b))^m}{m!}\right]$$

4. NUMERICAL EXAMPLE

Impact of various values of ordering moment and quantity on the net income in the inventory system with parameters $\mu=2$, $d=2$, $p=9.5$, $b=2$, $r=13$, $q=0.5$, $c=3$, $h=1.5$, $T=12$ is shown in table 1.

Table 1 **Impact of ordering moment and quantity on the net income**

	$l=1$	$l=2$	$l=3$	$l=4$	$l=5$	$l=6$	$l=7$	$l=8$
$x=0.0$	52.09	51.20	49.44	46.78	43.16	38.52	32.79	25.93
$x=0.5$	52.84	52.69	51.69	49.78	46.91	43.01	38.04	31.93
$x=1.0$	53.59	54.19	53.93	52.77	50.64	47.49	43.26	37.90
$x=1.5$	54.33	55.66	56.14	55.72	54.33	51.92	48.42	43.80
$x=2.0$	55.04	57.10	58.29	58.59	57.92	56.22	53.44	49.53
$x=2.5$	55.73	58.46	60.34	61.31	61.32	60.31	58.21	54.97
$x=3.0$	56.35	59.71	62.22	63.82	64.45	64.06	62.58	59.95
$x=3.5$	56.91	60.82	63.88	66.04	67.23	67.38	66.43	64.32
$x=4.0$	57.38	61.77	65.31	67.93	69.58	70.18	69.65	67.91
$x=4.5$	57.77	62.55	66.47	69.48	71.49	72.41	72.15	70.61
$x=5.0$	58.08	63.16	67.38	70.67	72.92	74.02	73.83	72.27
$x=5.5$	58.31	63.63	68.06	71.52	73.87	74.94	74.60	72.77
$x=6.0$	58.48	63.96	68.52	72.02	74.27	75.08	74.32	71.98
$x=6.5$	58.60	64.17	68.74	72.11	74.01	74.27	72.84	69.85
$x=7.0$	58.67	64.25	68.68	71.65	72.89	72.32	70.07	66.46
$x=7.5$	58.70	64.16	68.19	70.40	70.65	69.09	66.08	62.06
$x=8.0$	58.64	63.75	66.96	68.02	67.08	64.60	61.13	57.10
$x=8.5$	58.42	62.71	64.57	64.17	62.16	59.19	55.74	52.10
$x=9.0$	57.74	60.43	60.47	58.80	56.28	53.43	50.48	47.49
$x=9.5$	55.85	56.04	54.52	52.44	50.23	47.98	45.73	43.48
$x=10.0$	50.68	49.18	47.68	46.18	44.68	43.18	41.68	40.18

Maximal value of net income 75.129 is achieved for ordering moment 5.8315 and ordering quantity 6.

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