Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 59-10 (2025) 2147–2152

Generalization of the problem of sustainable container distribution by alternatively fueled vehicles

Mikhail Y. Kovalyov * **Mikhail N. Lukashevich** ** **Sergei Markov** *** **Erwin Pesch** ** **Sezgi Tekil-Ergün** ****

* *United Institute of Informatics Problems, National Academy of Sciences of Belarus, Surganova 6, 220012 Minsk, Belarus(e-mail: kovalyov_my@newman.bas-net.by).*
** *Information Systems, Faculty III, University of Siegen, 57068 Siegen, Germany (e-mail: mikhail.n.lukashevich@gmail.com, erwin.pesch@uni-siegen.de)*
*** *Faculty of Applied Mathematics and Informatics, Belarusian State University, 220030 Minsk, Belarus (e-mail: markov@bsu.by)*
**** *INFORM GmbH,52076 Aachen, Germany(e-mail: tekilsezgi@gmail.com)*

Abstract: We generalize a previously studied container pickup and delivery problem and relax several constraints by introducing heterogeneous vehicle speeds, distinct fuel consumption rates, partial refueling and operational costs, as well as allowing multiple visits to the same node. In some practical cases the earlier existed constraints result in no feasible solution or high transportation cost. A Mixed Integer Linear Programming model is developed for the more general problem. Preliminary computer experiments have shown that, in the same computing environment, the solution time of the new model is slightly shorter than the one of the old model for most of the same instances, and for a few instances it is slightly longer.

Keywords: Pickup and delivery, full and empty container transportation, green vehicle routing problem, mixed-integer programming.

1. INTRODUCTION

We generalize the container pickup and delivery problem studied by Tekil-Ergün et al. (2022a,b) and develop a *Mixed Integer Linear Programming (MILP)* model for the more general problem. The problem under study can be formulated as follows. There are cargo containers to be picked up and delivered by alternatively fueled vehicles of four types from and to different locations. The containers are of four standard types which are empty (type $20E$) and full (type $20F$) 20-foot containers, and empty (type $40E$) and full (type $40F$) 40-foot containers. A vehicle of type 1 can transport at most one 20-foot container, a vehicle of type 2 can transport at most two 20-foot containers, a vehicle of type 3 can transport at most one 40-foot container, and a vehicle of type 4 can transport at most one 20-foot container and one 40-foot container. The mentioned containers can be empty or full. We denote the set of container types as $\Pi = \{20E, 20F, 40E, 40F\}$ and the set of vehicle types as $M = \{1, 2, 3, 4\}$. Let $Cap_m^{(H)}$ represent the capacity of an m-type vehicle in terms of the number of H-foot containers, where $H \in \{20, 40\}$, $m \in M$.

The set of containers that can be transported by a vehicle is called a feasible load of this vehicle. Each empty or full container is associated with a fixed service time o, which is the unloading time if the container is unloaded from a vehicle and it is the loading time if the container is loaded on a vehicle. We assume that the same container can be dropped off and loaded to a different vehicle, or to the same vehicle at a later visit only if it is empty and only in a special empty container relocation node d^*. Thus, the intermediate relocation of empty containers is possible only in node d^*. Like in Tekil-Ergün et al. (2022a,b) it is assumed that there is an unlimited stock of empty containers of any type in d^*.

The road network is given by a pair (N, Q), where N and Q are the sets of nodes and arcs, respectively. Set N represents client locations that supply and demand containers, vehicle departure and arrival depots d^{out} and d^{in} (which can be physically the same depot), vehicle refuel locations and the single empty container relocation location d^*. Each arc $(i, j) \in Q$ represents a shortest journey between locations i and j. Each node i includes one or more service zones of a set Z_i. For nodes other than d^*, d^{out} and d^{in}, at most one vehicle can be serviced in each zone at the same time.

It is convenient to associate all possible visits of node $i \in N$ with a visiting sequence $V_i = \{1, \ldots, v_i^{\max}\}$ by the vehicles. The number of zones and the number of possible visits of node i do not exceed the number of containers to be loaded and unloaded in this node. The visits are numbered in

[1] Corresponding author: mikhail.n.lukashevich@gmail.com.

10.1016/j.ifacol.2025.09.361

increasing order of the vehicle arrival times. Thus, $v \in V_i$ can be viewed as a position in the visiting sequence of node i by all vehicles. The same vehicle can occupy several visit positions. If a visit is realized then we call it real, otherwise, we call it artificial.

Each node d^*, d^{out} and d^{in} has a single service zone and any number of vehicles can be serviced in this zone at the same time. We assume a single visit position for each of the departure and arrival depot nodes: $v^{\max}_{d^{out}} = v^{\max}_{d^{in}} = 1$. The single visit position for d^{out} or d^{in} can be occupied by any number of vehicles simultaneously. It is convenient to associate node d^* with an upper bound $v^0_{d^*}$ on the number of possible visits of this node by the same vehicle (not by all vehicles).

During the same visit of any node, each vehicle can unload and load any subset of containers such that its load remains feasible at any time. Unloading and loading of a container of the same type on the same vehicle at the same visit is a useless operation. Such operations are avoided in an optimal solution by introducing a non-zero cost $c^{(dp)}$ for each container dropping-picking operation. The useful unloading and loading operations of the same vehicle at the same visit are performed sequentially in any order.

All the container transfers and vehicle journeys must be completed within a planning time interval $[0, T]$. Each node $i \in N$ is associated with a *node time window* $[a_i, b_i]$, $0 \le a_i < b_i \le T$, within which setup and loading-unloading operations at this node must be performed. For the departure node $d^{out} \in N$ and arrival node $d^{in} \in N$ we have $a_{d^{out}} = a_{d^{in}} = 0$ and $b_{d^{out}} = b_{d^{in}} = T$. There is a *setup time* p, $0 \le 2p \le \min_{j \in N \setminus \{d^{in}, d^{out}\}}\{b_j - a_j\}$, which is needed to set up a vehicle for arrival to or departure from any node, unless the vehicle (artificially) goes directly from the departure depot to the arrival depot. Driving ranges of the vehicles are limited, and they need refueling (recharging) for continuous operation.

Each vehicle k of type m, $m \in M$, where M is the set of types, is associated with a *vehicle time window* $[A_k, B_k]$, $0 \le A_k < B_k \le T$, *initial drive time capacity* O^0_k at the departure from the departure depot, a fixed *activation* cost $c^{(ac)}_m$, a *unit-drive-time* cost $c^{(t)}_m$, and a *full-charge driving time range* f_m. The activation cost applies if the vehicle goes from the departure depot to any node different to the arrival depot. The unit-drive-time cost applies to each unit of the driving time. All the operations of an activated vehicle must be completed within its time window. We assume that at the beginning of the planning horizon all the vehicles are standing in the departure depot.

The node set N is partitioned into the following disjoint subsets: a set $DE^{(H)}$ of empty H-foot container demand nodes, a set $SE^{(H)}$ of empty H-foot container supply nodes, a set $DF^{(H)}$ of full H-foot container demand nodes, a set $SF^{(H)}$ of full H-foot container supply nodes, $H \in \{20, 40\}$, a set R of *refuel* nodes, the departure depot node d^{out}, the arrival depot node d^{in}, and the empty container relocation node d^*. It is convenient to introduce set $C = \cup_{H \in \{20,40\}}(DE^{(H)} \cup SE^{(H)} \cup DF^{(H)} \cup SF^{(H)})$ of *client nodes*. Thus, $N = C \cup R \cup \{d^{out}, d^{in}, d^*\}$.

The arc set is $Q = \{(i, j) \mid i \in C \cup R \cup \{d^{out}, d^*\}, j \in C \cup R \cup \{d^{in}, d^*\}, i \ne j\}$. Each arc $(i, j) \in Q$ is associated with a *drive time* t_{ijm} of an m-type vehicle, $m \in M$, which is the shortest time for such a vehicle to drive from node i to node j. By this definition, drive times satisfy the Euclidean metric *triangle inequality*: $t_{ijm} \le t_{ihm} + t_{hjm}$ for any vehicle type $m \in M$ and three distinct nodes i, h, j from N. We assume without loss of generality that $(i, j) \in Q$ implies $t_{ijm} \le f_m$ for at least one $m \in M$ because otherwise no vehicle can drive along the arc (i, j).

Each refuel node $i \in R$ is associated with the minimal refuel time $\delta^{\min}$ and the m-type vehicle *refuel speeds* e_{mi}, $e_{mi} \ge f_m/(b_i - a_i - 2p)$, $m \in M$. The refuel speed is measured in driving capacity time units per refuel time unit. For example, $e_{mi} = 30$ means that 1 minute of refuel time adds 30 drive time minutes to the vehicle. Each node $i \in DE^{(H)}$ is associated with a demand quantity $d^{(H)}_i$. Each node $i \in SE^{(H)} \cup \{d^*\}$ is associated with a supply quantity $s^{(H)}_i$ of empty H-foot containers and an upper bound $n^{(H)}_i \le s^{(H)}_i$ on the number of these containers remaining at i after the transportation plan has been realized.

Since clients are interested in the contents of full containers, and these contents are usually different, there is a one-to-one correspondence between nodes of the sets $SF^{(H)}$ and $DF^{(H)}$ such that each *origin node* $i \in SF^{(H)}$ corresponds to a unique *destination node* $f(i) \in DF^{(H)}$. Define the sets of *origin-destination arcs* as $F^{(H)} = \{(i, f(i)) \in Q \mid i \in SF^{(H)}\}$, $H \in \{20, 40\}$. Each node $i \in SF^{(H)}$ is associated with a single full H-foot container to be delivered from i to $f(i)$, $H \in \{20, 40\}$. It implies that the node $i \in SF^{(H)}$ possesses a single full H-foot container and the node $f(i) \in DF^{(H)}$ demands this container. The "full container" request is satisfied by a single visit. Therefore, there is a single service zone in each "full container" location i and $f(i)$ and the number of visits of any such zone is at most one: $|Z_i| = |Z_{f(i)}| = v^{\max}_i = v^{\max}_{f(i)} = 1$ for $i \in SF^{(H)}$, $H \in \{20, 40\}$.

Denote by $m(k)$ the type of vehicle k. For vehicle k, denote by $\widehat{X}_{ki}$ and $\widehat{Y}_{ki}$ times of its arrival to and departure from node i. If vehicle k drives from node i to node j, then $\widehat{X}_{kj} = \widehat{Y}_{ki} + t_{ij,m(k)}$. Assume that vehicle k unloads r containers, $r \in \{0, 1, 2\}$, and loads h containers, $h \in \{0, 1, 2\}$, at node $i \ne d^{in}$. The respective events happen in the following sequence: arrival, arrival setup, r unloading operations and h loading operations in any order, departure setup and departure. The following relations must be satisfied in order to respect the node time window $[a_i, b_i]$:

$$\max\{a_i, \widehat{X}_{ki}\} + 2p + (r + h)o \le \min\{b_i, \widehat{Y}_{ki}\}.$$

Assume now that the vehicle k refuels at a node $i \in R$. The respective events happen in the following sequence: arrival, arrival setup, refuel, departure setup and departure. Denote by $\widehat{I}_{ki}$ and $\widehat{O}_{ki}$ the remaining drive time capacity of vehicle k at its arrival to and departure from refuel node i, respectively, in units of the remaining drive time capacity. Denote by δ_{ki} the refuel time of this vehicle at this visit. We have $\widehat{O}_{ki} = \min\{f_{m(k)}, \widehat{I}_{ki} + e_{m(k),i}\delta_{ki}\}$. The following relations must be satisfied for this visit in order to respect the node time window $[a_i, b_i]$:

$$\max\{a_i, \widehat{X}_{ki}\} + 2p + \delta_{ki} \le \min\{b_i, \widehat{Y}_{ki}\}.$$

The objective is to find the number of activated vehicles of all types, their driving timetables, unloading-loading and refuel plans such that these vehicles start unloaded from the departure depot d^{out}, return unloaded to the arrival depot d^{in}, feasibly drive with respect to the drive time limits and vehicle and node time windows, are feasibly assigned to the service zones, all container demands and upper bounds on the number of remaining empty supply containers are satisfied, and the total vehicle activation, drive-time-dependent and container dropping-picking cost is minimized. We denote this problem as Container-VRP.

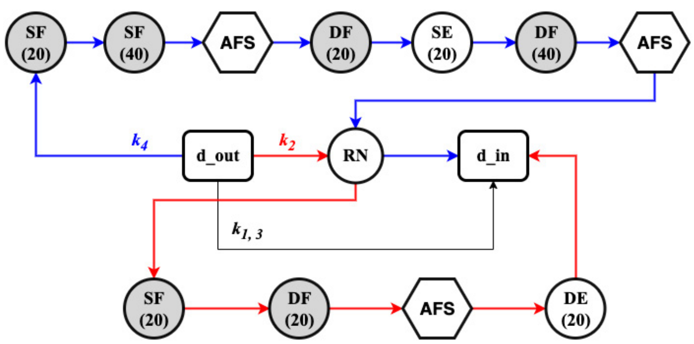

Fig. 1. The routes of container transportation.

In **Figure 1** we illustrate a solution to an instance of the Container-VRP, in which there are three full containers to be delivered (two full 20-foot containers and one 40-foot container), one empty 20-foot container to be picked up, and one empty 20-foot container to be dropped off. In this figure, the transportation route for container delivery is depicted, with various nodes representing different aspects of the logistics network. The nodes are labeled as follows:

- **SF**: Full container supply nodes.
- **SE**: Empty container supply node.
- **DE**: Empty container demand node.
- **DF**: Full container demand nodes.
- **RN**: Relocation node d^*.
- **AFS**: Refuel nodes.

Containers are indicated by their sizes: 20-foot containers are marked as "(20)" and 40-foot containers as "(40)". The vehicles are denoted as k_1 (type 1), k_2 (type 2), k_3 (type 3), and k_4 (type 4). The solution consists of the routes for two vehicles (k_2 and k_4), while k_1 and k_3 are not used. Their routes consist solely of a direct connection from the departure node to the arrival node.

The literature on vehicle routing with container pickup and delivery includes Coslovich et al. (2006), Demir et al. (2014), Ileri et al. (2006), Jula et al. (2006), Fazi et al. (2020), Larsen et al. (2023), Lurkin and Schyns (2015), Tekil-Ergün et al. (2022a), Tekil-Ergün et al. (2022b), Wang and Regan (2002), Yang and Daham (2020) and Yang et al. (2021). Works of Yang and Daham (2020), Yang et al. (2021) and Fazi et al. (2020) may be seen as the most relevant to Tekil-Ergün et al. (2022a,b) and our current work. However, the container transportation problem in Yang and Daham (2020), Yang et al. (2021) and Fazi et al. (2020) is significantly simpler as pickup and delivery locations are matched from the beginning and several other constraints don't exist. Nossack and Pesch (2013) consider a similar problem with a homogeneous fleet of trucks in a simplified network and no consideration of fuel consumption or refueling. Differences between the new problem and the earlier problem in Tekil-Ergün et al. (2022a,b) are listed in the next section. Section 3 presents our MILP model for the new problem.

2. COMPARISON OF OLD AND NEW PROBLEMS

Table 1 demonstrates the main differences between the problem studied in this paper and the problem in Tekil-Ergün et al. (2022a,b).

Table 1. Comparison of two problems.

Tekil-Ergün et al. (2022a,b)	This paper
All empty containers must be picked up.	For each pick-up location, no more than a given number of containers can be left.
A node in the network is associated with a single container supply or demand, implying a pseudo-polynomial number of nodes and single visits of the same node (except for the depot) by all vehicles.	A node in the network is associated with a single location and a fixed number of servicing zones, implying a polynomial number of nodes and multiple visits of the same node.
A vehicle visits the same node at most once, except for the depot.	No constraint on the number of visits of the same node by the same vehicle.
Any number of vehicles can be served simultaneously in the same location.	No two vehicles in the same zone can be served simultaneously. Vehicles in different zones can be served simultaneously.
A vehicle is always refueled to a full capacity.	Partial refueling is allowed.
Same refueling speed.	Different refueling speeds.
No minimal refueling time.	Minimal refueling time is given.
Same driving speeds.	Different driving speeds.
Same fuel consumption.	Different fuel consumptions.
No vehicle and fuel station related time windows.	Vehicle and fuel station related time windows exist.
Arrival to a node must be within a time window.	Arrival to and departure from a node must be within a time window.
Criterion is minimization of the total travel distance.	Criterion is minimization of the total vehicle activation, drive-time-dependent, and container drop/pick cost.

3. MILP MODEL FOR CONTAINER-VRP

It is convenient to introduce the following additional notation and terminology.

- Arc set linking pairs (visit, node): $E = \{((v, i), (u, j)) \mid v \in V_i,\ u \in V_j,\ (i, j) \in Q\}$.

If we say that a vehicle drives over an arc $((v, i), (u, j))$, it drives from the v-th visit of i to the u-th visit of j over the arc $(i, j) \in Q$. The following decision variables are used in our model.

- x^0_{kvi} – binary variable equal to 1 if vehicle k occupies position v in the visiting sequence of node i by all vehicles, $k \in K$, $v \in V_i$, $i \in N$, $i \ne d^*$.

- $x^0_{kvd^*}$ – binary variable equal to 1 if vehicle k occupies position v in its visiting sequence of relocation node d^*, $k \in K$, $v \in V_{d^*}$.
- x_{kviuj} – binary variable equal to 1 if vehicle k drives over the arc $((v,i),(u,j)) \in E$.
- $Pick^{(\pi)}_{kvi}$ – number of π-type containers that vehicle k picks at visit v of node i, , $k \in K$, $v \in V_i$, $i \in N$, $\pi \in \Pi$.
- $Drop^{(\pi)}_{kvi}$ – number of π-type containers that vehicle k drops at visit v of node i, $k \in K$, $v \in V_i$, $i \in N$, $\pi \in \Pi$.
- $Load^{(\pi)}_{kviuj}$ – number of π-type containers that vehicle k carries over the arc $((v,i),(u,j)) \in E$.
- X_{kviuj} – time of arrival of vehicle k to node (u,j) if it drives over the arc $((v,i),(u,j)) \in E$.
- Y_{kviuj} – time of departure of vehicle k from node (v,i) if it drives over the arc $((v,i),(u,j)) \in E$.
- I_{kviuj} – remaining drive time capacity of vehicle k at its arrival to node (u,j) if it drives over the arc $((v,i),(u,j)) \in E$.
- O_{kviuj} – remaining drive time capacity of vehicle k at its departure from node (v,i) if it drives over the arc $((v,i),(u,j)) \in E$. Note that $I_{kviuj} = O_{kujwh}$ for $j \notin R$.
- δ_{kvi} – refuel time of vehicle k at refuel node (v,i), $i \in R$. Equals zero if no visit.

Our MILP model can be written as follows. Denote $M_{ik} := \min\{b_i, B_k\}$.

$$\min \sum_{k\in K}\Bigg(\sum_{((v,i),(u,j))\in E} c^{(t)}_{m(k)} t_{ij,m(k)} x_{kviuj} + c^{(a)}_{m(k)}(1 - x_{k1d^{out}1d^{in}}) + c^{(dp)} \sum_{v\in V_i, i\in C\cup\{d^*\}} \sum_{\pi\in\Pi} (Drop^{(\pi)}_{kvi} + Pick^{(\pi)}_{kvi})\Bigg), \quad (1)$$

subject to

predefined variable values

$$x^0_{k1d^{out}} = 1, k\in K, \quad (2)$$

$$x^0_{k1d^{in}} = 1, k\in K, \quad (3)$$

$$x_{kviuj} = 0, ((v,i),(u,j)) \in E, \quad (4)$$
$$i \in SE^{(20)} \cup SE^{(40)} \cup SF^{(20)} \cup SF^{(40)},$$
$$j \in SE^{(20)} \cup SE^{(40)} \cup SF^{(20)} \cup SF^{(40)}, m(k) \in \{1,3\},$$

$$Drop^{(\pi)}_{k1d} = Pick^{(\pi)}_{k1d} = 0, k\in K,\ d \in \{d^{out}, d^{in}\},\ \pi \in \Pi, \quad (5)$$

$$Load^{(\pi)}_{k1duj} = Load^{(\pi)}_{kvi1d} = 0, k\in K,\ d\in\{d^{out}, d^{in}\},\ \pi\in\Pi, \quad (6)$$
$$((1,d^{out}),(u,j))\in E, ((v,i),(1,d^{in}))\in E,$$

$$Drop^{(HE)}_{kvi} = 0, k\in K, v \in V_i, \quad (7)$$
$$i \in N\backslash\{DE^{(H)} \cup \{d^*\}\},\ H \in \{20,40\},$$

$$Pick^{(HE)}_{kvi} = 0, k\in K, v \in V_i, \quad (8)$$
$$i \in N\backslash\{SE^{(H)} \cup \{d^*\}\},\ H \in \{20,40\},$$

$$Drop^{(HF)}_{kvi} = 0, k\in K, v \in V_i, i \in N\backslash DF^{(H)},\ H \in \{20,40\}, \quad (9)$$

$$Pick^{(HF)}_{kvi} = 0, k\in K, v \in V_i, i \in N\backslash SF^{(H)},\ H \in \{20,40\}, \quad (10)$$

upper bounds on initial drive time capacity

$$O_{k1d^{out}uj} \le O^0_k x_{k1d^{out}uj}, k\in K, ((1,d^{out}),(u,j)) \in E, \quad (11)$$

feasible driving over arc

$$x_{kviuj} \le x^0_{kvi}, x_{kviuj} \le x^0_{kuj}, k\in K, ((v,i),(u,j)) \in E, \quad (12)$$

limited number of useful dropping-picking operations

$$Drop^{(20E)}_{kvi} + Pick^{(20E)}_{kvi} \le x^0_{kvi}, \quad (13)$$
$$k \in \{K \mid m(k) \in \{1,4\}\}, v \in V_i, i\in C \cup \{d^*\},$$

$$Drop^{(20E)}_{kvi} + Pick^{(20E)}_{kvi} \le 2x^0_{kvi}, \quad (14)$$
$$k \in \{K \mid m(k) = 2\}, v \in V_i, i\in C \cup \{d^*\},$$

$$Drop^{(40E)}_{kvi} + Pick^{(40E)}_{kvi} \le x^0_{kvi}, \quad (15)$$
$$k \in \{K \mid m(k) \in \{3,4\}\}, v \in V_i, i\in C \cup \{d^*\},$$

load definition

$$\sum_{((v,i),(u,j))\in E} Load^{(\pi)}_{kviuj} = \sum_{((w,h),(v,i))\in E} Load^{(\pi)}_{kwhvi} \quad (16)$$
$$-Drop^{(\pi)}_{kvi} + Pick^{(\pi)}_{kvi}, k \in K, \pi \in \Pi, v \in V_i, i \in C \cup \{d^*\},$$

$$\sum_{((v,i),(u,j))\in E} Load^{(\pi)}_{kviuj} = \sum_{((w,h),(v,i))\in E} Load^{(\pi)}_{kwhvi}, \quad (17)$$
$$k \in K, \pi \in \Pi, v \in V_i, i \in R,$$

$$Drop^{(\pi)}_{kvi} \le \sum_{((w,h),(v,i))\in E} Load^{(\pi)}_{kwhvi}, \quad (18)$$
$$k\in K, ((w,h),(v,i)) \in E, \pi\in\Pi,$$

$$Load^{(HE)}_{kviuj} + Load^{(HF)}_{kviuj} \le Cap^{(H)}_{m(k)} x_{kviuj}, \quad (19)$$
$$k\in K, H\in\{20,40\}, ((v,i),(u,j)) \in E,$$

feasible drive time capacity of vehicles

$$O_{kviuj} \le f_{m(k)} x_{kviuj}, k\in K, ((v,i),(u,j)) \in E, \quad (20)$$

$$I_{kviuj} = O_{kviuj} - t_{ijm(k)} x_{kviuj},\ k\in K, ((v,i),(u,j)) \in E, \quad (21)$$

$$\sum_{((w,h),(v,i))\in E} I_{kwhvi} = \sum_{((v,i),(u,j))\in E} O_{kviuj}, \quad (22)$$
$$k \in K, v \in V_i, i \in C \cup \{d^*\},$$

$$I_{kviuj} + e_{m(k),j}\delta_{kuj} \le f_{m(k)} x^0_{kuj}, \quad (23)$$
$$k\in K, u \in V_j, j\in R, ((v,i),(u,j)) \in E,$$

$$\delta^{\min} x^0_{kvi} \le \delta_{kvi}, k\in K, v \in V_i, i\in R, \quad (24)$$

$$\sum_{((v,i),(u,j))\in E} O_{kviuj} = \sum_{((w,h),(v,i))\in E} I_{kwhvi} \quad (25)$$
$$+e_{m(k),i}\delta_{kvi}, k \in K, v \in V_i, i \in R,$$

linking departure and arrival times

$$X_{kviuj} \ge Y_{kviuj} + t_{ijm(k)} x_{kviuj}, \quad (26)$$
$$k\in K, j \in N, ((v,i),(u,j)) \in E,$$

bounding variables X and Y by means of variables x

$$X_{kviuj} \le T x_{kviuj}, Y_{kviuj} \le T x_{kviuj}, \quad (27)$$
$$k\in K, j \in N, ((v,i),(u,j)) \in E,$$

$$Y_{k1d^{out}uj} \ge p x_{k1d^{out}uj}, \quad (28)$$
$$k \in K, j \in N, j \ne d^{in}, ((1,d^{out}),(u,j)) \in E,$$

$$X_{kvi1d^{in}} \le (T - p) x_{kvi1d^{in}}, \quad (29)$$
$$k \in K, i \in N, i \ne d^{out}, ((v,i),(1,d^{in})) \in E,$$

node time window feasibility

$$\delta_{kvi} \le (b_i - (a_i + 2p))x^0_{kvi}, k\in K, v \in V_i, i\in R, \quad (30)$$

$$(a_i + 2p)x^0_{kvi} + \delta_{kvi} \le \sum_{((v,i),(u,j))\in E} Y_{kviuj}, \quad (31)$$
$$k\in K, v \in V_i, i\in R,$$

$$\sum_{((w,h),(v,i))\in E} X_{kwhvi} + \delta_{kvi} \le (b_i - 2p)x^0_{kvi}, \quad (32)$$
$$k\in K, v \in V_i, i\in R,$$

$$\sum_{((w,h),(v,i))\in E} X_{kwhvi} + 2px^0_{kvi} + \delta_{kvi} \quad (33)$$
$$\le \sum_{((v,i),(u,j))\in E} Y_{kviuj}, k\in K, v \in V_i, i\in R,$$

$$o\Big(\sum_{\pi\in\Pi} Drop^{(\pi)}_{kvi} + \sum_{\pi\in\Pi} Pick^{(\pi)}_{kvi}\Big) \le (b_i - (a_i+2p))x^0_{kvi}, \quad (34)$$
$$k\in K, v\in V_i, i\in C\cup\{d^*\},$$

$$(a_i + 2p)x^0_{kvi} + o\Big(\sum_{\pi\in\Pi} Drop^{(\pi)}_{kvi} + \sum_{\pi\in\Pi} Pick^{(\pi)}_{kvi}\Big) \quad (35)$$
$$\le \sum_{((v,i),(u,j))\in E} Y_{kviuj}, k\in K, v\in V_i, i\in C\cup\{d^*\},$$

$$\sum_{((w,h),(v,i))\in E} X_{kwhvi} + o\Big(\sum_{\pi\in\Pi} Drop^{(\pi)}_{kvi} + \sum_{\pi\in\Pi} Pick^{(\pi)}_{kvi}\Big) \quad (36)$$
$$\le (b_i - 2p)x^0_{kvi}, k\in K, v\in V_i, i\in C\cup\{d^*\},$$

$$\sum_{((w,h),(v,i))\in E} X_{kwhvi} + 2px^0_{kvi} \quad (37)$$
$$+ o\Big(\sum_{\pi\in\Pi} Drop^{(\pi)}_{kvi} + \sum_{\pi\in\Pi} Pick^{(\pi)}_{kvi}\Big)$$
$$\le \sum_{((v,i),(u,j))\in E} Y_{kviuj}, k\in K, v\in V_i, i\in C \cup \{d^*\},$$

vehicle time window feasibility

$$A_k x_{k1d^{out}uj} \le Y_{k1d^{out}uj}, ((1, d^{out}), (u, j)) \in E, k\in K, \quad (38)$$

$$X_{kvi1d^{in}} \le B_k x_{kvi1d^{in}}, ((v, i), (1, d^{in})) \in E, k\in K, \quad (39)$$

non-intersecting real visits

$$\sum_{k\in K} \sum_{((v,i),(u,j))\in E} Y_{kviuj} \quad (40)$$
$$\le \sum_{k\in K} \sum_{((w,h),(v+1,i))\in E} X_{kwh,v+1,i} + T(1 - \sum_{k\in K} x^0_{k,v+1,i}),$$
$$v = 1,\ldots, v_i - 1,\ i \in N\backslash\{d^{out}, d^{in}, d^*\},$$

satisfying full container supply and demand

$$\sum_{k\in K} Pick^{(HF)}_{k1i} = \sum_{k\in K} Drop^{(HF)}_{k1,f(i)} = 1, \quad (41)$$
$$i \in SF^{(H)},\ H \in \{20, 40\},$$

$$Pick^{(HF)}_{k1i} = Drop^{(HF)}_{k1,f(i)}, \quad (42)$$
$$k \in K, i \in SF^{(H)}, H \in \{20, 40\},$$

full container departure from i precedes its arrival to $f(i)$

$$\sum_{((1,i),(w,h))\in E} Y_{k1iwh} \le \sum_{((w,h),(1,f(i)))\in E} X_{kwh1f(i)}, \quad (43)$$
$$k \in K,\ i \in SF^{(H)}, H \in \{20, 40\},$$

satisfying empty container demand

$$\sum_{v\in V_i} \sum_{k\in K} Drop^{(HE)}_{kvi} = d^{(H)}_i,\ i \in DE^{(H)},\ H \in \{20, 40\}, \quad (44)$$

satisfying empty container supply and upper bounds

$$s^{(H)}_i - n^{(H)}_i \le \sum_{v\in V_i} \sum_{k\in K} Pick^{(HE)}_{kvi} \le s^{(H)}_i, \quad (45)$$
$$i \in SE^{(H)},\ H \in \{20, 40\},$$

at most one vehicle occupies the same visit position

$$\sum_{k\in K} x^0_{kvi} \le 1, v \in V_i, i \in N\backslash\{d^{out}, d^{in}, d^*\}, \quad (46)$$

real visits are consecutively indexed

$$\sum_{k\in K} x^0_{kvi} \ge \sum_{k\in K} x^0_{k,v+1,i}, \quad (47)$$
$$v = 1,\ldots, v_i - 1,\ i \in N\backslash\{d^{out}, d^{in}, d^*\},$$

$$x^0_{kvd^*} \ge x^0_{k,v+1,d^*},\ v = 1,\ldots, v_{d^*} - 1,\ k \in K, \quad (48)$$

vehicle occupies (v, i) if and only if

it comes to and leaves from (v, i)

$$\sum_{((h,w),(v,i))\in E} x_{kwhvi} = \sum_{((v,i),(u,j))\in E} x_{kviuj} = x^0_{kvi}, \quad (49)$$
$$k \in K, v \in V_i, i \in N\backslash\{d^{out}, d^{in}\},$$

each vehicle arrives once to d^{in}

$$\sum_{((v,i),(1,d^{in}))\in E} x_{kvi1d^{in}} = 1,\ k \in K, \quad (50)$$

each vehicle departs once from d^{out}

$$\sum_{((1,d^{out}),(u,j))\in E} x_{k1d^{out}uj} = 1,\ k \in K, \quad (51)$$

domains of variables

$$x_{kviuj} \in \{0, 1\}, 0 \le I_{kviuj} \le f_{m(k)}, 0 \le O_{kviuj} \le f_{m(k)}, \quad (52)$$
$$k \in K, ((v, i), (u, j)) \in E,$$

$$x^0_{kvi} \in \{0, 1\}, k\in K, v \in V_i, i\in N, \quad (53)$$

$$0 \le \delta_{kvi} \le f_{m(k)}, k\in K, v \in V_i, i\in R, \quad (54)$$

$$0 \le X_{kwh1i} \le B_k, 0 \le Y_{k1iuj} \le B_k, \quad (55)$$
$$k\in K, i\in \{d^{out}, d^{in}\}, ((w, h), (1, i)) \in E, ((1, i), (u, j)) \in E,$$

$$0 \le X_{kwhvi} \le M_{ik} - 2p - o, 0 \le Y_{kviuj} \le M_{ik}, \quad (56)$$
$$k\in K, v \in V_i, i\in C \cup d^*,$$
$$((w, h), (v, i)) \in E, ((v, i), (u, j)) \in E,$$

$$0 \le X_{kwhvi} \le M_{ik} - 2p - \delta^{\min}, 0 \le Y_{kviuj} \le M_{ik}, \quad (57)$$
$$k\in K, v \in V_i, i\in R, ((w, h), (v, i)) \in E, ((v, i), (u, j)) \in E.$$

For solving the problem instances, we used an academic version of the Python API of CPLEX. The experiments were conducted on a MacBook Pro equipped with an Intel Core i7 2.6 GHz processor and 16 GB of RAM under macOS Big Sur. Computer experiments with the data sets from Tekil-Ergün et al. (2022a,b) demonstrated that, in the same computing environment, the new model solves 87.5% of all instances faster than the old model, with an average run time reduction of approximately 30%. In only 12.5% of all instances, the new model exhibited slightly longer solution times, with an average run time increase of around 7%. There are instances from Tekil-Ergün et al. (2022a,b) for which one of the assumptions "at most one refueling", "at most one visit of the same node", "full capacity refueling" and "vehicle visits the same node at most once" results in no feasible solution or higher cost, while these effects do not exist for the new model. Table 2 demonstrates an instance of computer experiments in the 3f1p1d4s(A) data set, that encodes 3 full container pick-up requests (`3f`), 1 empty container pick-up request (`p1`), 1 empty container delivery requests (`1d`), 4 fuel stations (AFSs) (`4s`) and the data type (`A`), consists of six scenarios $T1, T2, \ldots, T6$ and 5 instances. This table contains the following information:

- The model from Tekil-Ergün et al. (2022a,b) is named Old and our model is named New.
- Average value of the objective function (cost) ($\overline{z}$).
- Maximum cost saving of the given model over the other model ($\Delta^z_{\max}$).
- Average run time in seconds (cpu_{mean}).
- Median run time in seconds (cpu_{median}).
- Average relative optimality gap ($\overline{gap}$).

Table 2. Computer experiments.

	$\overline{z}$	$\Delta^z_{\max}$	cpu_{mean}	cpu_{median}	$\overline{gap}$
			T1 (k=4)		
New	964	0	17.47	6.78	0
Old	964	0	22.79	16.86	0
			T2 (k=6)		
New	964	0	60.12	53.16	0
Old	964	0	140.57	105.14	0
			T3 (k=8)		
New	964	0	64.51	23.44	0
Old	964	0	136.66	116.73	0
			T4 (k=4)		
New	1222	5	24.87	8.82	0
Old	1223	0	27.36	17.22	0
			T5 (k=6)		
New	1222	5	182.46	90.85	0
Old	1223	0	75.29	28.99	0
			T6 (k=8)		
New	1222	5	195.50	44.12	0
Old	1223	0	382.91	105.40	0

4. CONCLUSION AND FUTURE WORK

This paper presents a generalized model for the sustainable container pickup and delivery problem by relaxing several restrictive constraints from previously studied models. Key generalizations include heterogeneous vehicle speeds, distinct fuel consumption rates, partial refueling and operational costs, as well as allowing multiple visits to the same node. These modifications improve the model's adaptability and relevance to practical logistical situations. A Mixed Integer Linear Programming model is developed, and computational experiments demonstrate that the new model achieves shorter solution times than the old model for most instances, with slightly longer times for a few cases. This work contributes to the literature on sustainable container logistics by providing a more realistic and adaptable modeling framework. However, limitations include the computational complexity of large-scale instances and the reliance on exact solution methods. Future research will focus on developing heuristic and metaheuristic algorithms to solve larger instances more efficiently, incorporating stochastic demand and travel times, and validating the model through real-world case studies. Additionally, further comparison with related models from the literature will be explored, along with an in-depth analysis of the impact of each relaxed constraint on solution quality and feasibility.

REFERENCES

Coslovich, L., Pesenti, R, Ukovich, W. (2006). Minimizing fleet operating costs for a container transportation company. *European Journal of Operational Research*, 171, 776-786.

Demir, E., Bektas, T., Laporte, G. (20141). A review of recent research on green road freight transportation. *European Journal of Operational Research*, 237(3), 775-793.

Ileri, Y., Bazaraa, M., Gifford, T., Nemhauser, G., Sokol, J., Wikum, E. (2006). An optimization approach for planning daily drayage operations. *Central European Journal of Operations Research*, 14, 141-156.

Jula, H., Dessouky, M., Ioannou, P., Chassiakos, A. (2005). Container movement by trucks in metropolitan networks: modeling and optimization. *Transportation Research Part E: Logistics and Transportation Review*, 41, 235-259.

Fazi, S., Fransoo, J.C., Van Woensel, T., Dong, J.-X. (2020). A variant of the split vehicle routing problem with simultaneous deliveries and pickups for inland container shipping in dry-port based systems. *Transportation Research Part E: Logistics and Transportation Review*, 142, 102057.

Larsen, R.B., Guo, W., Atasoy, B. (2023). A real-time synchromodal framework with co-planning for routing of containers and vehicles. *Transportation Research Part C: Emerging Technologies*, 157, 104412.

Lurkin, V., Schyns, M. (2015). The airline container loading problem with pickup and delivery, *European Journal of Operational Research*, 244(3), 955-965.

Nossack, J., Pesch, E. (2013). A truck scheduling problem arising in intermodal container transportation, *European Journal of Operational Research*, 230(3), 666-680.

Tekil-Ergün, S., Pesch, E., and Kuzmicz, K.A. (2022). Sustainable container distribution by alternatively fueled vehicles under customer and technical constraints. *IFAC-PapersOnLine*, 55(10), 836–841. 10th IFAC Conference on Manufacturing Modelling, Management and Control (MIM 2022).

Tekil-Ergün, S., Pesch, E., and Kuzmicz, K.A. (2022). Solving a hybrid mixed fleet heterogeneous dial-a-ride problem in delay-sensitive container transportation. *International Journal of Production Research*, 60(1), 297–323.

Wang, X., Regan A.C. (2002) Local truckload pickup and delivery with hard time window constraints. Transportation Research Part B: Methodological, 36, 97-112

Yang, X., Daham, H.A. (2020). A column generation-based decomposition and aggregation approach for combining orders in inland transportation of containers. *OR Spectrum*, 42, 261–296.

Yang, X., Daham, H.A., Salhi, A. (2021). Combined strip and discharge delivery of containers in heterogeneous fleets with time windows. *Computers & Operations Research*, 127, 105141.