An Effective Engineering Approach to Launch Vehicle Stability Analysis

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Ensuring the stability of the launch vehicle's flight is a crucial prerequisite for mission success. In practical engineering, establishing an accurate mathematical model and directly conducting stability analysis is extremely difficult, as the launch vehicle is a strongly coupled, high-order, time-varying nonlinear dynamical system integrating rigid body motion, structural elasticity, liquid sloshing, and variable mass characteristics. Traditional stability criteria for nonlinear systems, such as the Lyapunov method, are theoretically rigorous, but they have significant limitations in engineering applications due to issues such as the difficulty in constructing suitable Lyapunov functions and the huge amount of real-time computation required. By analyzing these linearized models, the stability of the rocket throughout the flight process can be effectively evaluated, providing a basis for control system design.

Keywords: launch vehicle; stability analysis; nonlinear dynamics; coupled dynamics; Lyapunov method

Эффективный инженерный подход к анализу устойчивости ракеты-носителя

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Обеспечение устойчивости полета ракеты-носителя является ключевым условием успеха миссии. В практической инженерии создание точной математической модели и прямой анализ устойчивости чрезвычайно сложны, поскольку ракета-носитель представляет собой сильно связанную, высокопорядковую, изменяющуюся во времени нелинейную динамическую систему, объединяющую движение твердого тела, структурную упругость, раскачивание жидкости и характеристики переменной массы. Традиционные критерии устойчивости для нелинейных систем, такие как метод Ляпунова, теоретически строги, но имеют значительные ограничения в инженерных приложениях из-за таких проблем, как трудности в построении подходящих функций Ляпунова и огромных объемов вычислений в реальном времени. Анализируя эти линеаризованные модели, можно эффективно оценить устойчивость ракеты на протяжении всего процесса полета, что служит основой для проектирования системы управления.

Ключевые слова: запускная установка; анализ устойчивости; нелинейная динамика; связанная динамика; метод Ляпунова

Introduction

As the primary means of transportation into space, the success of the flight process of a carrier rocket is directly related to the execution of major national space missions and the safety of space assets. Throughout the entire process from the rocket's takeoff from the ground to the delivery of the payload into the predetermined orbit, it is necessary to always maintain stable attitude to ensure that it can fly precisely along the preset nominal trajectory. Any minor instability may lead to severe deviation in flight attitude and even cause the rocket structure to overload and disintegrate, resulting in catastrophic consequences. Therefore, conducting an in-depth and accurate stability analysis of the carrier rocket is the core task and fundamental premise of its overall design and control system design.

However, conducting precise stability analysis on launch vehicles is a highly challenging task. Modern launch vehicles typically exhibit structural characteristics of large aspect ratio, thin-walled, and lightweight, making them complex rigid-flexible coupled elastomers. During high-speed flight, the vehicle body not only has to withstand severe aerodynamic torques, but the sloshing effect of the internal liquid propellant can also have a significant coupled impact on the vehicle's attitude [1]. Furthermore, the coupling between engine thrust pulsations, structural elastic vibrations of the vehicle body, and the propellant delivery system may also induce longitudinal "POGO" vibrations, posing a serious threat to system stability [2]. More importantly, as the fuel is continuously consumed during flight, the mass, center of mass position, moment of inertia of the rocket change rapidly over time, making the entire system essentially a high-order, multivariable, strongly coupled, and time-varying nonlinear dynamical system [3]. These complex dynamical characteristics are intertwined, making it difficult to establish a global mathematical model that can be directly used for analytical purposes.

In the field of control theory, although there are mature stability criteria for nonlinear systems such as Lyapunov's second method, applying them directly to complex engineering systems like launch vehicles faces a significant gap between theory and practice. Constructing a suitable Lyapunov function systematically for high-dimensional rocket dynamics systems remains an unresolved challenge, lacking a universal engineering approach [4]. Even if it could be constructed offline, the enormous computational load required for real-time analysis during flight far exceeds the capabilities of current on-board computers, making it impossible to meet real-time requirements. Therefore, it is crucial to seek an analytical method that can fully reflect the dynamic characteristics of the system and is easily implementable in engineering.

To address this challenge, a linearization analysis method based on small perturbation theory is commonly adopted in engineering practice. The core idea of this method is that the control system of a launch vehicle is designed to precisely track a pre-designed nominal trajectory. Therefore, during normal flight, the deviation of the actual state of the vehicle from the nominal state can be regarded as a small perturbation. Based on this assumption, complex nonlinear dynamic equations can be expanded into Taylor series along multiple flight state points along the nominal trajectory, neglecting higher-order terms, thus obtaining a series of Linear Time-Varying (LTV) or Linear Time-Invariant (LTI) small deviation models describing the local dynamic characteristics of the system [5]. This method successfully transforms a complex global nonlinear problem into a series of linear problems that can be independently analyzed in different flight phases. For these linearized models, classical control theory tools such as root loci and frequency response (e.g., Bode plots, Nyquist diagrams) can be easily applied for in-depth stability margin analysis and controller parameter design.

The research presented in this article aims to provide clear engineering ideas and theoretical support for the stability analysis and control system design of launch vehicles.

Stability Criteria for Linear Systems and Nonlinear Systems

Ideally, the launch vehicle should follow the predetermined trajectory. However, due to various disturbances during actual flight, errors are bound to occur. Therefore, it is necessary to design an attitude control system to eliminate the deviation between the actual flight trajectory and the predetermined trajectory. Under the control of the rocket guidance system, the actual flight trajectory of the rocket is quite close to the predetermined trajectory, and the deviation of flight parameters is small. Therefore, the stability analysis of the linear system obtained by the small disturbance method can well approximate the dynamic characteristics of the nonlinear system and has been effectively verified in engineering practice [7].

There are usually two methods for determining the stability of a linear system: one is the root locus method, which examines the situation of the characteristic roots corresponding to the differential equations describing the dynamics of the rocket body. Based on the distribution of open-loop zeros and poles and the root locus gain, a root locus diagram is drawn to identify the closed-loop poles. If the drawn root locus is entirely located on the left side of the S plane, it indicates that no matter how the system parameters are changed, all characteristic roots have negative real parts, and the system is stable. If the root locus is entirely in the right half of the S plane, it indicates that the system is unstable no matter how the system parameters are changed. If it is on the imaginary axis, it indicates critical stability, which means continuous oscillation. Since high-order linear systems have many characteristic roots, it is difficult to draw a root locus diagram, so the root locus method is suitable for stability analysis of second-order or multi-order systems. The other method is to determine the stability of the entire system by evaluating the open-loop amplitude-phase characteristics of the system. The frequency domain analysis method is mainly based on the Nyquist stability criterion. If the open-loop system is stable, i.e., $P = 0$, then the necessary and sufficient condition for the closed-loop system to be stable is that the Γ_{GH} curve does not enclose the point (-1, j0). If the open-loop system is unstable, i.e., $P \neq 0$, then the necessary and sufficient condition for the closed-loop system to be stable is that the curve rotates P times around the point (-1, j0) in a counterclockwise direction. A commonly used method in engineering design is to first calculate the frequency characteristics of the open-loop transfer function and then design the closed-loop system based on the Nyquist stability criterion [8].

During flight, the mass of the launch vehicle continuously decreases, and environmental parameters undergo continuous changes, making it a time-varying nonlinear system. The stability of a time-varying nonlinear system is typically analyzed using Lyapunov stability theory. According to the Lyapunov stability definition, for a continuous nonlinear system $\dot{x} = f(x, t)$, where x is an n-dimensional state vector, when there exists an equilibrium state x_e in the state space such that $f(x_e, t) = 0$, the system will reach equilibrium at this point without external forces. During the process of the system transitioning from unstable to stable, its energy continuously decreases, and the equilibrium point satisfies the conditions of energy $E > 0$, $(dE/dt) < 0$.

Lyapunov constructed an energy function $V(x)$ described by state variables, and its equilibrium point satisfies:

$$\begin{cases} V(x) > 0(x \neq 0) \\ V(x) = 0(x = 0) \end{cases} \tag{1}$$

and satisfying

$$\dot{V}(x) < 0 \tag{2}$$

the stability of the system can be determined. Based on the above analysis, Lyapunov proposed the stability theorem for nonlinear systems:

a) If $V(x)$ is a positive definite function and $\dot{V}(x)$ is a semi-negative definite function, then the equilibrium state $x_e = 0$ is stable;

b) If $V(x)$ is a positive definite function, $\dot{V}(x)$ is a negative definite function, and $\dot{V}(x) \neq 0$ for all nonzero solutions of the nonlinear system, then the equilibrium state $x_e = 0$ is asymptotically stable;

c) If $x_e = 0$ is asymptotically stable, and when $\|x\| \to \infty$, $V(x,t) \to \infty$, then $x_e = 0$ is globally asymptotically stable.

Currently, there is no universal method to construct an energy function, and the launch vehicle is a complex system with rigid-elastic-sloshing coupling. It is very difficult to find an energy function that accurately describes the effects of elastic vibration and liquid sloshing, making it challenging to meet the requirements of real-time stability analysis in practical engineering applications [6, 7].

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