БЕЛОРУССКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

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ENGLISH FOR MATHEMATICIANS

АНГЛИЙСКИЙ ЯЗЫК ДЛЯ МАТЕМАТИКОВ

В трех частях

Часть 1

Допущено
Министерством образования Республики Беларусь
в качестве учебного пособия
для студентов учреждений высшего образования
по специальностям «компьютерная математика
и системный анализ», «механика
и математическое моделирование»,
«математика и компьютерные науки», «математика»

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К31

Р е ц е н з е н т ы:
кафедра теории и практики перевода Минского государственного лингвистического университета (заведующий кафедрой кандидат филологических наук, доцент *Е. Г. Карапетова*);
доктор филологических наук, доцент *О. А. Лущинская*

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Представлены основные элементы арифметики, алгебры и геометрии. Аутентичный учебный материал сопровождается учебно-познавательными заданиями, направленными на развитие речевых умений. Усвоение приведенной информации будет способствовать межкультурной профессиональной коммуникации.

Предназначено для студентов учреждений высшего образования, обучающихся по специальностям «компьютерная математика и системный анализ», «механика и математическое моделирование», «математика и компьютерные науки», «математика».

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ПРЕДИСЛОВИЕ

«English for Mathematicians» — первая часть учебного пособия, в которой приведены основные элементы арифметики, алгебры и геометрии. Усвоение представленного материала (рассчитано на 70 академических часов) направлено на развитие иноязычной профессиональной коммуникативной компетенции.

Книга состоит из четырех блоков: «Unit 1. Matheducation», «Unit 2. Arithmetic», «Unit 3. Algebra», «Unit 4. Geometry» и трех приложений. Все блоки единообразно структурированы и включают следующие разделы: «Building the Concept», «Exploring the Topic», «Problem-solving».

Раздел «Building the Concept» содержит задания, выполнение которых поможет изучить понятийный аппарат по теме и сформировать на его основе умения профессионального общения. Усвоение специальной терминологии осуществляется на базе аутентичных текстовых и видеоматериалов.

В разделе «Exploring the Topic» даны тексты и задания к ним, выполнение которых будет способствовать расширению словарного запаса, усвоению речевого материала и его использованию при написании эссе и подготовке докладов/презентаций.

Раздел «Problem-solving» включает текстовой материал, изучение которого направлено на решение конкретной образовательной задачи проблемного характера.

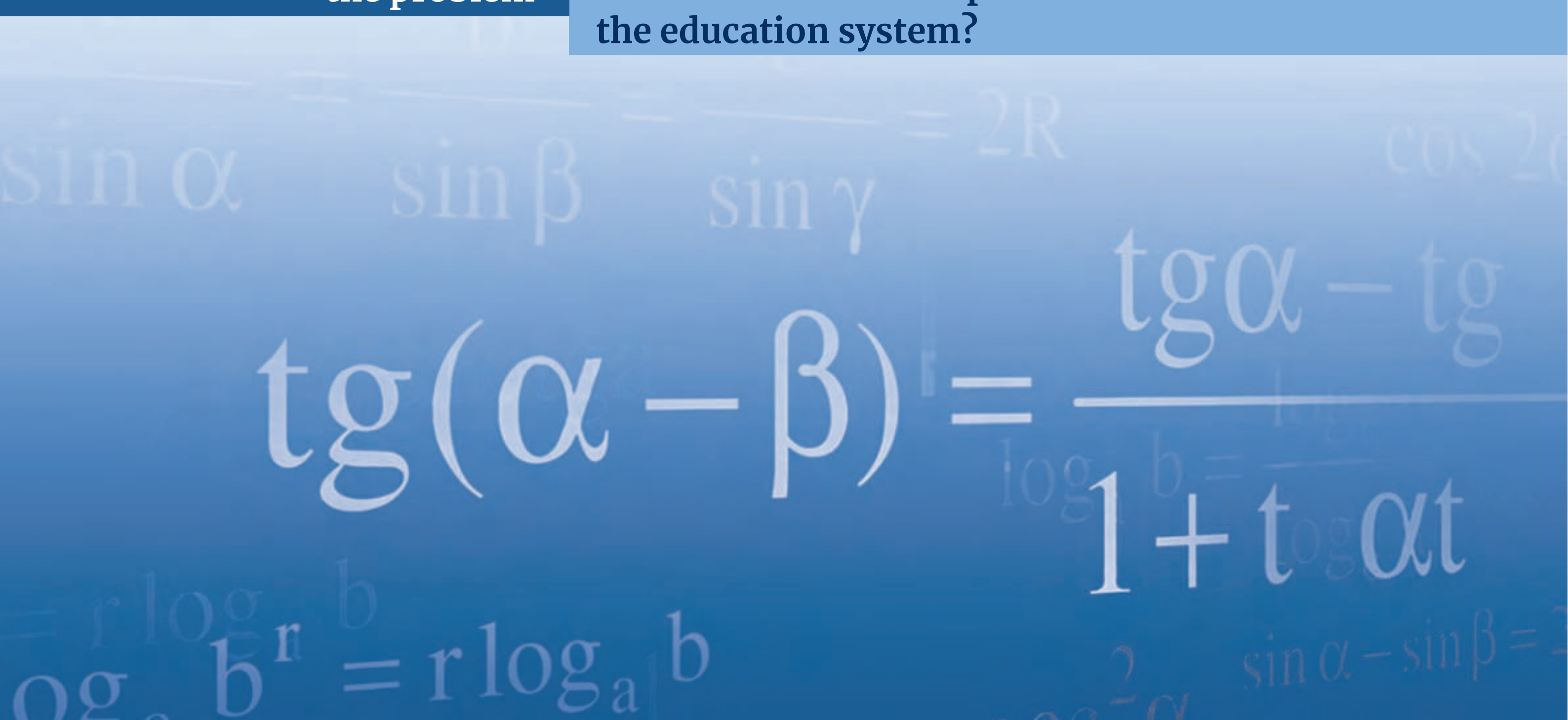
Помимо четырех блоков в учебном пособии представлены три приложения: «Building Autonomy», «Building Competence», «Building Confidence». Приложение «Building Autonomy» содержит материалы для проведения управляемой самостоятельной работы в дистанционном формате. «Building Competence» включает комплексы языковых упражнений для закрепления лексики и отработки языкового материала по изучаемым темам четырех блоков. В приложении «Building Confidence» даны рекомендации по подготовке докладов/презентаций, написанию эссе/статей, проведению дебатов.

MATHEDUCATION

UNIT 1

In this unit you'll

examine	BSU history; structure and scientific potential
look closely at	diversity of mathematical science
focus on	the reasons to choose the Mechanics and Mathematics Faculty
study	a system of higher education abroad
write an essay about	outstanding mathematicians
make a presentation about	the greatest achievements of the BSU
solve the problem	Education: How to improve the education system?



ACTIVE VOCABULARY

ADJECTIVES

elite
comprehensive
specialized
sustainable
state-funded
ancillary
extensive

NOUNS

department
admission
personnel
dormitory
flagshi
facility

VERBS

to possess
to maintain
to resume
to employ
to receive
to provide
to rename into
to conduct
to promote
to encompass
to implement

COLLOCATIONS

University sta
full cycle
job placement
training process
higher education
public association
academic mobility
quality of education
professional competencies

BUILDING THE CONCEPT

THE BELARUSIAN STATE UNIVERSITY

Task 1. ***Work in pairs and comment on the following quotations.***

Education is the most powerful weapon which you can use to change the world.

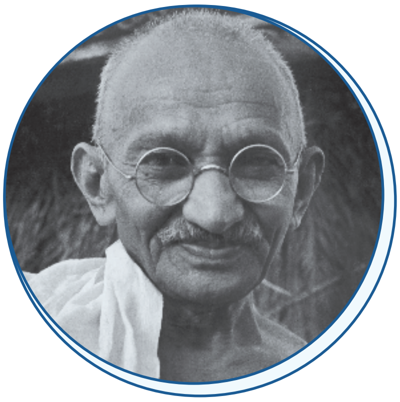
Nelson Mandela

Education is the key that unlocks the golden door to freedom.

George Washington Carver

Education's purpose is to replace an empty mind with an open one.

Malcolm Forbes



Live as if you were to die tomorrow. Learn as if you were to live forever.

Mahatma Gandhi

An investment in knowledge pays the best interest.

Benjamin Franklin

Task 2. ***Look at the following pairs of words and say if you think these are the same or different and why:***

- university and institute;
- high school and higher education;
- faculty and college;
- academic community and academic learning;
- training and education.

Task 3. ***In pairs, discuss the following questions: Why did you choose the BSU? What influenced your choice?***

Task 4. ***With your partner, name as many facts about the BSU as possible. Organize your ideas using the scheme.***

Task 5. A. ***Pair the verbs in column A with suitable words or phrases in column B.***

A	*B*
1. To teach	a) to the capital of the republic
2. To deliver	b) students
3. To be awarded	c) a course
4. To be housed	d) BSU diplomas
5. To be recognized	e) in international rankings
6. To train	f) the potential
7. To foster	g) a lecture
8. To exploit	h) the resource base
9. To be	i) projects
10. To strengthen	j) at students' disposal
11. To participate	k) the achievements
12. To carry out	l) as the leading higher education institution
13. To obtain	m) the title
14. To be involved	n) in the local school
15. To relocate back	o) worldwide fame
16. To gain	p) in civic and patriotic events

B. ***Work in pairs. Discuss who or what might carry out these actions in the BSU.***

Task 6. A. ***Match the words from the box with their meanings.***

1. A place where people study for an undergraduate or postgraduate degree.
2. A person who has a first degree from a university or college
3. A dedicated space for the primary purpose of watching movies, playing video games and engaging with other forms of entertainment media.
4. A group of people whose common interest is pedagogical training.
5. Buildings, equipment, and services provided for a particular purpose.
6. An organization or location where science is done.
7. A formal talk on a serious subject given to a group of people, especially students.
8. A person in charge of a university.
9. A workplace for the conduct of scientific research
10. Something that has been made or built from parts, especially a large building.

B. ***Complete the sentences using the words from the box above.***

1. The institute has become known as a ... ... outside the republic.
2. His ... ... were ordered and delivered with precise timing.
3. You also have a library, music room, and ... choose from.
4. The community center has some of the newest equipment and best ... in town.
5. Our clear vocation for teaching and the passion of the entire ... are the best guarantee of success.
6. ... and vice-rectors, as well as deans and directors of the departments are elected by the staff for five-year terms and may be re-elected onc
7. She was rejected the first time she applied to the ..., but when she reapplied the following year she was accepted.
8. There are numerous opportunities for young ... .
9. The company has a complex organizational ... .
10. Aside from theoretical studies, we also perform experimental studies in our own ... .

Task 7. A. ***Look through the following statements about the BSU and say if you consider them to be True or False.***

1. The first national university in Belarus was opened on August 0, 1921.
2. Regular classes began at its three faculties: Workers, Medicine and Social Sciences.
3. Twenty BSU students and teachers were awarded the title of the Hero of the Soviet Union.
4. The main academic building was housed in the local hospital.
5. On October 16, 1945 lectures resumed.
6. The BSU comprises 9 buildings and 166 facilities with a total area of more than 42,000 sq. m.

7. The Belarusian State University has equipped its academic buildings with state of art facilities — 70 computer laboratories and 4 media rooms are at students' disposal.
8. One of the strategic priorities for the BSU is to participate in international rankings.
9. The BSU trains only Belarusian students.
10. Throughout its history, over 1,000 foreign graduates have left the Belarusian State University.

B. ***Now read the text and check how many false sentences you have managed to figure out.***

The Belarusian State University is a leading scienti c, educational, innovative and cultural center of the Republic of Belarus. The University exploits its potential based on best local and international experience to satisfy intellectual, cultural and social needs and interests of the Belarusian society, and promotes the sustainable development of Belarus.

The history of the BSU is full of amazing facts. Once Albert Einstein himself intended to get a job here as his assistant, Jacob Grommer, had already been working at the BSU. For some period of time, Georgy Zhukov, the future Soviet marshal, taught a course on pre-conscription training here. Famous Belarusian writers and poets Y. Kolas, P. Hlebka, K. Krapiva, P. Brouka, among others, used to lecture at the BSU.

The rst national university in Belarus was opened on October 30, 1921. The historian and slavist Vladimir Picheta became its first rector. On October 31, 1921 the first lecture was delivered under the roof of the Belarusian State University. Regular classes began at its three faculties: Workers, Medicine and Social Sciences.

During the Great Patriotic War the University staff and students fought the enemy at the fronts and in partisan detachments. Twelve BSU students and teachers were awarded the title of the Hero of the Soviet Union.

On May 15, 1943 the Council of Peoples Commissars of the USSR adopted a resolution on the resumption of work of the Belarusian State University. The new location of the BSU was the Skhodnya station of the Oktyabrskaya Railway near Moscow. The main academic building was housed in the local school.

After the end of military operations, the Belarusian State University relocated back to the capital of the republic, Minsk. On October 16, 1944 lectures resumed. In December, the new University structure was approved: administration, academic, personnel and accounting departments, library, offic of the registrar, dormitory, special department, deans' offices In addition to the History, Chemistry, Biology and Geography faculties working in evacuation, the BSU opened Part-Time Faculty and the Faculty in Journalism.

After Belarus gained independence in 1991, the Belarusian State University was officiall recognized as the leading higher education institution in the new country. With the government's support, since the late 1990s the BSU has strengthened its resource base, renovated its fixed assets, restructured its educational and research system, and created a developed infrastructure.

The University gave birth to a number of independent specialized institutions in Belarus: Belarusian State Medical University, Belarusian State Economic University, Belarusian National Technical University, among others.

Today, the BSU maintains its position as a leading higher education institution in Belarus, which honours its foundations and traditions.

During its existence, the BSU has trained several generations of worthy professionals for Belarus and other countries and continues this mission up today by strengthening its prestige and fostering its achievements year by year.

The Belarusian State University is the Belarus' flagship higher education institution and the largest research and industrial conglomerate encompassing:

- 16 Faculties, 11 Educational Institutes and Educational institutions;
- 19 academic buildings;
- 42 research laboratories at the faculties;
- 6 research institutes;
- 6 innovative manufacturing enterprises;
- 3 training and experimental stations;
- 6 museums;
- 1 Research and Technology Park;
- 10 dormitories.

The BSU comprises 166 buildings and 9 facilities with a total area of more than 420,000 sq. m.

The Belarusian State University is the umbrella organization implementing state-funded research and state-funded scientific and technical programs. A number of enterprises have been created on the University premises.

The BSU comprises numerous academic, research, industrial, social and cultural, administrative, ancillary units, including:

- Administration;
- BSU Council;
- Faculties and educational institutions;
- Research institutions;
- Directorates, divisions, centers, services, office
- University wide departments;
- BSU enterprises;
- Councils, public associations;
- Museums.

The Belarusian State University has equipped its academic buildings with state of art facilities — 70 computer laboratories and 4 media rooms are at students' disposal. The university owns a modern sporting facility located in the center of the capital. It comprises a swimming pool, a game room, wrestling and weight rooms, a physical therapy room, a gym, a skiing center, a massage room, a sauna, and a stadium.

One of the strategic priorities for the BSU is to participate in international rankings. This allows the University to become a full- edged member of the international educational community, while bringing academic and scientifi research processes into accordance with international rules and standards in the field of education

The Belarusian State University is the national leader in education, training elite professionals. The BSU is a brand and a beacon of quality in teaching and research. According to the most infl ential international ranking agencies, the Belarusian State University ranks among the top 1.5 % universities wordwide and among the best 1 % universities in Eastern Europe and Central Asia.

The University's academics are justly considered one of the best in our country. Multimedia tools along with distance technologies eUniversity, Moodle and eLearning are actively used in the training process. Students have access to a unique collection of books at the Fundamental library, numerous research laboratories, scientific and educational centers. and creates conditions for developing their start-ups. Each year the BSU carries out about 50 international scientific and engineering projects within Horizon 2020, CERN, UN, 7th EU Framework Program, TEMPUS, Erasmus +, MOST Program, among others.

The BSU is introducing University 3.0 concept and encouraging the development of research and industry clusters in order to combine education, science and industry. The University actively and fruitfully cooperates with Belarusian real sector partners. Many BSU departments work with Great Stone industrial park, Belarus High-Tech Park, leading industrial enterprises, financial institutions, and research centers of our country.

The BSU has extensive relations with its partners abroad — about 450 cooperation agreements with universities, research and commercial organizations all over the world. This promotes academic mobility for BSU students and teachers: students study abroad and develop unique professional competencies, and BSU researchers obtain scientific results recognized by the world scientific community. In addition, innovative research and development centers and institutes are being set up together with foreign partners, among them is the joint Belarus-China Institute of Dalian Polytechnic University and Belarusian State University that is successfully functioning.

Annually highly skilled graduates obtain BSU diplomas. Among them are outstanding scientists who have gained worldwide fame, public gures, politicians, teachers, writers, inventors and athletes. The University scientists' developments are widely implemented in medicine, nuclear energy, space, aviation, IT and many other industries.

The BSU trains both Belarusian and international students. As early as 1961, the University opened a Special Faculty that trained foreign students, one of the firs in the USSR and the first in Belarus. Throughout its history, over 10,000 foreign graduates have left the Belarusian State University. Many of them have become scientists, businesspersons, entrepreneurs, educators, and outstanding artists.

BSU students are actively involved in civic and patriotic events, volunteer movement; they are always in the thick of events of national importance and thus they contribute to the sustainable development of Belarusian society. The University is home to diverse on-stage performance groups, societies, public associations, student governance bodies, and sports clubs. BSU Student Union, Belarusian Republican Youth Union, BSU Creative Union, trade union divisions of the university, BSU Student Assembly and others help students to fulfil their potential. Non-resident and foreign students are provided with housing in one of the on-campus dormitories, where the best possible conditions for living and study are maintained.

https://bsu.by/en/ustav-i-gimn-bgu.php

Task 8. ***Answer the following questions.***

1. What faculties were the initial ones of the BSU?
2. What happened to the University during the Great Patriotic War?

3. How can the BSU educational process be characterized?
4. What institutions did the BSU give birth to?
5. What scientific, research and industrial subdivisions are ther at the BSU?
6. What facilities does the Belarusian State University own?
7. Why are the University's academics considered one of the best in our country?
8. How many foreign graduates have left the BSU?
9. What events are BSU students involved in?

Task 9. ***Say what the following numbers refer to.***

1921	*1961*	*1991*	*420,000*
10,000	*70*	*1943*	

Task 10. ***Make a short summary of the text (about 10—12 sentences).***

Task 11. ***Support the idea that the BSU retains a special place not only in Belarus but on a global scale. Use the following hints.***

1. The Belarusian State University gave birth to … .
2. The BSU maintains its position … .
3. One of the strategic priorities for the BSU is … .
4. The University's academics are considered … .
5. The BSU has extensive relations with … .
6. Graduates have a possibility … .
7. Scientific and research activity of the BSU .
8. Students may … .

Task 12. ***You've read about the BSU — its past and present. In pairs, make predictions for next 10, 20, 30, 50 years about possible changes that the university may experience.***

Task 13. ***Get ready to take part in the presentation contest. Prepare a presentation about the greatest achievements of the BSU.***

THE FACULTY OF MECHANICS AND MATHEMATICS

Task 1. ***Work in pairs. Look at the following quotations about mathematics and say whether you agree with them or not. Give arguments.***

1. "Mathematics has a threefold purpose. It must provide an instrument for the study of nature. But this is not all: it has a philosophical purpose and, I daresay, an aesthetic purpose." *Henri Poincaré*

2. “One reason why mathematics enjoys special esteem, above all other sciences, is that its laws are absolutely certain and indisputable, while those of other sciences are to some extent debatable and in constant danger of being overthrown by newly discovered facts.” *Albert Einstein*
3. “Either mathematics is too big for the human mind, or the human mind is more than a machine.” *Kurt Gödel*
4. “In mathematics the art of proposing a question must be held of higher value than solving it.” *Georg Cantor*
5. “In order to understand the universe, you must know the language in which it is written. And that language is mathematics.” *Galileo*
6. “Mathematics is the queen of the sciences and number theory is the queen of mathematics.” *Carl Friedrich Gauss*
7. “Mathematics is like oxygen. If it is there, you do not notice it. If it would not be there, you realize that you cannot do without.” *Lex Schrijver*

Task 2. ***In pairs, discuss the following questions.***

1. Why have you decided to study mathematics?
2. Why have you chosen the faculty of Mechanics and Mathematics of the BSU?
3. What do you know about the faculty of Mechanics and Mathematics?

Task 3. A. ***Pair the verbs in column A with suitable words or phrases in column B.***

A	*B*
1. To study	a) its long-standing traditions and foundations
2. To be well known	b) further qualification
3. To be	c) for internship
4. To confir	d) for the State Prize winners
5. To train	e) with modern technological facilities
6. To occupy	f) in postgraduate studies
7. To use	g) at the faculty
8. To be equipped	h) their knowledge
9. To put	i) at the front of research
10. To participate	j) future experts
11. To preserve	k) one’s idea into practice
12. To be sent	l) leading positions
13. To apply	m) the title
14. To enroll	n) in various scientific project
15. To gain	o) modern educational technologie

B. ***Work in pairs. Discuss who or what might carry out these actions.***

Task 4. A. ***Match the words from the box with their meanings.***

1. A student who is studying for their first university degree (= ualification)
2. A particular subject of study.
3. A division of a school or college dealing with a particular fie d of knowledge.

4. Skill in, understanding of, or information about something, which a person gets by experience or study.
5. The work that a teacher does in helping students to learn.
6. The systematic investigation into and study of materials and sources in order to establish facts and reach new conclusions.
7. A course of study in which you study at home, receiving, and sending off work by post, email, etc.
8. A particular department at a college or university, or the teachers in that department.

9. The study and knowledge of the structure and behavior of natural things in an organized way.
10. Education at a college or university.

B. ***Complete the sentences using the words from the box above.***

1. Our ... comprises multifaceted staff who have varied experience
2. Students who happen to be some distance away from technical schools can prepare themselves by a ... .
3. He emphasized that all the people taking part in the ... were volunteers.
4. Several of the teachers were ineffectual at maintaining .
5. All ... may be divided into two categories: pure or theoretical knowledge and applied or practical knowledge.
6. The school is experimenting with new ... methods.
7. Sixty ... rated the quality of essays which included both well and poorly written samples.
8. Jane shows a talent for math and ... .
9. Since 1995 the ... has organized teaching of foreign students in English.
10. ... ... institutions need to be approached and the findings of this research discussed.

Task 5. A. ***Look through the following statements about the Faculty of Mechanics and Mathematics and say if you consider them to be True or False.***

1. The faculty of Mechanics and Mathematics has existed for more than 60 years.
2. The faculty of Physics and Mathematics was opened in 1922.
3. In October 1985, the faculty of Physics and Mathematics was divided into two: Physics and Mathematics.
4. The MMF includes 10 departments.
5. The MMF possesses a research laboratory “Applied Mechanics”.
6. The faculty of Mechanics and Mathematics conducts training in ve specialties.
7. The MMF doesn’t maintain scientific contacts with many universities around the world.
8. In addition to numerous annual events, film screenings and board games are held every week by faculty activists.

B. ***Now read the text "The Faculty of Mechanics and Mathematics" and check how many false sentences you have managed to figure out.***

THE FACULTY OF MECHANICS AND MATHEMATICS

The faculty of Mechanics and Mathematics has existed for more than 60 years and has a long and proud history of excellence in teaching and research. The training of mathematicians in Belarus began in 1923 with the opening of the faculty of Physics and Mathematics at the Belarusian State University. In October 1958, the faculty of Physics and Mathematics was divided into two: Physics and Mathematics. In May 1970, on the basis of three departments of the Faculty of Mathematics, the faculty of Applied Mathematics was separated. In 1975, the faculty of Mathematics, in connection with the introduction of the Department of Mechanics, was renamed into the faculty of Mechanics and Mathematics. Now the faculty includes 10 departments:

- Bio- and Nanomechanics Department;
- Web Technologies and Computer Simulation Department;
- Higher Algebra and Information Security Department;
- Geometry, Topology and Mathematics Teaching Methodology Department;
- Mathematical Cybernetics Department;
- Theoretical and Applied Mechanics Department;
- Function Theory Department;
- Functional Analysis and Analytical Economics Department;
- Differential Equations and System Analysis Department
- General Mathematics and Computer Science Department.

The faculty also possesses a center for educational and information technologies, and a research laboratory "Applied Mechanics".

It employs 26 professors and doctors of science, 3 academicians, 65 associate professors and candidates of science. About 1000 full-time students, about 300 part-time students, 69 undergraduates, over 40 post-graduate students and 3 doctoral students study at the faculty.

The main disciplines are: Introduction to Mathematics, Analytical Geometry, Algebra and Number Theory, Elementary Mathematics, Mathematical Analysis, Functional Analysis, Numerical Methods, Programming Methods And Information Science, Computer Mathematics, Discrete Math, Web Programming Differentia Geometry and Topology, Differential Equations, Complex Analysis, Foreign Language, Sociology, Economics, Political Science, Philosophy, Belarusian (Professional Vocabulary), Physics, Theoretical Mechanics, Physical Training.

Over the years, the faculty has always been at the forefront of research in the fiel of mathematics, information technologies and mechanics and it is well known for the State Prize Winners, Honoured scientists and higher education workers.

Annually the faculty of Mechanics and Mathematics con rms the title of one of the most elite faculties of Belarusian universities. Everyday its departments train future experts in mathematics, mechanics, systems analysis and information technologies (about 1,500 undergraduate and Master's course students of full time and correspondence course). Its graduates occupy leading positions in the sphere of science and education, business and management, economics and IT sphere. Among them there are such figures as Michael Zhuravkov (Minister of Education), Sergey Chizhik (First Deputy Chairman of the Presidium of the National Academy of Sciences), Sergey Ablameyko (BSU Rector), Alexander Moshensky (General Director, LLC Santa Impex Brest) and others.

Currently, the Faculty conducts training in four specialties.

1. Mathematics (Research and educational activity).
2. Computer Mathematics and Systems Analysis.
3. Mathematics and Information Technologies (for directions):
   - Web-programming and internet technologies;
   - Mathematical and mobile devices software.
4. Mechanics and mathematical modeling.

The faculty of Mechanics and Mathematics is a full cycle hub of fundamental and applied research, production and promotion of scientific and technological developments. The University uses modern educational technologies and creates all the conditions for teaching and research. The faculty is equipped with modern technological facilities. Multimedia tools along with distance technologies eUniversity, Moodle and eLearning are actively used in the training process. It helps students to put their ideas into practice, provides every opportunity for them to participate in various scientific projects and conferences. The MMF preserves its long-standing traditions and foundations, provides opportunities for comprehensive social, creative and athletic self-fulfilment of students

The MMF maintains scientific contacts with many universities around the world. In summer, the best students of the Faculty are sent for an internship at the Otto-von-Guericke University (Magdeburg, Germany). Also, the graduates have a large field to apply their knowledge. Thanks to close cooperation with leading IT companies, such as EPAM Systems, Itransition, HiQo Solutions, IBA Group, Qulix Systems, etc., the students have an excellent opportunity to get practical and pre-diploma professional practice, work in laboratories of leading Hi-Tech Park resident-companies with further job placement. Students also have an opportunity to learn from case studies of leading national companies (Belaruskali JSC, Altimed medical implants manufacturer, and many others). Graduates who have shown high abilities in research work receive recommendations for admission to the Master`s course, and can also enroll in postgraduate studies, getting the opportunity to gain further qualifications through scientific and research project

The students of the MMF are among the most active in the social and scientific activities of the university. The faculty has the Student Union, the Council of Student Monitors, the Council for the Quality of Education, the Creative Union, the Trade Union and the Belarusian Republican Youth Union. In addition to numerous annual events, film screenings and board games are held every week by faculty activists. Studying at the Faculty of Mechanics and Mathematics is interesting and fun: it is here that the highest quality of education and active student life meet!

https://mmf.bsu.by/en/about-faculty/

Task 6. ***Answer the questions.***

1. When did the training of mathematicians in Belarus begin?
2. When was the faculty of Mathematics renamed into the faculty of Mechanics and Mathematics?
3. How many departments does the faculty include today? What are they?
4. What are the main disciplines at the faculty?
5. In what spheres do graduates of the MMF occupy leading positions?
6. What opportunities do the best students of the faculty have?
7. What do you know about social and cultural life of students?

Task 7. ***Say what the following numbers refer to.***

60	1970	26	1000	40	4
1923	10	65	69	3	
1958	1975	3	300	1,500	

Task 8. ***Make a short summary of the text (about 10—12 sentences).***

Task 9. ***In pairs, discuss the following questions.***

1. How has your life changed since you entered the university?
2. What are some qualities of a good student?
3. What is the biggest difficult   in being a student of the faculty of Mechanics and Mathematics?
4. What extracurricular activities would you like to be involved in?
5. If you became the dean of your faculty, what changes would you make? Why?
6. What subjects do you think are the most important to study at the faculty of Mechanics and Mathematics?
7. Where can the graduates of the faculty of Mechanics and Mathematics work?
8. Which class or subject is most important for your future job?

Task 10. ***You are going to present the MMF to future applicants. In pairs, shoot a video "5 reasons to choose the faculty of Mechanics and Mathematics of the BSU".***

EXPLORING THE TOPIC

MATHEMATICS TODAY

The book of nature is written
in the language of mathematics.

Galileo Galilei

Task 1. ***Have a look at the title of the text. What do you think you can learn from it? What do you know about Mathematics? What would you like to know? Fill in columns 1 and 2 in the grid below. Then read the part of the text assigned to you and fill in columns 3 and 4.***

I know ... (What do you already know about mathematics?)	*I want to know ...* (Write down any new questions you have.)	*I have learned ...* (State 3 main ideas you have learned from the text.)	*I know ... because ...* (Record 1 supporting fact or detail for each of your main ideas in the previous column.)

MATHEMATICS TODAY

1. ____

Mathematics is the science that deals with the study of elemental practices of counting and measuring structure, order and relation with logical reasoning and quantitative calculation. (1) ...

(2) ... Engineers, chemists, business professionals, and educators all use mathematics to calculate measurements, determine business profits, and build critical thinking skills in young learners. Since so many professions use math, a degree in the field prepares graduates for many different care paths.

Math majors both in our country and abroad study statistics, applied mathematics, and theoretical mathematics, preparing graduates for roles like mathematician, statistician, analyst, actuary, and math teacher. (3) ... Earning a bachelor's degree in mathematics qualifies a candidate for entry-level roles in most fields, making mathematics one of the most versatile majors. Math majors can use their analytical and critical thinking skills to solve problems in business, engineering, technology, government, and healthcare. A degree in mathematics could be more rewarding for you as it not only develops your logical thinking, problem-solving and decision-making skills but also make you one of the most valued employees across many industries. With so many career options for math majors, many degree-holders fin the pathway worthwhile.

(4) ... Almost all areas of human activity make more and more use of mathematics. They use all branches of mathematics, not just traditional applied mathematics. Mathematical activity like research, applications, education, exposition, has changed a lot in the last some years. Some of these changes, like the use of computers, are very perceptible and they are being applied in mathematical education fairly broadly. Many new forms of mathematical activity like algorithms and programming, modeling, conjecturing, expository writing and lecturing, are acquisition significance

Currently, eff rts are being undertaken worldwide to facilitate collaborative research across traditional academic fields and to help train a new generation of interdisciplinary mathematicians and scientists. (5) ... For example, curricula for the social sciences programs now include sophisticated mathematics over and above the traditional descriptive statistics. Curricula of some universities in the developed countries have inter-disciplinary programs where mathematics students and students from other sciences work jointly on projects. The aim is to prepare graduates for the new approaches and practices in their fields nd careers.

As evidenced by the discoveries of the last half of the 21st century, mathematics can enrich not only physics and the other discipline of sciences, but also medicine and the biomedical sciences and engineering. It can also play a role in such practical matters as how to speed the flow of traffi on the Internet or sharpen the transmission of digitized images, how to better understand and possibly predict patterns in the stock market and even how to enrich the entertainment world through contributions to digital technology. (6) ... Mathematics has to do with human genes, the world of finance and geometric motions. For example, science now has a huge body of genetic information, and researchers need mathematical methods and algorithms to search the data as well as clustering methods and computer models to interpret the data. Finance is very mathematical; it has to do with derivatives, risk management, portfolio management and stock options. All these are modeled mathematically, and consequently mathematicians are having a real impact on how those businesses are evolving. Motion driven by the geometry of interfaces is omnipresent in many areas of science from growing crystals for manufacturing semiconductors to tracking tumors in biomedical images.

2. ____________________

The convergence of mathematics and the life sciences, which was not foreseen a generation ago, is a remarkable opportunity for application. (7) ... All these, have posed a big challenge on the mathematics curricula at all levels of the education systems, teacher preparation and pedagogy. The 21st century mathematics thinking is to further strengthen efforts to bridge the division lines within mathematics, to open up more for other disciplines and to foster the line of inter-discipline research.

A modern mathematician is a person who knows how to solve complex problems related to processing large volumes of difficu and diverse information. (8) ... If you are determined to do Mathematics, get deep knowledge, develop logical thinking and use it all through complicated tasks, there are several universities in Belarus to choose from, that will lead you through the world of mathematics.

Mechanics and Mathematics Faculty, Belarusian State University

(9) ...: Mathematics (Research and educational activity); Computer Mathematics and Systems Analysis; Mathematics and Information Technologies (for directions):

Web-programming and internet technologies and Mathematical and mobile devices software; Mechanics and mathematical modeling.

Faculty of Applied Mathematics and Informatics, Belarusian State University

This faculty is also popular in the Belarusian State University. It trains IT-specialists who are thoroughly versed in science and can apply their knowledge in practice. Applicants choose from six popular specialties: Applied Mathematics, Informatics, Actuarial Mathematics, Economic Cybernetics, Applied Informatics, Computer Security.

Faculty of Physics and Mathematics, Belarusian State Pedagogical University of Maxim Tank

If you want to explain mathematics to others, pay attention to the profession of a teacher. In a pedagogical university, you will become a teacher of mathematics and computer science.

Faculty of Physics and Mathematics, Brest State University of A. S. Pushkin

The university trains teachers of mathematics and computer science, mathematicians-programmers, mathematicians-economists, and only in the full-time department.

Faculty of Mathematics and Information Technology, Vitebsk State University of P. M. Masherov

Vitebsk University at this faculty teaches five full-time specialties, and this faculty is suitable for those who want to work in IT in the future and have a good understanding of science.

Faculty of Mathematics and Programming Technologies, Gomel State University named after Francysk Skaryna

Teachers, mathematicians-programmers and mathematicians-economists are trained here.

Faculty of Mathematics and Informatics, Yanka Kupala`s Grodno State University

Grodno University trains programmers, mathematicians-programmers, teachers, as well as managers-economists of information systems.

Faculty of Mathematics and Natural Science, Mogilev State University of A. A. Kuleshov

(10) .... There is also an opportunity to enter the "Information Technology Software" specialization.

https://link.springer.com/article/10.1007/s10649-021-10043-2

Task 2. A. ***Make a summary of your part of the text and present to your partner.***

B. ***Listen to your partner's summary and fill in columns 3 and 4 of the chart in Task 1.***

Task 3. ***Give the headings to the part of the text you've read.***

A. Opportunities for Mathematics in the Life Sciences.
B. The Versatility of a Mathematics Degree.

Task 4. ***Read the text once again and fill in the gaps with sentences from the list below. There are 2 extra sentences.***

1. Through mathematical modelling, numerical experiments, analytical studies and other mathematical techniques, mathematics can make huge contributions to many fields
2. The university trains teachers of mathematics
3. He is able to correctly and systematically organize the process of thinking, to see the possible ways of problem solution.
4. Math is present in day to day life and is being used even when people don't realize they are using mathematical reasoning.
5. Mathematics has paved its way in having a role in everything we do including architecture, sports, money, art, mobile devices and so on.
6. Disciplines that hardly used mathematics in their curricula are now demanding substantial doses of knowledge and skills in mathematics.
7. Students can get here not only a good education, but also to find future employment.
8. The full-time department of the MMF of BSU presents a large selection of specialties related to mathematics.
9. Every branch of mathematics has a potential for applicability in other fields of mathematics and other disciplines.
10. Graduates also work as data scientists, actuaries, risk managers, accountants.
11. Many disciplines draw on mathematics.
12. A student is studying mathematics as his principal field

Task 5. ***Discuss the questions below with your partner.***

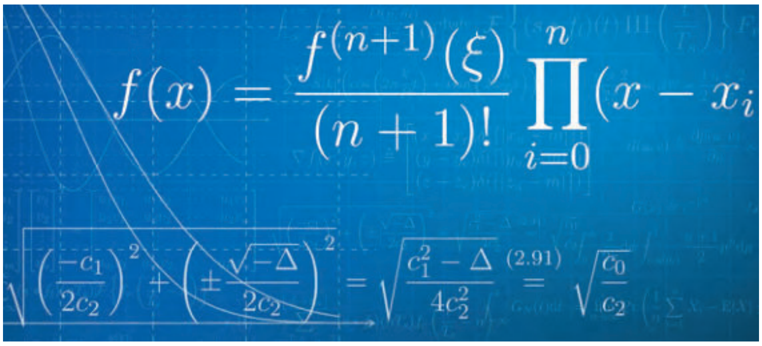
1. What are some common misconceptions about mathematics?
2. How important do you think mathematics is in today's world?
3. Why do you think math is considered an important subject in universities?
4. How has the study of math evolved over time, and how does it continue to impact our daily lives?
5. Do you think that having strong math skills can make someone more competitive in the job market?
6. How do you think universities can encourage more students to pursue math-related majors and careers?

Task 6. A. ***Discuss with your partner if the statements below are True or False.***

1. Humans have always known about numbers and counting.
2. There is a mathematical reason why we use a ten digit number system.
3. Binary is a number system that uses three digits.
4. Arithmetic would be much easier if we used a twelve digit number system.
5. Logic is not an important foundation of mathematics.
6. If x squared is even, then x is even.
7. Euclid's algorithm calculates the smallest common divisor of two numbers.

8. Cryptography is a recent development that only dates back to the invention of computers.
9. The NSA is the largest employer of mathematicians in the world.
10. The discovery of Pi was made easily and quickly.

B. ***Watch the video about "History of mathematics" to check your ideas.***



https://drive.google.com/file d/18oeVDocbDeahBqCrn5myDDUY _p9JWk1E/view?usp=sharing

Task 7. ***Arrange the sentences below in the order they appear in the video. Watch the video again if needed.***

1. It can apply to public key cryptography and it has various applications in physics.
2. The set is countably infinite
3. The fact that you can swap two numbers and get the same result means the group is commutative.
4. Rational numbers are either decimals that end or decimals that go on forever.
5. In 1736 Leonard Euler published a paper on the seven bridges of Königsberg, which is regard as the first paper in graph theory
6. This was the beginning of chaos theory a field of math dealing with dynamic systems that are very sensitive to initial conditions.
7. In the 17th century calculus was introduced to the world.
8. Physical signal itself can be quite complicated and not tell us much by using Fourier analysis.
9. The mathematician Euclid published a series of 13 books known as "Elements".
10. A graph is made up of nodes and edges that connect them.

Task 8. ***Answer the questions.***

1. What is the age-old question about numbers?
   A. Did we invent numbers or are they already there?
   B. How many digits should a number system have?
   C. Why do we use a ten digit number system?
   D. How do we use binary numbers?
2. Why do we use a ten digit number system?
   A. Because it is the most efficient syste
   B. Because it is the easiest system to learn.
   C. Because it is based on the number of fingers we have
   D. Because it is the oldest number system.

3. What is binary?
   A. A number system with nine digits.
   B. A number system with ten digits.
   C. A number system with two digits.
   D. A number system with twelve digits.
4. Why do mathematicians say that arithmetic would be much easier if we used 12 digits instead of 10?
   A. Because 12 is divisible by more numbers than 10.
   B. Because 12 is a more aesthetically pleasing number.
   C. Because 12 is easier to count on your fingers
   D. Because 12 is a lucky number.
5. What is the oldest foundation of mathematics?
   A. Algebra.
   B. Geometry.
   C. Logic.
   D. Calculus.
6. What is the Euclidean algorithm?
   A. A series of steps that solves a problem.
   B. A mathematical theorem.
   C. A type of encryption.
   D. A method for breaking codes.
7. What is cryptography?
   A. The study of integers.
   B. The study of prime factors.
   C. Techniques that ensure secure communication.
   D. Techniques for breaking codes.
8. What is the largest employer of mathematicians in the United States?
   A. The National Security Agency
   B. The Federal Bureau of Investigation
   C. The Central Intelligence Agency
   D. The Department of Defense
9. What is the purpose of encryption?
   A. To make information easier to understand.
   B. To protect sensitive information.
   C. To make information more accessible.
   D. To prevent people from communicating.
10. What was the purpose of encrypting written letters hundreds of years ago?
   A. To protect sensitive information.
   B. To make information easier to understand.
   C. To prevent people from communicating.
   D. To make information more accessible.

Task 9. ***Work in pairs. Take turns presenting the information of the video, answering the questions below.***

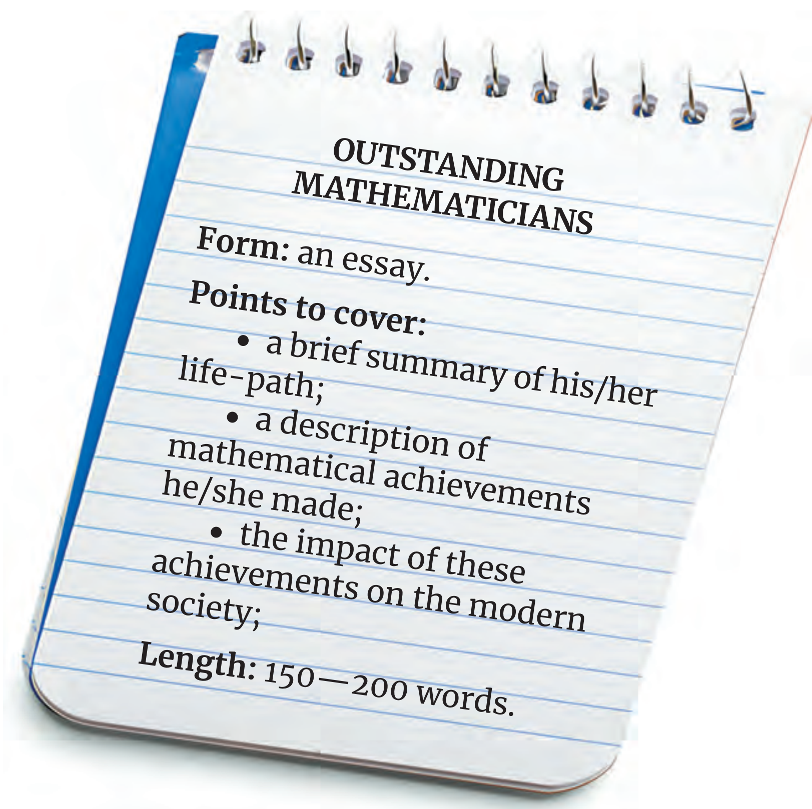
Student A

1. What does the beginning of mathematics start with?
2. What foundations of Mathematics is one of the oldest?
3. What is cryptography?
4. In what spheres is calculus used? Can you add some more to the list?
5. To what fields does topology apply

Student B

1. When did Joseph Fourier publish a paper on heat flow
2. When was Boolean algebra discovered?
3. What is set theory concerned with?
4. What does Markov chain include?
5. When did game theory become known?
6. Who proved Fermat's theorem in 1994?

Task 10. ***Write an essay about one of the mathematicians mentioned in the video. Consider the requirements listed below.***



PROBLEM-SOLVING

MATHEMATICS EDUCATION ACROSS THE WORLD

Task 1. *Read the fact file below. Do you agree with the information?*

WHERE IN THE WORLD SHOULD YOU GO TO STUDY MATH?

While some may groan at the thought of ever taking another math class in their life, others are enthralled with the prospect of working out that impossible equation or proof. For the latter half, focusing on mathematics as an undergraduate, graduate, or post-graduate may sound like a dream come true.

There are many Universities to choose from around the world that offer fantastic math departments. English for publication purposes. Earning a mathematics degree from the best mathematics programs in the world helps graduates stand out on the job market. At the same time, there are several that rise above the rest, hitting the top of several ranking assessments.

Task 2. ***What educational problems do you encounter? Make a list of problems and support them with particular examples of your learning experience.***

Task 3. ***Compare your list with your partner's. Do you have a lot in common?***

Task 4. ***Discuss possible reasons for the problems on your list.***

Task 5. ***In groups, think about a possible educational system that will help students overcome the problems on your list. Make a model of higher education, dwelling on the points below.***

EDUCATIONAL SYSTEM

1. Courses to be taken.
2. Methods of instruction implemented in the educational process.
3. Students' roles (passive/active learners, partners, creators, etc.).
4. Educational modes (in class/online/hybrid).
5. Other characteristics of your educational system.

Task 6. ***Hold a panel discussion justifying your educational models. Are the educational systems presented by all the groups similar? What are the differences?***

Task 7. ***Are the models of education you've created realistic? Compare them to the real ones.***

1. Choose one of the universities below and analyze its educational process.
2. Fill in line 1 in the chart below with the information you can find on the officia university websites.
3. Make a presentation (a video clip, a poster) and get ready to share the information with your groupmates.
4. Hold a conference exchanging the results of your research with your groupmates.

University/ Country	Specialties/ Subjects	Model of Instruction	Exams and Grades System	Students' Tips
1.				
2.				

Task 8. ***Fill in all the other lines in the chart with the information you get from your groupmates, presenting educational process in other universities.***

Task 9. ***Compare the educational system you created with those in the chart. What are the similarities/differences?***

Task 10. ***Which educational system, presented in the chart do you find effective? Why?***

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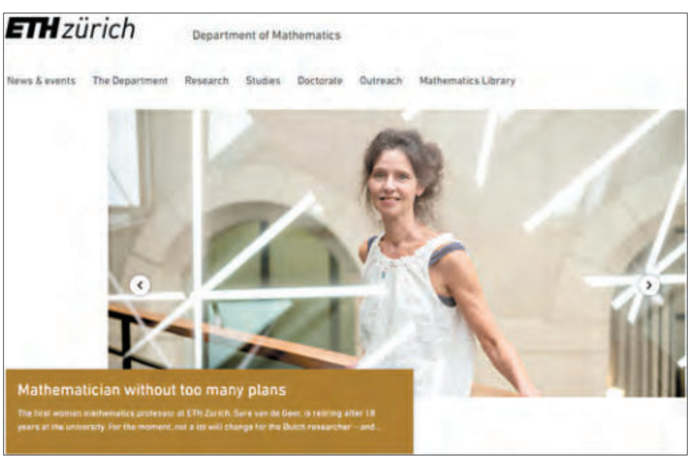
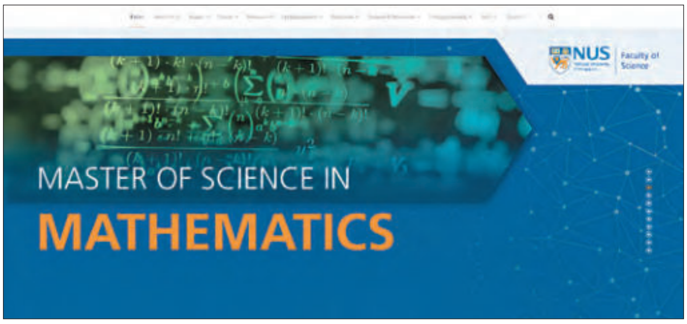


https://math.ethz.ch

NATIONAL UNIVERSITY OF SINGAPORE



https://www.math.nus.edu.sg

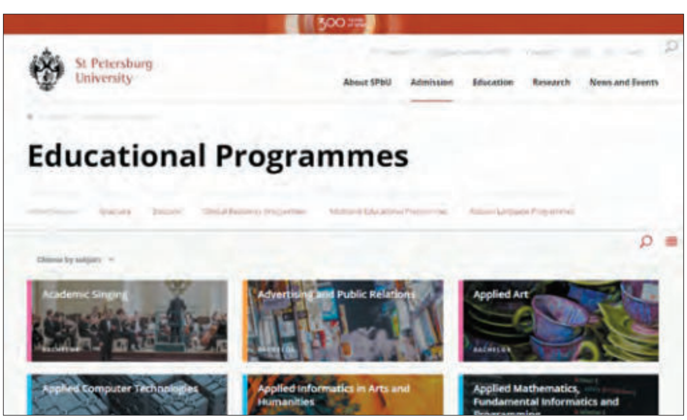
KYOTO UNIVERSITY

https://www.sci.kyoto-u.ac.jp/en/education/undergraduate

TECHNICAL UNIVERSITIES OF MUNICH

https://english.spbu.ru/admission/programms/undergraduate

ST PETERSBURG UNIVERSITY



https://www.cit.tum.de/en/cit/home/

ARITHMETIC 2 UNIT

In this unit you'll

examine	numbers, their types and properties
look closely at	rational and decimal numbers
focus on	arithmetic operations and their order
study	the history of numerals and numeral systems
write an essay about	the role of rational numbers in solving real-world problems
make a presentation about	types of number systems
solve the problem	Order of operations: What advantages or disadvantages are there?

ACTIVE VOCABULARY

VERBS

to add
to capture
to come up with
to consider
to defin
to denote
to determine
to multiply
to occur
to represent
to simplify
to subtract

NOUNS

addition
brackets
decimal
denominator
digit
equation
fraction
integer
numerator
quantity
quotient
ratio
remainder
subtraction
subtrahend
value

ADJECTIVES

associative
commutative
distributive
recurring

COLLOCATIONS

closure property
decimal point
imaginary numbers
irrational numbers
least common denominator
mathematical concept
natural numbers
number line
number system
number theory
quadratic equation
rational numbers
real numbers
repeating decimals
set notation
terminating decimals
the square root of
whole numbers

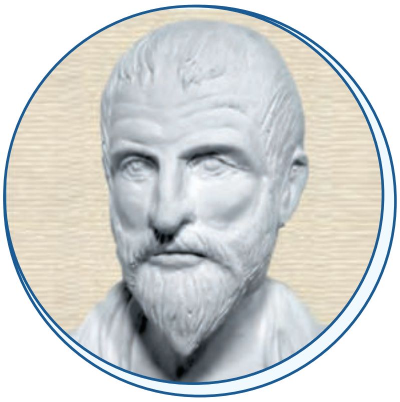
BUILDING THE CONCEPT

WHAT IS A NUMBER?

Task 1. A. ***Comment on the following quotations.***

Perfect numbers like perfect men are very rare.

Rene Descartes



Wherever there is number, there is beauty.

Proclus

Number rules the universe.

Pythagoras

B. ***Have a discussion "Numbers: For & Against".***

1. Split into 2 groups.
2. Group A, make a list of arguments ***For***. Group B, make up a list of arguments ***Against***. Use the arguments below as an example.
3. Have a discussion. Take turns presenting your group's arguments.

FOR

1. Numbers provide a universal way of expressing quantities and measurements.
2. They facilitate mathematical calculations, essential in fields such as science, engineering and finance
3. Numbers help tracking data and analyzing trends, crucial for business decision making.
4. Number fluency is vital for functioning in society, including managing money or scheduling appointment.
5. Understanding numbers can enhance critical thinking skills and cognitive development.

AGAINST

1. Misinterpreting numerical data can lead to wrong conclusions and decisions.
2. Heavy reliance on technology that handles numbers can cause lack of basic calculation skills.
3. Not everyone has equal access to educational opportunities to learn math and number concepts.
4. The prevalence of numbers in modern life can create pressure or anxiety for those who struggle with math.
5. Personal biases may in uence interpretantion and use of numbers, potentially leading to unintended consequences or discrimination.

Task 2. A. ***How well do you know numbers? Do the quiz below.***

1. What is the highest level of the number system?
   A. Natural numbers.
   B. Real numbers.
   C. Rational numbers.
   D. Irrational numbers.

2. Which of the following is NOT a real number?
   A. 0.
   B. −5.
   C. 3.14.
   D. Square root of −1.

3. How do you know if a number is a real number?
   A. if it can be written as a fraction.
   B. if it is a whole number.
   C. if it can be polleted on a number line.
   D. if it is a repeating decimal.

4. What is the symbol used to represent real numbers?
   A. R.
   B. N.
   C. Z.
   D. Q.

5. Which of the following is irrational number?
   A. 0.25.
   B. 1/3.
   C. Square root of 2.
   D. 5.

6. What is the difference between rational and irrational numbers?
   A. rational numbers can be written as a fraction, while irrational numbers cannot.
   B. irrational numbers can be written as a fraction, while rational number cannot.
   C. rational numbers are positive, while irrational are negative.
   D. irrational numbers are whole numbers, while rational numbers are decimals.

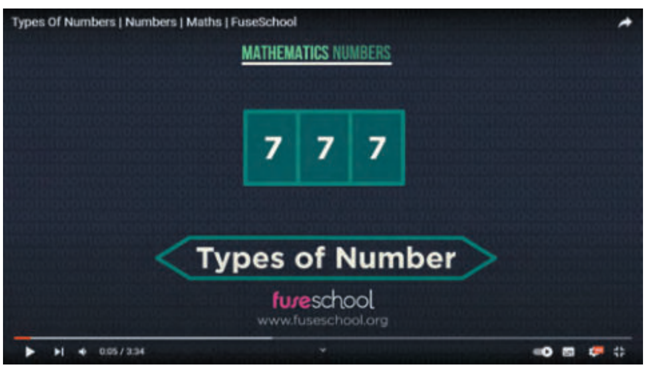
7. What is the difference between natural numbers and integers?
   A. natural numbers include zero, while integers do not.
   B. integers include zero, while natural numbers do not.
   C. natural numbers are negative, while integers are positive.
   D. integers are whole numbers, while natural numbers are decimals.

8. What is the difference between whole numbers and natural numbers?
   A. whole numbers include zero, while natural numbers do not.
   B. recurring decimals have repeating digits, while terminating decimals do not.
   C. terminating decimals are negative, while recurring decimasls are positive.
   D. recurring decimals are whole numbers, while terminating decimals are decimals.

9. How can you remember the order of natural numbers, whole numbers, integers and rational numbers?
   A. now what is real.
   B. natural, fractional, integers, rational.
   C. real, irrational, rational, integers.
   D. whole, natural, integers, rational.

10. What is the difference between terminating and recurring decimals?
   A. terminating decimals have repeating digits, while recurring decimals do not.
   B. recurring decimals have repeating digits, while terminating decimals do not.
   C. terminating decimals are negative, while recurring decimasls are positive.
   D. recurring decimals are whole numbers, while terminating decimals are decimals.

B. ***Watch the video to check your ideas.***



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Task 3. A. ***Match the words (1—10) with the options (a—j) to make mathematical collocations.***

1. Mathematical	a) numbers
2. Positive	b) theory
3. To represent	c) concept
4. Distributive	d) equation
5. Number	e) integers
6. To perform	f) quantity
7. Number	g) decimals
8. Non-repeating	h) property
9. Natural	i) line
10. Quadratic	j) calculation

B. ***Fill in the gaps with the appropriate word collocations from the task above.***

1. The ... ..., x, y, and z, are consecutive terms of an arithmetic progression.
2. When you multiply these two binomials, you're really just doing the ... twice.
3. The theoretical importance of equations with integral coefficient is quite great as they are closely connected with many problems of ... .
4. Infinity is a ..., with no physical reality
5. Irrationals are represented by infinite, .
6. The ninth, final axiom is a second order statement of the principle of mathematical induction over the ... .
7. There are three basic shapes of a ...: linear, concave, and convex.
8. Elementary arithmetic also includes fractions and negative numbers, which can be represented on a ... .
9. One was a dream about a machine that would ... .
10. Numbers were invented by people to ... .

Task 4. ***Match the words with their meanings.***

1. Theory	a) a quality that something has.
2. Operation	b) the process of removing one number from another.
3. Fraction	c) a mathematical process, such as addition, in which one set of numbers is produced from another.
4. Property	d) a mathematical statement in which you show that two amounts are equal using mathematical symbols.
5. Equation	e) a number that results from dividing one whole number by another.
6. Factor	f) a number expressed using a system of counting based on the number ten.
7. Decimal	g) the number below the line in a fraction.
8. The square root of	h) of a particular number, a number that is squared to produce that particular number.
9. Denominator	i) a formal statement of the rules on which a subject of study is based or of ideas that are suggested to explain a fact or event.
10. Subtraction	j) any whole number that is produced when you divide a larger number by another whole number.

Task 5. ***Below you can see pictures of types of numbers. Can you recollect their names and say what distinguishes one type from another?***

1, 2, 3, 4, 5, 6, 7

0, 1, 2, 3, 4

$\sqrt{-2}$, $\sqrt{-5}$, ...

..., −4, −3, −2, −1, 0, 1, 2, 3, 4, ...

$\sqrt{2}$, $\sqrt{3}$, $\sqrt{.434343}$, π...

−2/5, 0.54, 1/5, 13/4, ...

Task 6. ***Read the text and match numbers with their types.***

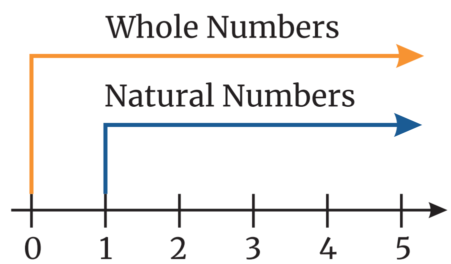
NUMBERS

If we think about it for a moment, numbers are everywhere in our daily lives. They help us to think logically and to keep track of the things we do. For example, numbers help us with simple tasks like calculating the time that it takes you to get from home to the university, the amount of money that you need to pay for your shopping, and the amount of bags that you need to carry your shopping home. In addition to this, they are also especially useful to solve more complex problems in the world of sciences and engineering, like calculating the amount of fuel that will be needed to get a rocket into space, or the number of lorries that a warehouse needs to transport their customer's orders safely and on time.

Numbers are considered the heart of Mathematics, and rightly so because without numbers Maths would simply not exist.

A number is a mathematical concept that represents quantity, which has many applications such as counting, measuring, labelling, and performing calculations that help us to solve problems, among others.

Numbers can be classified into different groups according to the **types** of numbers that they include:



Natural numbers are also known as counting numbers, because these are the numbers that you first learn how to count with. They include all positive numbers greater than zero. That is 1, 2, 3, 4, 5, 6, and so on. They are represented with the letter N. The set notation for natural numbers is as follows: N = {1, 2, 3, 4, 5, ...}.

Whole numbers are basically the natural numbers plus zero. They do not include negative numbers, fractions or decimals.

They are represented with the letter W, and their set notation is shown below: W = {0, 1, 2, 3, 4, 5, ...}.

Natural numbers have four different **properties**.

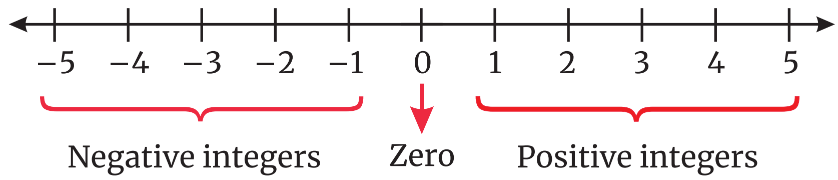
Closure property — this means that when two or more natural numbers are multiplied or added together, it will result in a natural number. For example, $2 + 2 = 4$ or $3 \times 2 = 6$.

Associative property — suggests that when three natural numbers are added or multiplied together, it will result in the same answer no matter how they are grouped. For example, $3 + (2 + 5) = 10$ and $(3 + 2) + 5 = 10$. This also works when they are multiplied, $3 \times (2 \times 5) = 30$ and $(3 \times 2) \times 5 = 30$.

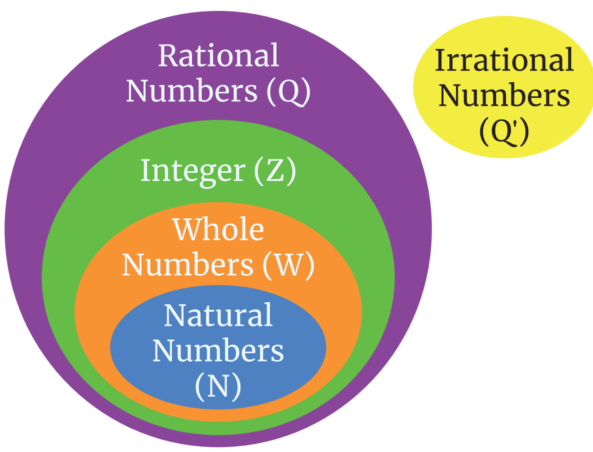
Commutative property — this property says that when two natural numbers are multiplied or added together, they will result in the same answer no matter their order. For example, $4 + 8 = 12$ and $8 + 4 = 12$. This also works when they are multiplied, $4 \times 8 = 32$ and $8 \times 4 = 32$.

Distributive property — when three natural numbers are being multiplied using brackets, you can also do this by multiplying the numbers separately. For example, $5(2 + 3) = 25$ and $5 \times 2 + 5 \times 3 = 25$.

The integer numbers include all positive numbers, zero and negative numbers. Again, integers do not include fractions or decimals. They are represented with the letter Z, and their set notation is as follows: Z = {..., −4, −3, −2, −1, 0, 1, 2, 3, 4, ...}.



Integers can be generated from the set of counting numbers and the subtraction operation. For example, when you subtract a larger natural number from a smaller one, you have a negative number. When a natural number is also subtracted from itself, you have zero.



The result of adding, subtracting, or multiplying integers is always an integer. This cannot be true with dividing integers. Dividing 5 by 2 will give you 2.5, which isn't an integer.

Positive integers are known as natural numbers. An important characteristic of natural numbers can be seen in the equation $a + x = b$. This only has a solution if $b > a$, as a and x can only be positive and their addition will produce a larger number. In the realm of integers, the equation $a + x = b$ will always have an answer.

Rational numbers include all numbers that can be expressed as a fraction in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. This group of numbers includes fractions and decimals. Rational numbers are represented with the letter Q.

All integers, natural and whole numbers are rational numbers, as they can be expressed as a fraction with a denominator of 1.

Irrational numbers are numbers that can't be expressed as a fraction of two integers. Irrational numbers have *non-repeating decimals* that never end, and have no pattern whatsoever. They are represented with the letter Q′.

There are numbers with *non-terminating decimals* that are actually rational. This is the case of numbers with non-terminating decimals that repeat in a pattern, as they can be expressed as a fraction of two integers. For example, $\frac{1}{9} = 0.\overline{1}$, the bar above the decimal 1 means that it repeats forever. So, it is a rational number.

Real numbers include all the numbers that you can think of, which you can find in the real world, apart from imaginary numbers. Real numbers are represented with the letter R, and they include all rational and irrational numbers, therefore the set of real numbers can be represented as $R = Q \cup Q'$.

Apart from real numbers, mathematicians came up with a special type of number to be able to solve the square root of negative numbers, included in simple quadratic equations such as $x^2 + 9 = 0$.

Imaginary numbers are the root of negative numbers.

We know that we can't take the square root of negative numbers, because there is no number that when squared will result in a negative number. In this case, we need to use imaginary numbers. To do this, we say that $\sqrt{-1} = i$.

https://www.hellovaia.com/explanations/math/pure-maths/types-of-numbers/

Task 7. ***In pairs, answer the following questions.***

1. How do numbers help people in daily life?
2. Why are numbers considered the heart of Mathematics?
3. What is a number?
4. What types of numbers are there?
5. What is the difference between natural and whole numbers
6. What properties do natural numbers have?
7. What numbers do integers include?
8. How can rational numbers be expressed?
9. What are real numbers?
10. Why can't we take the square root of negative numbers?

Task 8. ***Discuss with your partner if the statements below are True or False.***

1. A number is a physical concept that represents quantity, which has many applications.
2. Numbers can be classified according to the types of numbers
3. Whole numbers are closely related to natural numbers.
4. Natural numbers have five different propertie
5. Only integer numbers include positive numbers and zero.
6. Rational numbers are integers, natural and whole numbers.
7. Irrational numbers are numbers that can be expressed as a fraction of two integers.
8. Real numbers include all numbers apart from rational numbers.
9. Imaginary numbers are the root of negative.

Task 9. ***Make a short summary of the text (8—10 sentences).***

Task 10. ***Discuss the questions with your partner.***

1. What is the significance of numbers in our daily lives
2. How do you think numbers have evolved over time?
3. Can you represent a quantity without using numbers?
4. Do you think there is a universal language of numbers that everyone can understand? Why or why not?
5. Have you ever encountered any interesting number patterns or sequences? If so, please share.
6. Are there any superstitions or beliefs related to certain numbers in your culture?
7. How has technology changed the way we use and interact with numbers?
8. How can we plot numbers on a line?

9. How can a number line be used to represent different types of n mbers?
10. Why is it important to perform calculations accurately?

Task 11. ***Be ready to deliver a lecture for children about numbers and their types.***

RATIONAL AND DECIMAL NUMBERS

Task 1. A. ***Look through the list of Fun Facts about numbers. Are there any facts you haven't heard before? Do you find any facts surprising? Why/why not?***

1. The number 0 was invented in India around 400 AD.
2. The number system we use today, based on digits 0—9, is called Hindu-Arabic numerals.
3. In Roman numerals, the symbol for 1,000 is M.
4. The number 40 is used frequently in Jewish and Christian religious texts, symbolizing a period of testing or trial.
5. Googol is a number written as a 1 followed by 100 zeros.
6. The Fibonacci sequence, in which each number is the sum of the two preceding number, appears frequently in nature.
7. Infinity is not a number but a concept representing something that is without limit or boundless.
8. The number e, also known as Euler's number, is an irrational number with infinit decimal places.
9. The number system used by computers is binary, which only uses two digits, 0 and 1.
10. The number 666 is known as the "number of the beast", according to the book of Revelation in the Bible.

B. ***Do you know any other Fun Facts about numbers? Discuss them in small groups.***

Task 2. A. ***Match the words (1—10) with the options (a—j) to make mathematical collocations.***

1. To be expressed	a) fractions
2. The set of	b) numbers
3. Common	c) the numerator
4. Decimal	d) rational numbers
5. Recurring	e) as a fraction

6. Irrational
7. Division
8. Multiply
9. Mathematical
10. Convert

f) denominator
g) form
h) value
i) decimal
j) method

B. ***Fill in the gaps with the appropriate word collocations from the task above.***

1. A … … such as 0.111111… is in fact a fairly straightforward number, and is equivalent to the fraction.
2. An irrational number cannot … … .
3. 12 is a … … of 1/3 and 1/4.
4. Indian mathematicians later became the first to use it as a … … in computations.
5. … the mixed … to improper fractions and then perform the multiplication.
6. So, to write it out in its … …, you'd have an on-going series of digits starting with 3.14159 and continuing forever!
7. So … … and the denominator by square root of 2 and you'll get square root of 2 over 2.
8. The … … is said to be closed with respect to the four arithmetic operations.
9. The mathematicians of the nineteenth century realized that the distinction between rational and … … could profitably be refine
10. From this viewpoint, the use of variable … … is one of possible ways of this search.

Task 3. A. ***Discuss with your partner if the statements below are True or False.***

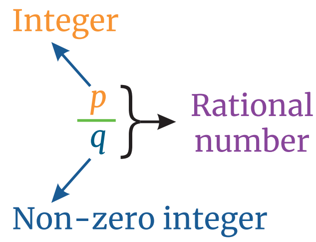
1. If a number can be expressed as a fraction where both the numerator and the denominator are integers, the number is a rational number.
2. Decimals are a set of numbers lying between integers on a number line.
3. The numbers to the right of the decimal point are the integers or whole numbers and the numbers to the left of the decimal point are decimal fractions.
4. There is only one way to read a decimal number.
5. In non-recurring decimal numbers, digits never repeat after a xed interval.
6. To compare the decimal or fractional part of any decimal number, consider one digit at a time from the left of the decimal point.
7. While dividing a number by the long division method, if we get zero as the remainder, the decimal expansion of such a number is called non-terminating.
8. A rational number is terminating if it can be expressed in the form: $p/(2n \times 5m)$.
9. 0 is a irrational number as it can be written as a fraction of integers like 0/1, 0/−2, … etc.
10. Irrational numbers are numbers that can be expressed as fractions of integers.

B. ***Read the text "Rational and Decimal Numbers" to check your ideas.***

RATIONAL AND DECIMAL NUMBERS

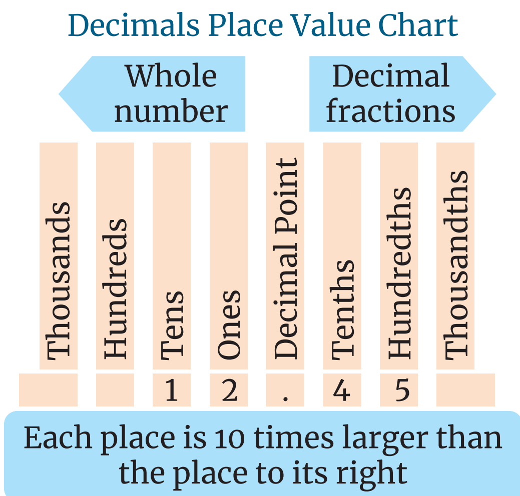
Rational numbers are in the form of p/q, where p and q can be any integer and $q \neq 0$. This means that rational numbers include natural numbers, whole numbers, integers, fractions of integers, and decimals (terminating decimals and recurring decimals).

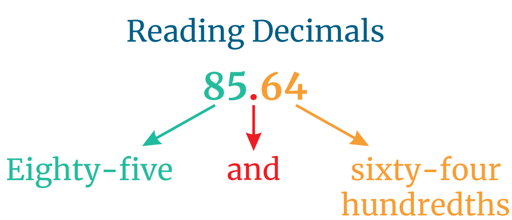
Rational numbers are well related to the concept of fractions which represent ratios. In other words, if a number can be expressed as a fraction where both the numerator and the denominator are integers, the number is a rational number.



The differen **types** of rational numbers are given as follows:

- integers like −2, 0, 3, etc.;
- fractions whose numerators and denominators are integers like 3/7, −6/5, etc.;
- terminating decimals like 0.35, 0.7116, 0.9768, etc.;
- non-terminating decimals with some repeating patterns (after the decimal point) such as 0.333..., 0.141414..., etc.





Decimals are just another way to represent fractions in mathematics. With the help of decimals, we can write more precise values of measurable quantities like length, weight, distance, money, etc. The numbers to the left of the decimal point are the integers or whole numbers and the numbers to the right of the decimal point are decimal fractions. There are *two ways to read* a decimal number. The first way is to simply read the whole number followed by “point”, then to read the digits in the fractional part separately. It is a more casual way to read decimals. For example, we read 85.64 as eighty-five point six-four. The second way is to read the whole number part followed by “and”, then to read the fractional part in the same way as we read whole numbers but followed by the place value of the last digit. For example, we can also read 85.64 as eighty-five and sixty-four hundredths.

Let us have a look at how the decimals are categorized based on their **type** here.

- Terminating decimals mean it does not reoccur and end after a finite number of decimal places. For example: 543.534234, 27.2, etc.
- Non-terminating decimals mean that the decimal numbers have infinit digits after the decimal point. For example, 54543.23774632439473747..., 827.79734394723... etc. The non-terminating decimal numbers can be further divided into 2 parts:
  – In *recurring decimal numbers*, digits repeat after a fixed interval. For example, 94346.374374374..., 573.636363... etc.
  – In *non-recurring decimal numbers*, digits never repeat after a fixed interval. For example, 743.872367346..., 7043927.78687564... and so on.

The decimal representation of a rational number is converting a rational number into a decimal number that has the same mathematical value as the rational number. A rational number can be represented as a decimal number with the help of the long division method. We divide the given rational number in the long division form and the quotient which we get is the decimal representation of the rational number.

The non-terminating but repeating decimal expansion means that although the decimal representation has an infinite number of digits, there is a repetitive pattern to it. The rational number whose denominator is having a factor other than 2 or 5, will not have a terminating decimal number as the result.

For *adding and subtracting* rational numbers, we use the same rules of addition and subtraction of integers. Let us understand this with the help of an example.

Example. Solve 1/2 – (–2/3).

Solution. Let us solve this using the following steps.

Step 1. As we simplify 1/2 – (–2/3), we will follow the rule of addition and subtraction of numbers which says that the subtraction fact can change to an addition fact and the sign of the subtrahend gets reversed. This will make it 1/2 + 2/3.

Step 2. Now, we need to add these fractions 1/2 + 2/3.

Step 3. Using the rules of addition of fractions, we will convert the given fractions to like fractions to get common denominators so that it becomes easier to add them. For this, we need to nd the LCM (least common denominator) of the denominators 2 and 3 which is 6. Then we will convert the fractions to their respective equivalent fractions which will make them 3/6 + 4/6. This will give the sum as 7/6 which can be written in the form of a mixed fraction 1.16116.

The multiplication and division of rational numbers can be done in the same way as fractions. To multiply any two rational numbers, we multiply their numerators and their denominators separately and simplify the resultant fraction. Let us understand this with the help of an example.

Example. Multiply 3/5 × –2/7.

Solution. Let us solve this using the following steps.

Step 1. In order to multiply 3/5 × (–2)/7, we will first multiply the numerators and then multiply the denominators.

Step 2. In this case, when we multiply the numerators, it will be 3 × (–2) = –6.

Step 3. When we multiply the denominators, it will be 5 × 7 = 35. Therefore, the product will be –6/35.

When we need to *divide any two fractions*, we multiply the first fraction (which is the dividend) by the reciprocal of the second fraction (which is the divisor). Let us understand this with the help of an example.

Example: Divide 3/5 ÷ 2/7.

Solution: Let us solve this using the following steps.

Step 1. In order to divide 3/5 ÷ 2/7, we will rst write the reciprocal of the second fraction. This will make it 3/5 × 7/2.

Step 2. Now, we will multiply the numerators. This will be 3 × 7 = 21.

Step 3. Then, we will multiply the denominators, it will be 5 × 2 = 10. Therefore, the product will be 21/10 or 21102110.

The numbers which are NOT rational numbers are called irrational numbers. The di erence between rational and irrational numbers can be understood from the following figure and table given below

Rational Numbers	Irrational Numbers
These are numbers that can be expressed as fractions of integers. **Examples:** 1/2, 0.75, –31/5, etc.	These are numbers that cannot be expressed as fractions of integers. **Examples:** $\sqrt{5}$, π, etc.

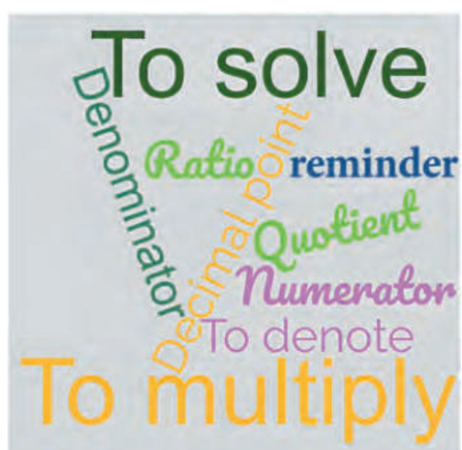
Rational Numbers	Irrational Numbers
They are terminating decimals.	They are NEVER terminating decimals that do not have an accurate value.
They can be non-terminating decimals with repetitive patterns of decimals or recurring decimals. **Example:** 1.414, 414, 414 ... has repeating patterns of decimals where 414 is repeating.	They should be non-terminating decimals with NO repetitive patterns of decimals. **Example:** $\sqrt{5}$ = 2.236067977499789696409173... has no repeating patterns of decimals.
The set of rational numbers contains all-natural numbers, all whole numbers, and all integers.	The set of irrational numbers is a separate set and it does NOT contain any of the other sets of numbers.

https://mathblog.com/reference/arithmetic/decimals/

Task 4. ***In pairs, answer the following questions.***

1. What are rational numbers?
2. What types of rational numbers are there?
3. How can rational numbers be identified
4. What are decimals?
5. What types of decimals are there?
6. What is the decimal representation of a rational number?
7. What rule do we use when subtracting rational numbers?
8. How can the multiplication of rational numbers be done?
9. How can the division of rational numbers be done?
10. What is the difference between rational and irrational numbers

Task 5. ***Match the words from the box with their meanings.***



1. A comparison of two numbers calculated by dividing.
2. The dot used between the whole numbers and the tenths of a decimal.
3. The number below the line in a fraction.
4. To add a number to itself a particular number of times.
5. The result of dividing one number by another.
6. To represent something.
7. The number above the line in a fraction.
8. To find an answer to a problem
9. The amount that is left when one number cannot be exactly divided by another.

Task 6. ***Make a summary of the text (8—10 sentences).***

Task 7. ***Agree or disagree with the statements below. Prove your ideas.***

1. Rational decimal numbers can be expressed as a fraction of two integers.
2. All terminating decimals are rational numbers.
3. The repeating part of a repeating decimal can also be expressed as a fraction.
4. Irrational numbers cannot be expressed as a ratio of two integers.
5. Every rational number has a terminating or repeating decimal representation.
6. Rational numbers can be positive, negative, or zero.
7. Rational numbers form a closed set under addition, subtraction, multiplication, and division operations.
8. The set of rational numbers is denoted by the letter Q.
9. All integers are rational, but not all rational numbers are integers.
10. Any repeating decimal can be simplified into a fraction using algebraic equations.

Task 8. ***Write an essay on the topic "The role of rational numbers in solving real-world problems". Make it about 100—150 words long, use the information from the text above as needed.***

NUMBER SYSTEMS

Task 1. A. ***What number systems do you know? In pairs make a list of the number systems you can think of.***

B. ***Watch the video "A Brief History of Number Systems" to check your ideas.***

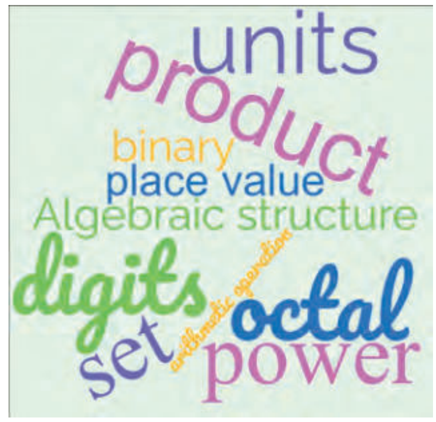
https://drive.google.com/file d/1rpBkAAtmbrYs9TBtYt_ I69lRRQ_edD_8/ view?usp=sharing

Task 2. ***Answer the questions. Watch the video again if needed.***

1. What were some early methods of counting used by humans?
2. How did different civilizations develop ways of recording highe numbers?
3. What is positional notation and how does it work?
4. What is the significance of the number zero in positional notat on?
5. How did the Hindu-Arabic numeral system become the most commonly used number system in the world?
6. Why did the Hindu-Arabic system use base ten?
7. What is a vigesimal system and why did the Aztecs use it?

8. What is a duodecimal system and why do some people think it would be a good idea to use it?
9. How are different number systems used in everyday life
10. Can you think of a different way to represent a large number

Task 3. ***Complete the sentences with the words from the box.***



1. It is also called factorial base, although factorials do not function as base, but as … of digits.
2. 3 to the … … 4 is usually written as 34.
3. We use the same notation for a sequence of … and the real number it represents.
4. The result of any … … is always a number.
5. Real numbers are a kind of field which is an essential … … where arithmetic processes are defined
6. Binary is easily converted to the … numeral system.
7. What would pi be in … numbers?
8. The … of 6 and 3 is 18.
9. Tens go in the left-hand column and … in the right.
10. A … of numbers is a collection or group of numerical values that share a common characteristic or property.

Task 4. ***Read the text "What Is a Number System?" to find out more about number systems.***

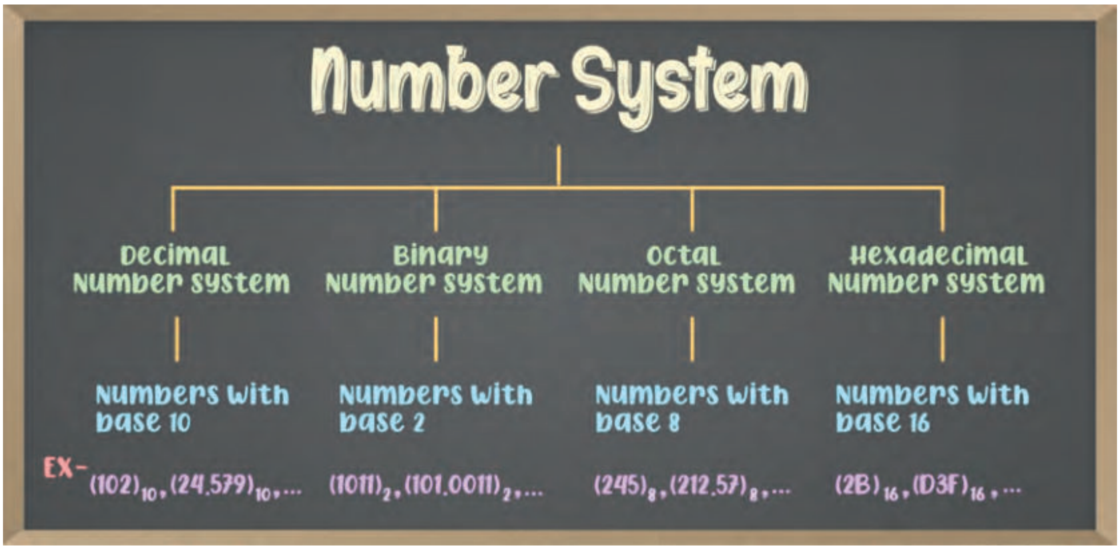
WHAT IS A NUMBER SYSTEM?

A number system is a method of showing numbers by writing, which is a mathematical way of representing the numbers of a given set, by using the numbers or symbols in a mathematical manner. The numeral system represents a useful set of numbers, reflects the arithmetic and algebraic structure of a number, and provides standard representation. The digits from 0 to 9 can be used to form all the numbers. With these digits, anyone can create infinite numbers. For example, 156,3907, 3456, 1298, 784859 etc.

Based on the base value and the number of allowed digits, number systems are of many types. The four common **types** of number system are decimal number system, binary number system, octal number system, hexadecimal number system.

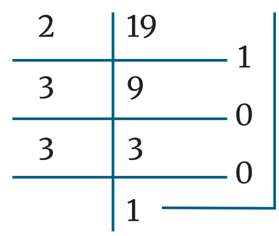
Number system with a base value of 10 is termed a ***Decimal number system***. It uses 10 digits i.e. 0—9 for the creation of numbers. Here, each digit in the number is at a specific place with place value a product of different powers of 10. Here, the place value is termed from right to left as first place value called units, second to the left as tens, so on hundreds, thousands, etc. Here, units have the place value as 100, tens have the place value as 101, hundreds as 102, thousands as 103, and so on.

Number system with base value 2 is termed as ***binary number system.*** It uses 2 digits i.e. 0 and 1 for the creation of numbers. The numbers formed using these two digits are termed binary numbers. The binary number system is very useful in electronic devices and computer systems because it can be easily performed using just two states ON and OFF i.e. 0 and 1.



Decimal numbers 0—9 are represented in binary as: 0, 1, 10, 11, 100, 101, 110, 111, 1000, and 1001. For example, 14 can be written as 1110, 19 can be written as 10011, 50 can be written as 110010.

Logic operations are the backbone of any digital computer, although solving a problem on computer could involve an arithmetic operation too. The introduction of the mathematics of logic by George Boole laid the foundation for the modern digital computer. He reduced the mathematics of logic to a binary notation of '0' and '1'.



Octal number system is one in which the base value is 8. It uses 8 digits i.e. 0—7 for the creation of Octal Numbers. Octal Numbers can be converted to Decimal values by multiplying each digit with the place value and then adding the result. Here the place values are 80, 81, and 82.

Number system with base value 16 is termed as ***hexadecimal number system.*** It uses 16 digits for the creation of its numbers. Digits from 0—9 are taken like the digits in the decimal number system but the digits from 10—15 are represented as A—F i.e. 10 is represented as A, 11 as B, 12 as C, 13 as D, 14 as E, and 15 as F. Hexadecimal Numbers are useful for handling memory address locations.

Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

https://byjus.com/maths/number-system/

Task 5. ***Answer the questions.***

1. What is a number system?
2. What common types of number systems are there?
3. How is number system with a base value of 10 termed?
4. How is number system with a base value of 2 called?
5. What did George Boole do for the modern digital computer?
6. What advantages of binary number systems are there?
7. How is number system with a base value of 8 termed?

8. How can octal numbers be converted to decimal values?
9. How is number system with a base value of 16 termed?
10. How are digits represented in hexadecimal number system?

Task 6. ***Look through the following statements and say if you consider them to be True or False.***

1. The digits from 0 to 9 can be used to form all the numbers.
2. There are three common types of number system.
3. Number system with a base value of 2 is called a decimal number system.
4. Decimal number system uses 10 digits i.e. 0—9 for the creation of numbers.
5. Number system with a base value of 10 is called a decimal number system.
6. The binary number system is very useful in electronic devices and computer systems.
7. Octal numbers can be converted to decimal values by each digit with the place value and then subtracting the result.
8. Number system with base value 8 is termed as hexadecimal number system.
9. Hexadecimal numbers are useless for handling memory address locations.

Task 7. ***Match the words with the definitions.***

1. Number	a) a number or symbol that represents an amount.
2. Symbol	b) the way that someone or something is shown or described.
3. Infinit	c) the science of numbers, forms, amounts, and their relationships.
4. Creation	d) a fact, idea, etc. that provides support for something.
5. Value	e) a unit that forms part of the system of counting and calculating.
6. Device	f) the act of creating something, or the thing that is created.
7. Mathematics	g) a particular way of thinking, especially one that is reasonable and based on good judgment.
8. Foundation	h) a sign, shape, or object that is used to represent something else.
9. Logic	i) without limits; extremely large or great.
10. Representation	j) an object or machine that has been invented for a particular purpose.

Task 8. ***Work in pars. Discuss the questions.***

1. What is a number system?
2. What are common types of number systems?
3. What are the differences between these types
4. What is the difference between a decimal and binary number syst m?
5. How do you convert a decimal number to a binary number?
6. Can you explain what a hexadecimal number system is used for?
7. Why are computers designed to use binary numbers instead of decimal numbers?
8. What is the significance of the base number in a number system
9. How does the octal number system differ from other number syst ms?

10. Can you give an example of a non-positional number system?
11. What is the purpose of using different number systems in mathem tics?
12. Are there any cultures or societies that use unique number systems, and if so, how do they work?

Task 9. ***Make a short summary of the text (about 10—12 sentences).***

Task 10. ***Agree or disagree with some students' opinions on number systems given below. Justify your choice.***

Samantha:

I find number systems fascinating! It's amazing how different cultures use different symbols to represent numbers. For example, in Japan, they use kanji characters to represent numbers rather than Arabic numerals like we do in the Western world. It opens up a whole new world of understanding and cultural appreciation.

Emily:

I have always loved numbers and the logic behind them. It's like a puzzle that I am constantly solving. When I was in high school, I participated in math competitions and even won a few. Understanding different number systems is just another exciting challenge for me.

John:

I don't really know what to think about number systems. On the one hand, I can see the importance of understanding different ways of counting and representing numbers, especially in a globalized world where we interact with people from all over. But on the other hand, I don't see how it's relevant to my life or my work in any practical way.

Task 11. ***Make a presentation to speak about any type of number system.***

EXPLORING THE TOPIC

ARITHMETIC OPERATIONS

Task 1. ***Agree or disagree with some students' opinions on arithmetic operations. Which opinion do you support? Justify your choice.***

Jenny:

I really enjoy division because it's like a puzzle to me. Whenever I'm trying to split up a group of things, I feel like I'm solving a mystery or a riddle. It's also really helpful when I want to divide something equally among a group of people.

Mike:

I absolutely hate multiplication. I can never remember all of the times tables and I always have to use my calculator. It just takes too much brain power. I really prefer addition and subtraction because they're more straightforward.

Amy:

I'm not sure which one I like best. Sometimes I feel like addition is the easiest, but other times I think multiplication is more useful. I guess it just depends on what I'm trying to accomplish. If I'm shopping and trying to determine the total cost of items, addition is best. But if I'm baking and need to double a recipe, multiplication is key.

Tom:

Arithmetic operations used to make me feel stupid. I struggled a lot in math class and felt like everyone else was picking it up so quickly. But now I'm a CPA and I use these operations all the time! It's funny how you can fully understand something later in life that confused you so much when you were younger.

Task 2. ***Have a look at the title of the text. What do you think you can learn from it? What do you know about Arithmetic operations? What would you like to know? Fill in columns 1 and 2 in the grid below. Then read the part of the text assigned to you and fill in columns 3 and 4.***

I know ... (What do you already know about arithmetic operations?)	*I want to know ...* (Write down any new questions you have.)	*I have learned ...* (State 3 main ideas you have learned from the text.)	*I know because ...* (Record 1 supporting fact or detail for each of your main ideas in the previous column.)

ARITHMETIC OPERATIONS

(1) ... It mainly consists of operations such as addition, subtraction, multiplication, and division. In our everyday life, we use arithmetic operations to find out total business income and expenses, to make a monthly or annual budget, to measure lengths, etc.

We cannot solve the problem without using the rules of arithmetic operations. (2) ... There is a particular symbol used for each of the four arithmetic operations which are given in the image below.

Let's discuss the above arithmetic operations in detail.

The addition is a basic mathematical skill of finding or calculating the total of two or more numbers, or we can say in simple words adding things together. It is denoted by the symbol '+'. (3) ... The order of numbers does not matter in the addition.

For example: 367 + 985 = 1352 (367: augend or summand; 985: addend or summand; 1352: sum).

The subtraction arithmetic operation shows the difference between two numbers. It is denoted by the symbol '–'. Subtraction is mostly used to find out what is left when things are taken away or in other words, taking one number away from another number.

For example: 20 – 9 = 11 (20: minuend; 9: subtrahend; 11: difference)

The repeated addition is known as **multiplication**. It is represented by the symbol '×'. Multiplication as an arithmetic operation helps us to find out the total when a number is repeating itself a number of times. For example, 2 times 3 is 6. Mathematically, we can write it as 2 × 3 = 6. (4) ... The product is the term we use for the result of the multiplication of multiplicand and the multiplier.

For example: 20 × 31 = 620 (20: multiplier or factor; 31: multiplicand or factor; 620: product).

The division is an act of dividing something into equal parts or groups. It is one of the four basic arithmetic operations, which gives a fair result of equal sharing. The division is an inverse of multiplication. For example, 2 groups of 3 pencils each make 6 pencils (2 × 3) in multiplication, and in the case of division 6 pencils divided into 2 equal groups give 3 pencils in each group. It is represented by the symbol '÷'. So, here we can write it as 6 ÷ 2 = 3 (6: dividend; 2: divisor; 3: quotient).

1. ____

With whole numbers, we can easily perform the four basic arithmetic operations. (5) ... Such numbers do not have any fractional or decimal parts. The addition of two or more whole numbers always leads to an increase in the final sum. For example, if we add three numbers 4, 5, and 6, we will get 4 + 5 + 6 = 9 + 6 = 15. So, here 15 is greater than all the three addends. The addition of any number with 0 always results in the same number, and if we add 1 to any whole number, we get its consecutive number or successor.

In the case of whole numbers, we always subtract a smaller quantity from a larger quantity to get a difference that is less than the minuend. (6) ... Multiplication of two or more whole numbers can be done by using multiplication tables. The product is always greater than both the numbers except in the case of multiplication with 1 and 0. A number multiplied to 0 always results in 0 and multiplication with 1 gives us the same number as the product.

The division of two whole numbers may or may not result in whole numbers. (7) ... If it is not, then it will result in a decimal number as the quotient.

2. ____________________

A rational expression is an algebraic fraction whose numerator and denominator are both polynomials.

Operations on rational numbers are carried out in the same way as the arithmetic operations like addition, subtraction, multiplication, and division on integers and fractions. Arithmetic operations on rational numbers with the same denominators are easy to calculate but in the case of rational numbers with different denominators, we have to operate after making the denominators the same. (8) ... .

The golden rule for adding and subtracting fractions together is:

(9) ... .

This essentially leads to two types of use cases such as rational expressions with common denominators, rational expressions with different denomi ators.

To add or subtract expressions with like denominators, simply add or subtract (depending on the sign) the numerators keeping the common denominator constant.

If we have to perform addition or subtraction on expressions with differen denominators, we first manipulate the expressions so that they end up having the same denominators.

You can follow the following procedure for adding and subtracting fractions with unlike denominators:

Step 1. Replace the denominator of each term with the Lowest Common Multiple (LCM) of all the denominators.

Step 2. Replace the numerator of each term with

$$\text{Original numerator} \times \frac{\text{Lowest Common Denominator}}{\text{Original Denominator}}.$$

Step 3. Now that all the denominators are the same, add or subtract the numerators together to obtain the resultant numerator over the common denominator.

Step 4. Simplify the expression if necessary.

Since addition and subtraction of polynomials with unlike denominators involves calculating the LCM of the denominators, which are polynomials, it is necessary to be comfortable with calculating the LCM of given polynomials.

(10) ... Before moving on to examples of adding and subtracting polynomials with different denominators, let us first work through an example of evaluating the LCM of 2 polynomials.

3. ____

Closure property states that when any two rational numbers are added, subtracted, multiplied or divided, the result is also a rational number.

Associative property. For adding or multiplying three rational numbers, they can be rearranged internally without any effect on the final answer. This property does not hold true for subtraction and division of rational numbers.

Commutative property states that two rational numbers can be added or multiplied irrespective of their order. This property does not hold true for subtraction and division of rational numbers.

Additive/Multiplicative identity. 0 is the additive identity of any rational number. When we add 0 to any rational number, the resultant is the number itself.

1 is the multiplicative inverse of any rational number. When we multiply 1 to any rational number, the resultant is the number itself.

Distributive property. Two rational numbers combined with the addition or subtraction operator can be multiplied to a third rational number separately by putting the addition or subtraction sign in between.

https://www.cuemath.com/numbers/arithmetic-operations/

Task 3. A. ***Make a summary of your part of the text and present to your partner.***

B. ***Listen to your partner's summary and fill in columns 3 and 4 of the chart in Task 1.***

Task 4. ***Give the headings to the part of the text you have read.***

A. Operations on Rational Numbers.
B. Arithmetic Operations with Whole Numbers.
C. Properties of Operations on Rational Numbers.

Task 5. ***Read the text once again and fill in the gaps with sentences from the list below. There are 2 extra sentences.***

1. Whole numbers are a set of numbers that starts from 0 and go on up to infinity
2. When we add two or more numbers it results in a single term.
3. The arithmetic operations include four basic rules that are addition, subtraction, multiplication, and division.
4. If the quotient is a whole number, it means that the dividend is a multiple of the divisor.
5. If the fractions to be added or subtracted have the same denominators, the corresponding numerators can simply be added or subtracted keeping the denominator constant.
6. Multiplicand and multiplier are the terms used in the multiplication process.
7. Subtraction of 0 from any number always results in the same number, and deducting 1 from a number gives its predecessor.

8. The process of finding the LCM of algebraic polynomials is no different from finding the LCM of a given set of integers
9. Irrational numbers are numbers that can't be expressed as a fraction of two integers.
10. Arithmetic operations are the basics of mathematics.
11. Rational numbers are expressed in the form of fractions, but we do not call them fractions as fractions include only positive numbers, while rational numbers include both positive and negative numbers.
12. In recurring decimal numbers, digits repeat after a fixed inte val.

Task 6. ***Prove that arithmetic operations:***

- are used in daily life;
- are useful in all the other parts of mathematics;
- are very important for the development of mathematical thinking and its successful application in various areas of life.

Task 7. ***Answer the questions below.***

1. What are arithmetic operations?
2. How is addition denoted?
3. What does subtraction show?
4. What is multiplication?
5. What is inverse operation of multiplication?
6. How can we perform arithmetic operations with whole numbers?
7. What is the golden rule for adding and subtracting fractions together?
8. What is the procedure for adding or subtracting fractions with unlike denominators?
9. What are the properties of operations on rational numbers?

Task 8. ***Discuss the following cases in mini-groups.***

1. Some people believe that arithmetic operations are no longer relevant in today's world because of calculators and computers. To what extent do you agree or disagree with this opinion?
2. In many countries, students are taught arithmetic operations at a young age. Is this beneficial or detrimental to their overall education
3. With the rise of technology, some people argue that arithmetic operations should no longer be taught in schools. Do you think this is a positive or negative development?
4. The ability to perform arithmetic operations is considered an essential skill. Do you agree or disagree with this statement?
5. Some people believe that arithmetic operations should be taught more creatively to make them more engaging for students. To what extent do you agree or disagree with this opinion?

Task 9. ***Write a social media post explaining the importance of learning arithmetic operations. Make it about 120—150 words long, using the information from the text "Arithmetic operations".***

THE ORDER OF OPERATIONS

https://www.educator.com/mathematics/basic-math/pyo/order-of-operations.php

Task 1. ***Look at the pictures. What arithmetic operations are shown in them?***

$10 \div 2 = 5$

$5 + 2 = 7$

$5 \times 2 = 10$

$5 - 2 = 3$

Task 2. A. ***Have a look at the title of the video. What do you think you can learn from it? What do you know about order of operations? What would you like to know? Fill in columns 1 and 2 in the grid below.***

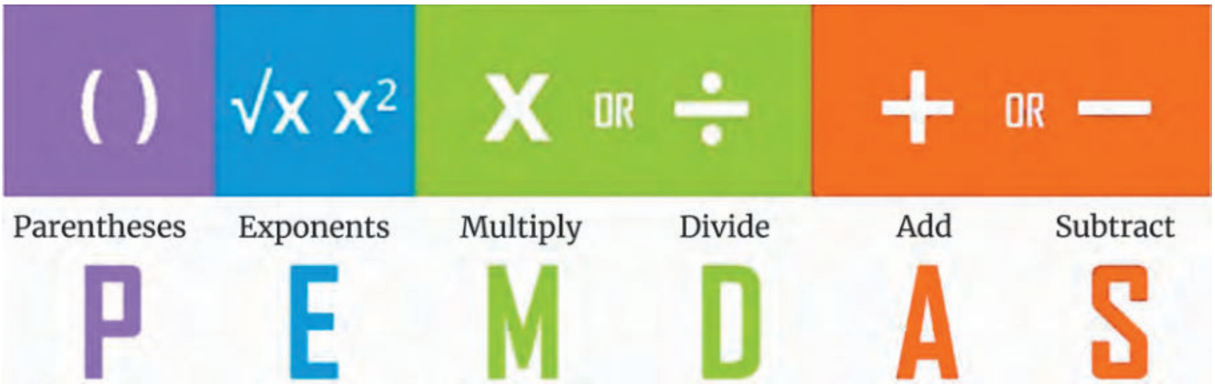
I know ... (What do you already know about order of operations?)	*I want to know ...* (Write down any new questions you have.)	*I have learned ...* (State 3 main ideas you have learned from the video.)	*I know ... because ...* (Record 1 supporting fact or detail for each of your main ideas in the previous column.)

B. ***Fill in columns 3 and 4 after watching the video "Order of operations".***

Task 3. ***Choose the best summary of the video.***

1. The video explains the order of operations in solving mathematical equations. However, the speaker does not emphasize the importance of following the order of operations. The example equation provided is solved incorrectly, with the speaker skipping steps and not following the order of operations. The final answer is incorrect, proving that not following the order of operations can lead to wrong answers. The video is not helpful in teaching the correct way to solve equations.
2. The video explains the order of operations in solving mathematical equations, specifically focusing on equations with exponents. The speaker provides multiple examples of equations with exponents and demonstrates how to solve them step by step, following the order of operations. The speaker also explains the differenc between exponential notation and standard notation.

3. The video explains the order of operations in solving mathematical equations. The speaker emphasizes the importance of following the order of operations, which is PEMDAS (Parentheses, Exponents, Multiplication and Division, and Addition and Subtraction). The speaker provides an example equation and demonstrates how to solve it step by step, following the order of operations. The video finishes up with a quiz to test the viewer's understanding of the order of operations with exponents.



Task 4. A. ***Match the halves of the sentences.***

1. If you have parentheses,
2. PEMDAS
3. Since I have an exponent right here,
4. You are just going to solve out whichever comes firs

a) when it comes to multiplying and dividing.
b) I would have to solve this out before I do anything else.
c) Please Excuse My Dear Aunt Sally.
d) then always solve that first

B. ***Watch the video about "Order of operations" to check your ideas.***

Task 5. ***Arrange the sentences below in the order they appear in the video. Watch the video again if needed.***

1. No matter what, you are always going to solve within the parentheses first
2. There is no order for multiplication and division.
3. A plus in parentheses B minus C plus D squared plus E times F.
4. When we have several different operations we look at within a single problem, there is an order of which ones we have to do first
5. 6 times 5 minus 2 squared divided by 9, as long as you follow the order of operations, your answer will be 6.
6. When it comes to multiplying and dividing, you are just going to multiply or divide across whichever ones come first
7. If there is something you have to subtract before you have to add, then you just go ahead and do that.
8. Exponents come after parentheses; we are going to have to solve this before subtracting.
9. Operations we know are multiplying, dividing, adding, subtracting, things like that.
10. Let's look at this: 2 plus 3 times 5.

Task 6. ***In pairs, restore the contents of the text, answering the questions below in turns.***

Student A

1. What are arithmetic operations?
2. What is the first step in solving this problem
3. What operation comes after solving the parentheses?
4. Is it true that multiplications should be performed before divisions? Why or why not?
5. What phrase helps us to remember the order of operations?

Student B

1. What is the order of operations?
2. What do parentheses tell us?
3. What is the next step after solving the exponent?
4. What is the final answer to the C problem
5. What is the importance of following the order of operations?

Task 7. ***Speak about order of operations, solving the following example.***

$$a + (b - c) + d^2 + e(f).$$

Task 8. A. ***Mark the statements below as advantages or disadvantages of the order of operations.***

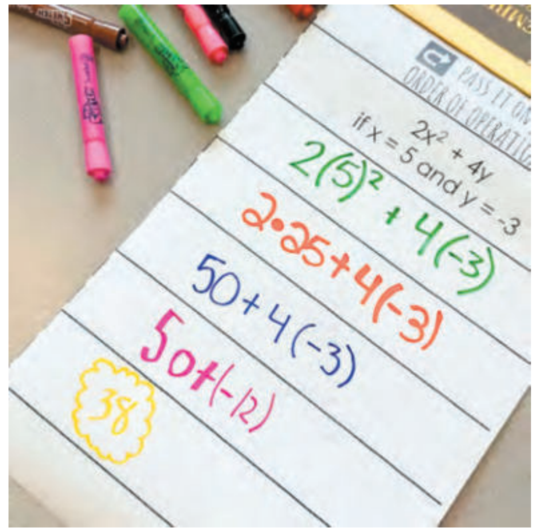
Advantages or Disadvantages

1. Helps to solve complex mathematical equations in a standardized efficien way.
2. Lack of understanding or incorrect application of the order of operations can lead to incorrect solutions.
3. Allows for clear communication of steps in problem-solving with other individuals.
4. Promotes critical thinking, logical reasoning, and analytical skills.
5. Depending too much on the memorization process instead of truly understanding how to use the order of operations could hinder learning capability as new concepts arise.
6. Individuals may get bored or uninterested in the subject matter if they do not fully understand why it is important or lack of motivation.
7. The hierarchy of math operations may break down when attempting more complex computations with different types of equations involved
8. Prevents ambiguity and confusion when multiple operations are performed together.
9. Enables systematic calculations that lead to accurate results.
10. Complex problems may require one to perform several substeps within an operation before moving on to completing another operation which could make solving challenging.

B. ***Compare your ideas in mini-groups. Brainstorm some more advantages and disadvantages of using the order of operations.***

Task 9. ***Comment on the quotations.***

1. "The order of operations is not a choice, it's a rule. It's not open to interpretation." *Danica McKellar*
2. "The order of operations is a convention, not a law of nature." *Steven Strogatz*
3. "The order of operations is a vital tool for solving mathematical problems, but it's important to remember that it's just a convention." *Keith Devlin*



Task 10. ***Get ready to deliver a lecture on the order of operations (about 150 words).***

PROBLEM-SOLVING

THE HISTORY OF NUMERALS AND NUMERAL SYSTEMS

Task 1. ***Read the introduction to the problem-solving and answer the questions in bold.***

Number systems have progressed from the use of fingers and tally marks, perhaps more than 40,000 years ago, to the use of sets of glyphs able to represent any conceivable number efficientl

The earliest known unambiguous notations for numbers emerged in Mesopotamia about 5000 or 6000 years ago.

But how did numeral systems evolve?
Who came up with the idea of number systems and for what reasons?
DIG DEEPER TO FIND THE ANSWERS TO THE QUESTIONS ABOVE!

Task 2. ***Read one of the texts below and fill in the table below.***

Type of numeral system	Peculiarities of development	Representation of numbers

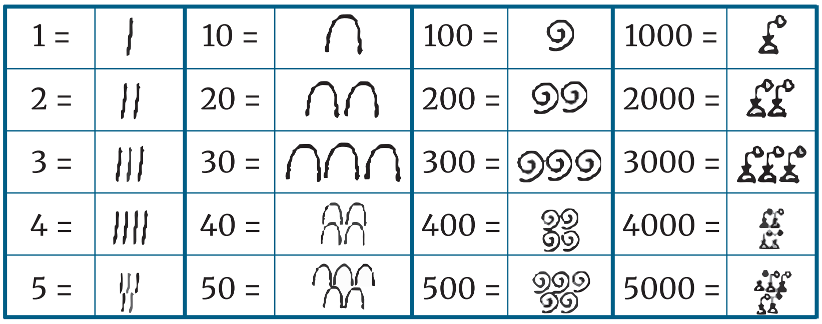
Task 3. ***Share the information with your groupmates.***

Task 4. ***Fill in the table above with the information your groupmates shared with you.***

Task 5. ***Study the table thoroughly and write an essay "Evolution of numeral systems".***

ANCIENT EGYPTIAN NUMBERS & NUMERAL SYSTEM

Likewise, Egyptians did not follow a numeral system like ours where there are 9 digits from 0 to 9 and bigger numbers are formed with these numbers. They had the unary system of Egyptian numbers, common among ancient civilizations.

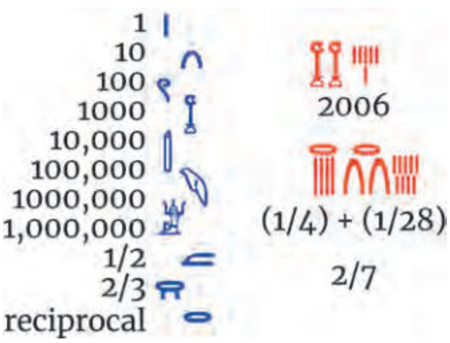


In this system, a simple line meant one; two lines meant two, three lines three and so on. When it reached 10, a new symbol like an inverted U was used. A coil of rope was the symbol for a hundred.

The symbol for a thousand is the lotus or water lily. It shows the leaf, stem and rhizome or root. A single large human finger symbolised the number ten thousand. The number one lakh was represented by a tadpole nearly turning to a frog.

God Heh or a man with both hands raised was the symbol for a million, sometimes used for infinity also. A symbol resembling a circle was again used to represent infinity because a journey through a circle never ends

Multiples of these values were expressed by repeating the symbol as many times as needed. As they did not have a concept of zero, there was no symbol for it.



The system of Ancient Egyptian numerals was used in Ancient Egypt until the early firs millennium AD. Egyptian Numbers and calculations were important to the Egyptians especially in the construction of Pyramids and monuments.

Egyptians knew addition, subtraction, some division and multiplication. They only multiplied and divided by two, so if they wanted to find ex 5, they would use ex 2 + ex 2 + e. 13 / 4 was done as $4 \times 2 + 4 = 12$, $13 - 12 = 1$, and so the answer was 3 1/4.

Being only able to multiply and divide by two, Egyptian math was unwieldy. To get whole numbers like 32, the Egyptians would have to write: 10 + 10 + 10 + 1 + 1. Although simple, the way the Egyptians wrote their maths made it long and repetitive.

The Egyptian sign 'gs' was used for the word 'side' or 'half' 1/2. The word 'hsb' meant 'fraction'. and represented plus and minus signs respectively. The Egyptians were somewhat familiar with both roots and square roots.

The Great Pyramid of Khufu from the Fourth Dynasty was a mathematical wonder because it was laid out with geometric precision. The Egyptians also made the 365-day calendar, which again proves their excellence in mathematical skills.

https://ancientegyptianfacts.com/ ancient-egyptian-numbers.html

GREEK NUMERALS

Greek numerals are a system of representing numbers using letters of the Greek alphabet. They are also known by the names Milesian numerals, Alexandrian numerals, or alphabetic numerals. In modern Greece, they are still in use for ordinal numbers, and in much of the same way that Roman numerals are in the West; for ordinary (cardinal) numbers, Arabic numerals are used.

At first, before it was used more, the Greek alphabet, Linear A and Linear B had used a different system with symbols for 1, 10, 100, 1000 and 10 000 operating with the following formula: | = 1, – = 10, ∘ = 100, ¤ = 1000, ☼ = 10 000.

The earliest alphabet-related system of numerals used with the Greek letters was a set of the acrophonic Attic numerals, operating much like Roman numerals (which derived from this scheme), with the following formula: Ι = 1, Γ = 5, Δ = 10, ΓΔ = 50, Η = 100, ΓΗ = 500, Χ = 1000, ΓΧ = 5000, Μ = 10 000 and ΓΜ = 50 000.

The acrophonic system was replaced by a new alphabetic system, sometimes called the Ionic numeral system, from the 4th century BC. Each unit (1, 2, ..., 9) was assigned a separate letter, each tens (10, 20, ..., 90) a separate letter, and each hundreds (100, 200, ..., 900) a separate letter. This requires 27 letters, so the 24-letter Greek alphabet was extended by using three obsolete letters: Ϝ (fau) for 6, ϟ (koppa) for 90, and ϡ (sampi) for 900.

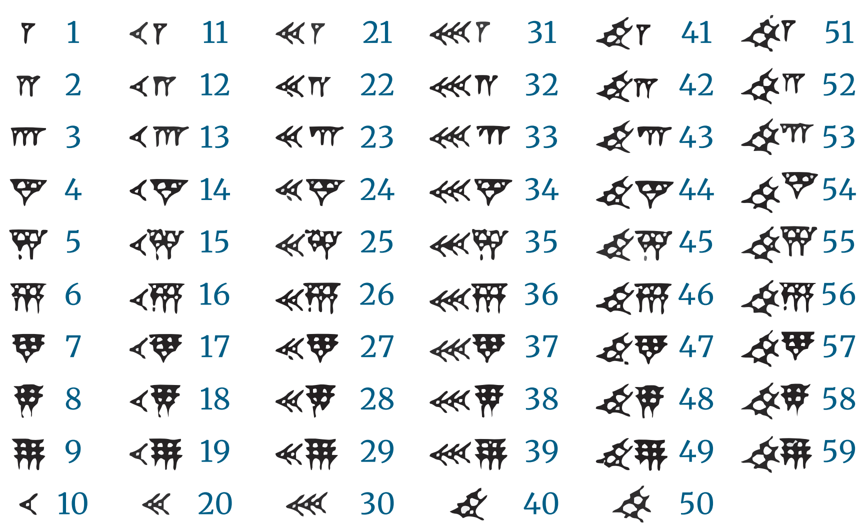
This alphabetic system operates on the additive principle in which the numeric values of the letters are added together to form the total. For example, 241 is represented as ΣΜΑʹ (200 + 40 + 1).

To represent numbers from 1,000 to 999,999the same letters are reused to serve as thousands, tens of thousands, and hundreds of thousands.

https://simple.wikipedia.su/wiki/Greek_numerals

THE BABYLONIAN NUMERALS

The Babylonians developed a system for writing down numbers, using symbols for singles, tens, and hundreds, showing that they probably used a decimal system for everyday life. This system allowed them to handle large numbers comfortably and perform all of the major arithmetical functions. However, there is no evidence that they used a number for zero, and they did not use fractions.



However, the Sumerians also used a base 60 system of counting, the reason why we still divide a circle into 360 degrees and count hours, minutes, and seconds. This sexagesimal system was used for weights and measures, astronomy, and for the development of mathematical functions. For example, one tablet lists the squares of all of the numbers up to 60^2, and sexagesimal numbering is used for the numbers greater than 60 — 64 is written as 60 + 4, 81 as 60 + 21...

This idea of using position to arrange integers, known as the principle of position, is the first known use of such a system, the basis of our decim l system.

This base 60 system, also allowed the Babylonians to use fractions, and they expressed a half as '30' (30 sixtieths) and a quarter as '15' (15 sixtieths). This system found its way into Greece and became the preferred way to express fractions until many centuries later, when the decimal system became the preferred language for mathematicians.

The accepted reason for the use of a sexagesimal system is that it was based in astronomy and the desire of the Babylonians to develop accurate calendars to chart the turning of the seasons and predict the best times for planting, extremely importantly in a culture with a strong agricultural base. Initially, the Babylonians believed that there were 360 days in a year, and this formed the basis of their numerical system; they divided this into degrees and this represented the daily movement of the sun around the sky. They then transferred this into measuring circles by dividing degrees into minutes. Our entire system of astronomy, geometry, and dividing the day into hours, minutes and seconds hails from this period of history.

In geometry, besides the development of degrees, the Babylonians contributed little, tending to use rough approximations, and there is little evidence that they used geometrical techniques for raising their buildings, preferring trial and error. Of course, so little is known about this sophisticated culture that evidence may yet turn up revealing more about their mathematical techniques.

https://explorable.com/babylonian-mathematics

ROMAN NUMERALS

Roman numerals are a numeral system that originated in ancient Rome and remained the usual way of writing numbers throughout Europe well into the Late Middle Ages. Numbers are written with combinations of letters from the Latin alphabet, each letter with a xed integer value. Modern style uses only these seven:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

The use of Roman numerals continued long after the decline of the Roman Empire. From the 14th century on, Roman numerals began to be replaced by Arabic numerals; however, this process was gradual, and the use of Roman numerals persists in some applications to this day.

One place they are often seen is on clock faces. For instance, on the clock of Big Ben (designed in 1852), the hours from 1 to 12 are written as:

I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII

The notations IV and IX can be read as "one less than five" (4) and "one less than ten" (9), although there is a tradition favouring representation of "4" as "IIII" on Roman numeral clocks.

Other common uses include year numbers on monuments and buildings and copyright dates on the title screens of movies and television programs. MCM, signifying "a thousand, and a hundred less than another thousand", means 1900, so 1912 is written MCMXII. For the years of the current (21st) century, MM indicates 2000. The current year is MMXXIII (2023).

Roman numerals use different symbols for each power of ten and no zero symbol, in contrast with the place value notation of Arabic numerals (in which place-keeping zeros enable the same digit to represent different powers of te).

This allows some flexibility in notation, and there has never been an officia or universally accepted standard for Roman numerals. Usage varied greatly in ancient Rome and became thoroughly chaotic in medieval times. Even the post-renaissance restoration of a largely "classical" notation has failed to produce total consistency: variant forms are even defended by some modern writers as offering improved "flexibility"

The following table displays how Roman numerals are usually written.

Individual decimal places

	Thousands	Hundreds	Tens	Units
1	M	C	X	I
2	MM	CC	XX	II
3	MMM	CCC	XXX	III
4		CD	XL	IV
5		D	L	V
6		DC	LX	VI
7		DCC	LXX	VII
8		DCCC	LXXX	VIII
9		CM	XC	IX

https://en.wikipedia.org/wiki/Roman_numerals

HINDU-ARABIC NUMERAL SYSTEM

The Hindu-Arabic numeral system or Hindu numeral system is a positional decimal numeral system developed between the 1st and 4th centuries by Indian mathematicians.

The system is based upon ten (originally nine) different glyphs. The symbols (glyphs) used to represent the system are in principle independent of the system itself. The glyphs in actual use are descended from Indian Brahmi numerals, and have split into various typographical variants since the Middle Ages.

The Hindu numeral system is designed for positional notation in a decimal system. In a more developed form, positional notation also uses a decimal marker (at first a mark over the ones digit but now more usually a decimal point or a decimal comma which separates the ones place from the tenths place), and also a symbol for “these digits recur ad in nitum.” In modern usage, this latter symbol is usually a vinculum (a horizontal line placed over the repeating digits). In this more developed form, the numeral system can symbolize any rational number using only 13 symbols (the ten digits, decimal marker, vinculum, and an optional prepended dash to indicate a negative number).

Despite the numeral system being described as the “Hindu-Arabic numeral system”, the system had been developed by Indian mathematicians and in use extensively throughout India, before being adopted by Persian mathematicians in India and passed on to the Arabs further west. The numeral system was transmitted to Europe in the Middle Ages. The use of Arabic numerals spread around the world through European trade, books and colonialism. Today they are the most common symbolic representation of numbers in the world.

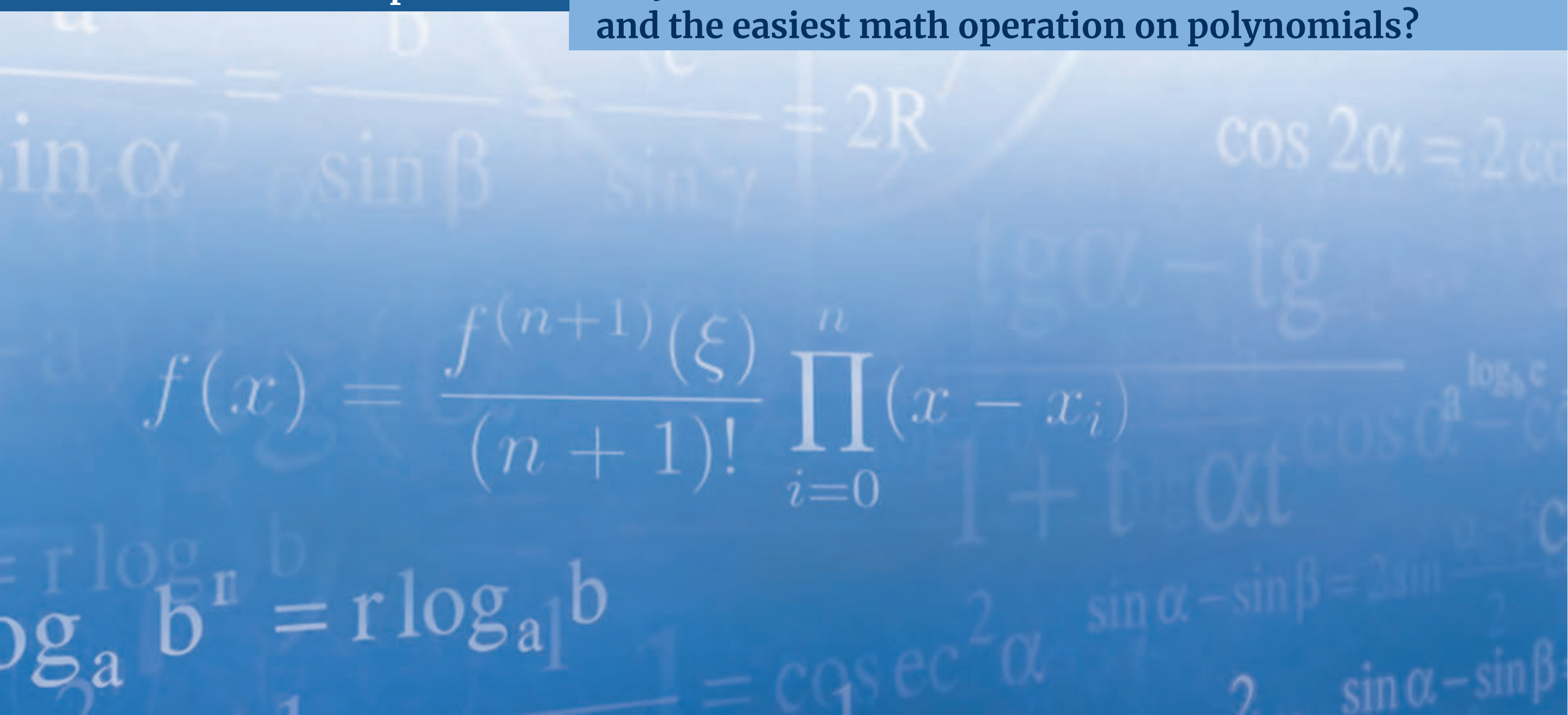
Although generally found in text written with the Arabic abjad (“alphabet”), numbers written with these numerals also place the most-significant digit to the left, so they read from left to right. The requisite changes in reading direction are found in text that mixes left-to-right writing systems with right-to-left systems.

https://mathgeek.fandom.com/wiki/
Hindu-Arabic_numeral_system

ALGEBRA 3 UNIT

In this unit you'll

- **examine** branches of algebra numbers
- **look closely at** mathematical language
- **focus on** polynomials
- **study** algebraic laws
- **write an essay about** the beauty of Euler's identity
- **make a presentation about** algebraic operations
- **solve the problem** Polynomials: What is the most difficult and the easiest math operation on polynomials?



ACTIVE VOCABULARY

ADJECTIVES

numerical
precise
pure
specifi
transformational

NOUNS

addition
associativity
closure
commutativity
constant
distributivity
division
egree
exponent
expression
fiel
lattice
mapping
matrice
multiplication
plane
polynomial
quantity
ring
sentence
subset
subtraction
sum
variable

VERBS

to attach
to communicate
to indicate
to merge
to reduce
to remain
to represent
to require
to yield

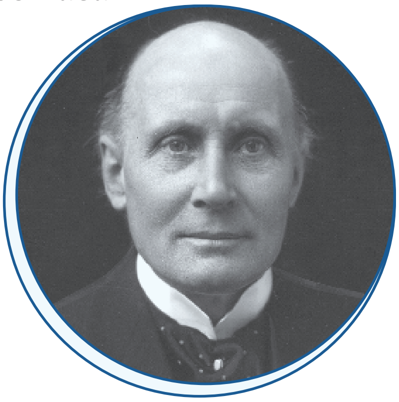
COLLOCATIONS

algebraic term
coordinate geometry
differential topolog
fundamental concept
generic topology
invariant theory
linear equation
quadratic equation
unknown values
vector spaces

BUILDING THE CONCEPT

ALGEBRA

Task 1. ***Do you agree with the following quotation? Prove your idea.***



> Algebra is the intellectual instrument which has been created for rendering clear the quantitative aspects of the world.
>
> *Alfred North Whitehead*

Task 2. A. ***Do you know any interesting facts about algebra? In pairs, make a list of facts.***

B. ***Compare your list with the one below. Are there any facts you haven't heard before? Do you find any of them surprising/unrealistic?***

1. The word «algebra» originally comes from Arabic and means «the reunion of broken parts.»
2. The ancient Babylonians were the first to use algebraic methods over 4,000 years ago.
3. Algebraic equations were first solved using words rather than s mbols.
4. Italian mathematician Girolamo Cardano introduced complex numbers in algebra in the 16th century.
5. The distributive property, $(a + b) \times c = a \times c + b \times c$, is a fundamental rule in algebra.
6. Algebraic expressions can be used for solving real-life problems, such as calculating distances or populations.
7. French mathematician René Descartes developed the concept of coordinate geometry in algebra.
8. Matrices and determinants are important concepts in linear algebra.
9. Algebra is widely used in computer programming, cryptography, physics, and engineering.

Task 3. A. ***Match the words (1—10) with the options (a—j) to make mathematical collocations.***

1. Unknown	a) ring
2. Mathematical	b) computations
3. Elementary	c) values

4. Quadratic	d) geometry
5. Numerical	e) expressions
6. Vector	f) algebra
7. Coordinate	g) spaces
8. Linear	h) equation
9. Polynomial	i) mappings

B. ***Fill in the gaps with the appropriate word collocations from the task above.***

1. The principal means of study in analytic geometry are the method of coordinates and the methods of ... .
2. The specific properties of ... ... are studied in linear algebra
3. It uses its own programming language designed for symbolic Mathematics as well as arbitrary-precision ... .
4. ... ... is a ring formed from the set of polynomials in one or more variables with coefficients in another ring, often a fie
5. He could solve any ... ... by his general rule (finding positive ro ts).
6. One of the most important concepts in the theory of vector spaces is that of a ... .
7. Group theory is one of the more ... .
8. It is common practice to omit multiplication signs in ... ... generalizing this result, Hilbert's Nullstellensatz provides a fundamental correspondence between ideals of polynomial rings and algebraic sets.
9. This is achieved by using other related known values that are located in sequence with the ... .

Task 4. A. ***What is Algebra? In pairs think of the difference between Arithmetic and Algebra.***

B. ***Watch the video "What is Algebra?" to check your ideas.***

https://drive.google.com/file/d 1wwbmXnn9DTDvXP6VV5bnk6 2ELXOBdigH/view?usp=sharing

Task 5. ***Answer the questions. Watch the video again if needed.***

1. What rules does algebra use?
2. What operations does algebra use? What element does algebra introduce?
3. What is an algebraic equation?
4. What is one of the main goals of algebra?

5. What rules of using symbols in equations are there?
6. What is the same unknown value?
7. Why is multiplication called a default operation in algebra?
8. What classes of algebraic equations are there?
9. What is graphing an equation?
10. In what spheres is algebra used?

Task 6. ***Read the text to find out more about Algebra.***

ALGEBRA

Being able to understand algebra not only helps you to represent algebraic expressions and find their solutions. It also allows you to improve your problem-solving skills, helping you to think critically and logically, identify patterns, and solve more complex problems involving numbers and unknown values.

Nowadays algebra is defined as the branch of mathematics that represents problems as mathematical expressions, using letters or variables (i.e. x, y or z) to represent unknown values that can change. The purpose of algebra is to fi d out what the unknown values are, to find a solution to a problem

Algebra combines numbers and variables using mathematical operations like addition, subtraction, multiplication and division to represent a speci c problem. The solutions to the problems are found by using predefined rules to manipulate each mathematical expression. The fundamentals of algebra are built around symbols that are used to represent the problems that require mathematical principles to solve, and the relationship between them. Letters such as a, b, c, x, y, and z are the most common variables to come across, that usually represent quantities. Lowercase Greek letters such as α, β, γ, θ are habitually used to represent angles and planes.

Algebraic expressions are expressions containing variables and constants. An algebraic expression should not be confused with an algebraic equation. When two algebraic expressions are merged together using an "equal to" sign then they form an algebraic equation. Thus, $5x + 1$ is an expression while $5x + 1 = 0$ will be an equation.

Algebra can be classi ed into di erent sub-branches according to the level of complexity of their algebraic expressions and where they are applied. These branches range from elementary algebra to more abstract and complex equations, which require more advanced mathematics. Elementary algebra deals with solving algebraic expressions to find a solution, and it is used in most fields like science, medicine, economics and engineering.

The various branches of algebra based on the use and complexity of the expressions are as such:

Pre algebra includes the fundamental concepts of arithmetic and algebra, such as the order of operations, basic operations with numbers, and simplifying expressions. Algebra assists in turning day-to-day problems into mathematical expressions that use algebraic techniques and algebraic expressions. Pre algebra specifically involves creating an algebraic expression for the provided problem statement.

The goal of **elementary algebra** is to find a solution by resolving algebraic expressions. Simple variables like x and y are expressed as equations in elementary algebra. The equations are divided into polynomials, quadratic equations, or linear equations depending on the degree of the variable. The formulas for linear equations are $ax + b = c$, $ax + by + c = 0$, and $ax + by + cz + d = 0$. Based on the number of variables, quadratic equations, and polynomials are subsets of elementary algebra. For a polynomial problem, the typical form of representation is $ax^n + bx^{n-1} + cx^{n-2} + \ldots + k = 0$, while for a quadratic equation, it is $ax^2 + bx + c = 0$.

Abstract algebra is a branch of mathematics that focuses on algebraic systems like groups, rings, fields, and modules, rather than on specific numerical computations. In abstract algebra, we do not study specific operations like addition and multiplication but instead study general properties of basic operations, such as associativity, commutativity, distributivity, and the existence of inverses. Groups, sets, modules, rings, lattices, vector spaces, and other algebraic structures are studied in abstract algebra.

Universal algebra can be used to explain all other mathematical forms using algebraic expressions in coordinate geometry, calculus, and trigonometry. In each of these areas, universal algebra focuses on equations rather than algebraic models. You can think of all other types of algebra as being a subset of universal algebra.

Linear algebra is a branch of algebra, nds uses in both pure and practical mathematics. It deals with the linear mappings of the vector spaces. It also involves learning about lines and planes. It is the study of linear systems of equations with transformational features. It is used in almost all areas of mathematics. It deals with the representation of linear equations for linear functions in matrices and vector spaces.

Commutative algebra is one of the types of algebra that studies commutative rings and their ideals. Both algebraic geometry and algebraic number theory require commutative algebra. Rings of algebraic integers, polynomial rings, and other rings are all present. Numerous other areas of mathematics, such as differential topology, invariant theory, order theory, and generic topology, make use of commutative algebra.

https://www.hellovaia.com/explanations/math/pure-maths/

Task 7. ***In pairs, answer the questions.***

1. How algebra can be classified
2. What does pre algebra include?
3. What is the goal of elementary algebra?
4. What is abstract algebra?
5. Where can universal algebra be used?
6. What does linear algebra deal with?
7. What does commutative algebra study?

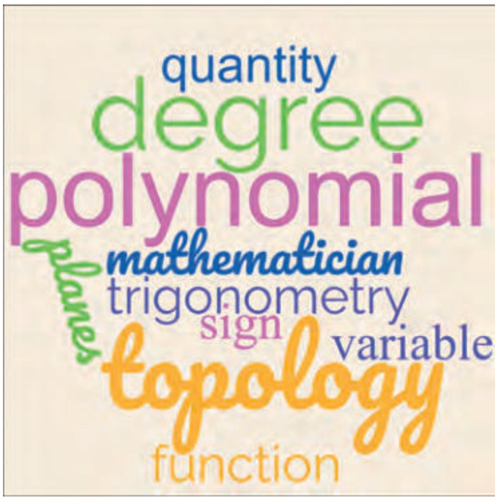
Task 8. ***Mark the sentences as True or False.***

1. Algebra combines numbers and variables using such mathematical operations as addition and subtraction.
2. Algebraic expressions are expressions containing only variables.
3. Elementary algebra deals with solving algebraic expressions to find a solution, and it is used in most fields like science, medicine, economic and engineering.
4. Algebra doesn't include the fundamental concepts of arithmetic and algebra, such as the order of operations, basic operations with numbers, and simplifying expressions.

5. For a polynomial problem, the typical form of representation is $ax^n + bx^{n-1} + cx^{n-2} + \ldots + k = 0$, while for a quadratic equation, it is $ax^2 + bx + c = 0$.
6. The equations are divided into polynomials, quadratic equations, or linear equations depending on the degree of the variable.
7. In abstract algebra we study specific operations like addition and multiplication.
8. Linear algebra involves learning about lines and angles.
9. Algebraic geometry and algebraic number theory require commutative algebra.

Task 9. ***Match the words from the box with their meanings.***

1. Someone who studies, teaches, or is an expert in mathematics.
2. A letter or symbol that represents any of a set of values.
3. The amount or number of something, especially that can be measured.
4. A written or printed mark that has a standard meaning.
5. A number or variable (= mathematical symbol), or the result of adding or subtracting two or more numbers or variables.
6. A unit of measurement of angles, often shown by the symbol ° written after a number.
7. A type of mathematics that deals with the relationship between the angles and sides of triangles, used in measuring the height of buildings, mountains, etc.
8. A flat or level surface that continues in all directions.
9. A quantity whose value depends on another value and changes with that value.
10. The way the parts of something are organized or connected.



Task 10. ***Make a summary of the text (8—10 sentences).***

Task 11. ***Work in pairs / groups. Discuss the questions.***

1. What is the most important branch of mathematics?
2. Can you explain the concept of algebra and its importance in mathematics?
3. What are the main topics of algebra?
4. Where can these branches be applied?
5. Can you give an example of how algebra is used in real-life situations?
6. What are some applications of algebra in fields such as physics, engineering, or computer science?
7. Are there any historical figures who made significant contributions to the development of algebra?
8. How can studying algebra help improve problem-solving skills and logical thinking?

Task 12. ***Choose one of the branches of Algebra and write an essay about its importance. Make it about 100—150 words long, use the information from the text above as needed.***

MATHEMATICAL LANGUAGE AND MATHEMATICAL SYMBOLS

Task 1. A. ***What mathematical symbols do you know? How were the symbols invented?***

B. ***How well are you aware of the topic? Do the quiz below.***

1. What book did Robert Recorde write to teach English students algebra?
   A. "The Whetstone of Witte".
   B. "Mathematical Symbols".
   C. "Algebra for Beginners".
   D. "The History of Mathematics".
2. Why did Robert Recorde replace the words "is equal to" with two parallel horizontal line segments?
   A. He thought it looked more visually appealing.
   B. He wanted to save time and effort in writing
   C. He believed that no two things can be more equal.
   D. He wanted to create a new symbol for equality.
3. What is the origin of the plus sign for addition?
   A. It was derived from the Latin word "et" meaning and.
   B. It was invented by Robert Recorde.
   C. It was introduced by Christian Kramp.
   D. It was a symbol used in ancient Greek mathematics.
4. Why did Christian Kramp introduce the exclamation mark for factorials?
   A. He needed a shorthand for expressions like factorials.
   B. He wanted to create a unique symbol for his mathematical work.
   C. He thought it would make calculations more visually interesting.
   D. He believed it would simplify mathematical equations.
5. What purpose do symbols serve in mathematics?
   A. They represent unknown quantities and relationships between variables.
   B. They condense repeated operations into a single expression.
   C. They provide instructions for performing calculations.
   D. All of the above.
6. How do symbols make mathematical calculations easier?
   A. They shorten lengthy calculations to smaller terms.
   B. They provide concise instructions for performing calculations.
   C. They represent quantities and relationships in a compact form.
   D. All of the above.
7. What is the author's main point about symbols in mathematics?
   A. Symbols are arbitrary and have no inherent meaning.
   B. Symbols are universal and would be the same in any civilization.
   C. Symbols are a language that needs to be memorized and applied.
   D. Symbols are a visual representation of mathematical concepts.

C. ***Watch the video "Where Do Math Symbols Come From?" to check your ideas.***

https://youtu.be/eVmo63xmnow?si=6k-1ENrYV8Gs4YXm

Task 2. A. ***Match the words (1—10) with the options (a—j) to make mathematical collocations.***

1. Logical	a) coefficie
2. To express	b) sentence
3. Certain	c) term
4. Inequality	d) ideas
5. Equal	e) complex thoughts
6. Mathematical	f) quantities
7. Basic	g) sign
8. To represent	h) symbols
9. Numerical	i) mathematics
10. Particular	j) a value

B. ***Fill in the gaps with the appropriate word collocations from the task above.***

1. Mathematical symbols are used to refer to … … concepts, and ideas, among other things.
2. One common example would be for the x-axis to represent time and the y-axis … … that changes over time.
3. Now it becomes an equation if we put an … … .
4. A constant multiplier of the variables in a term is called … … .
5. Some tutoring programs based on artificial intelligence already exist and can help students through … … writing, and other subjects.
6. An … … is denoted by $<$, which is drawn pointed toward the smaller number.
7. We have language and use it to … … … .
8. The numerical coefficient is the constant in a … .
9. However, if it is translated into a … language, one … can turn into a big book.
10. Albert Einstein once said: "Pure math is the poetry of … …".

Task 3. ***Read the text "Mathematical Language and Mathematical Symbols" and name the following symbols.***

$+ \quad \leq, \geq \quad \neq \quad /$

$- \quad \times \quad = \quad <, >$

MATHEMATICAL LANGUAGE AND MATHEMATICAL SYMBOLS

A mathematical language is a system used in the field of mathematics to communicate mathematical ideas, concepts, and theories among people. It is distinct and unique from the usual language most people are used to and used to communicate abstract, logical ideas.

There are three **characteristics** of mathematical language: being precise, being concise, and being powerful. ***Precision*** of mathematical language means the language is able to make very fine distinctions about and between things. ***Conciseness*** is the ability to say things briefl . Mathematical language being ***powerful*** is its ability to express complex thoughts with relative ease so that most people can understand them.

Mathematical symbols are used to refer to certain quantities, concepts, and ideas, among other things. The most commonly used symbols in basic mathematics are the numbers (0, 1, 2, 3, 4, 5, 6, 7, 8, 9), the four fundamental symbols of operation ($+$, $-$, $\times$, $\div$), the inequality symbols ($\neq$, $\leq$, $\geq$, $<$, $>$), and the equal sign ($=$).

A mathematical expression is a correct arrangement of mathematical symbols used to represent a mathematical object of interest. It does not state a complete thought and it does not make sense to ask if an expression is true or false. In mathematics areas, such as algebra, the most common expressions are numbers, sets, and functions.

A mathematical sentence is a correct arrangement of mathematical symbols that state a complete thought. Hence, it makes sense to ask if a sentence is true, false, sometimes true, or sometimes false.

There are **common words** that are used in algebra:

Variable is a symbol that represents a value or a number. Usually, small letters are used as variables. For instance, a, x, and y.

Constant is a number on its own, meaning the value is never definite and will never change. For instance, 4, 9, and 35.

Term is a constant, or a variable, or constant(s) and variable(s) multiplied together. For instance, 4, z, $5xy$, $87xy$, and xyz. The literal coefficien is the variable in a particular term. The numerical coefficient is the constant n a particular term.

Expression is also called an **algebraic expression**, it is a group of terms separated by the "plus" or "minus" symbols. For instance, $3x - 2$ and $x + 3$.

As mentioned, mathematics uses sentences. In algebra, these sentences can be written mathematically or verbally. Mathematical sentences are written using variables, constants, and an equal sign or inequality sign. On the other hand, verbal sentences are written in words using **phrases**.

1. *Addition* ("plus," "increased by," "added to," "the sum of," and "more than").
2. *Subtraction* ("decreased by," "subtracted from," "the di erence of," "less than," and "diminished by").
3. *Multiplication* ("multiplied by," "of," "the product of," and "times").
4. *Division* ("ratio of," "the quotient of," and "divided by").
5. *Inequality* ("is greater than," "is less than," "is at least," "is less than or equal," "is greater than or equal to," "is at most," and "is not equal to").
6. *Equality* ("the same as," "is equal to," and "equals").

Sentence	Verbal Sentence
$x + 2 = 12$	The sum of a number *x* and two is equal to twelve.
$9y = 18$	Nine times a number *y* equals eighteen.
$9 + 8 > 2$	The sum of eight and nine is greater than one.
$x - 8 \leq 2$	The difference of a number *x* and eight is less than or equal to two.
$8x + 7 = 90$	The sum of seven and eight times a number *x* is equal to ninety.

https://owlcation.com/stem/Mathematical-Language-and-Symbols

Task 4. ***In pairs, answer the questions.***

1. What is a mathematical language?
2. What three characteristics of mathematical language are there?
3. What are the most commonly used symbols in basic mathematics?
4. What is a mathematical expression?
5. What are the most common expressions?
6. What is a mathematical sentence?
7. Where are mathematical sentences and expressions commonly used in algebra?
8. How can mathematical sentences be written?

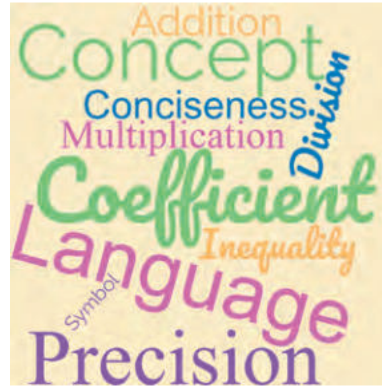
Task 5. ***Discuss with your partner if the statements below are True or False.***

1. Mathematical language is a system used in the eld of mathematics to communicate mathematical ideas, concepts, and theories among people.
2. There are four characteristics of mathematical language.
3. Mathematical symbols are used to refer to certain concepts and ideas.
4. In mathematics areas, such as algebra, the most common expressions are numbers, sets, and functions.
5. A mathematical sentence isn't the mathematical analogue of an English sentence.
6. Variable is a symbol that represents a value or a number.
7. Constant is a number on its own, meaning the value is never definite and will never change.
8. Mathematics doesn't use sentences.
9. In algebra sentences can be written mathematically or verbally.

Task 6. ***Match the words from the box with their meanings.***

1. An idea, theory, etc. about a particular subject.
2. The process of finding the number of times one number is contained within another.
3. A system of communication consisting of sounds, words, and grammar.
4. The quality of being exact.
5. The quality of being short and clear, and expressing what needs to be said without unnecessary words.
6. The process of calculating the total of a group of numbers.

7. A number, letter, or sign used in mathematics.
8. A value, in mathematics, that appears in front of and multiplies another value.
9. The process of adding a number to itself a particular number of times, or a calculation in which this is done.
10. A relationship between two expressions or values that are not equal to each other.



Task 7. ***Make a summary of the text (8—10 sentences).***

Task 8. ***What do you think about math symbols? Which opinion below do you support? Why?***

Mike:

I swear, mathematical symbols are the bane of my existence! They're like a puzzle that I can never seem to solve because there's always something missing. Every time I try to understand them, they just twist and turn and leave me feeling dizzy. It's like trying to navigate through a maze blindfolded!

Lily:

Honestly, I think mathematical symbols are awesome! They bring clarity to chaos and make everything so logical. The equal sign is like a superhero, swooping in to save the day by showing us when two things are the same. When I see those symbols, I feel like I'm on top of the world, ready to conquer any math problem that comes my way!

Charlie:

Mathematical symbols are such a mixed bag for me. Sometimes they're helpful, but other times they're like a foreign language that I can't decipher. I mean, sure, addition and subtraction signs are easy enough to understand, but anything beyond that? Forget it. It's like my brain goes into meltdown mode!

Emily:

Ugh, don't even get me started on mathematical symbols. They're the stuff of nightmares! Whenever I see them, I feel like I'm suffocating under a mountain of numbers and letters. It's like they're taunting me, saying, 'You'll never figure this out!' Honestly, I wish we could banish them from existence.

Task 9. ***Be ready to deliver a lecture for children about the mathematical language.***

ALGEBRAIC OPERATIONS

Task 1. ***Discuss the quotes below. Which one do you support? Justify your choice.***

Algebra is the written language through which God has written the universe.

Galileo Galilei

Algebra is the offer made by the devil to the mathematician. The me devil says: ‘I will give you this powerful machine, it will answer any question you like. All you need to do is give me your soul: give up geometry and you will have this marvelous machine.

Michael Atiyah

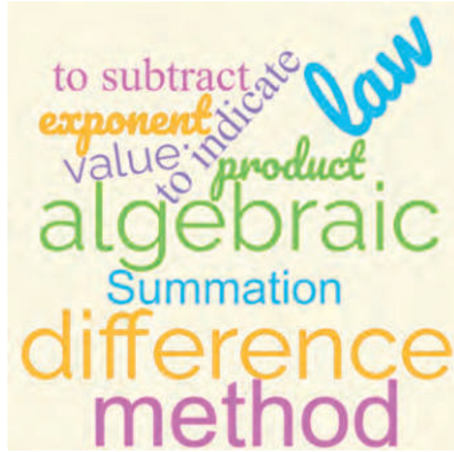
Task 2. ***Can you name any advantages or disadvantages of algebra? In pairs discuss the facts below and mark them as "+" or "–".***

1. Some learners may lack motivation when they find that algebraic operations do not always have real-world applications.
2. Professions that require mastery of advanced algebra may limit career options for those who struggle with this subject.
3. Learning algebra can lay a strong foundation for advanced math and scientific disciplines.
4. Precise calculations and equations become possible in several elds of study because of algebra.
5. Complex mathematical expressions can be simplified and manipulated with ease thanks to algebra.
6. For some students, understanding algebra can be difficult or co using.
7. Solving complex algebraic problems may consume significant amounts of time.
8. Errors in calculation or misinterpretation of concepts can lead to incorrect answers.
9. Algebra helps to develop critical thinking skills as well as problem-solving abilities.
10. By honing logical reasoning and abstract thinking, algebra improves overall cognitive functions.

Task 3. A. ***Match the words from the box with their meanings.***

1. Relating to algebra.
2. The result you get when two or more numbers are multiplied together.

3. When two or more numbers or amounts have been added together.
4. The result of subtraction.
5. A number or symbol that represents an amount.
6. A particular way of doing something.
7. To remove a number from another number.
8. A number or sign that shows how many times another number is to be multiplied by itself.
9. A general rule that states what always happens when the same conditions exist.
10. To show something, point to something, or make something clear.



B. ***Complete the sentences using the words from the box above.***

1. ... reasoning lies behind modern economics.
2. The ... of 13 and 7 is 20.
3. We used different ... of calculation, but we both got the same re ult.
4. A number system defines a set of ... used to represent quantity
5. The ... of 8 and 4 is equal to 4.
6. According to the ... of physics this would not be possible.
7. The ... 6 and 3 is 18.
8. The research ... that it is helpful for the teachers to focus on pupils' learning processes rather than only on their learning outcomes.
9. In 6^4 and y^n, 4 and n are the ... .
10. 4 ... from 10 equals 6.

Task 4. A. ***Look through the following statements about algebraic operations and say if you consider them to be True or False.***

1. There are four basic mathematical operations that are used in algebra.
2. The summation of two or more algebraic terms is done in multiplication.
3. The method of finding the difference between two algebraic terms is called division of algebraic terms.
4. Subtraction is also possible only between the unlike algebraic terms.
5. Multiplication of algebraic terms is indicated by "×" or $(a)(b)$.
6. Multiplication is possible between both like and unlike terms.
7. The division between algebraic terms is indicated by the "/" symbol.
8. Algebraic laws or properties include closure, commutative, associative and distributive properties.

B. ***Now read the text "Algebraic Operations" and check how many false sentences you have managed to figure out.***

ALGEBRAIC OPERATIONS

There are four basic mathematical operations that are used in algebra. These are addition, subtraction, multiplication, and division.

The summation of two or more algebraic terms is done in **addition**. The addition of algebraic terms is indicated by "+" symbol. The addition of algebraic terms to yield a single value is only possible if there are like algebraic terms else the expression remains as it is. For Example, $3x^2y + 5x^2y = 8x^2y$ as $3x^2y$ and $5x^2y$ are like algebraic terms while if we add $3xy^2$ and $5x2y$ then it will not yield a single value instead it will remain as it is i.e. $3xy^2 + 5x^2y$.

The method of nding the diff rence between two algebraic terms is called **subtraction** of algebraic terms. Subtraction of algebraic terms is indicated by "–" terms. Like addition, subtraction is also possible only between the like algebraic terms. For example, if we subtract $3x^2y$ from $5x^2y$ it will give a difference as $5x^2y - 3x^2y = 2x^2y$. But if subtract $3xy^2$ from $5x^2y$ it will not yield a single value as the two terms are unlike. Hence, the difference will be written as $x^2y - 3xy^2$.

Unlike addition and subtraction, **multiplication** is possible between both like and unlike terms. Multiplication of algebraic terms is indicated by "×" or $(a)(b)$. While performing the multiplication of algebraic terms, multiply the numerical coefficien as normal numbers and multiply the variables using the laws of exponents. For example, if we multiply $4x^2y$ with $5xyz$ then the product is given as $(4x^2y)(5xyz) = 20x^3y^2z$.

Like multiplication, **division** of algebraic terms is possible between both like and unlike terms while keeping in mind the laws of exponents for variables and normal division for numbers. The division between algebraic terms is indicated by the "/" symbol. For Example, the division of $6x^2y^2$ by $3xy^2$ is given as $6x^2y^2 / 3xy^2 = 2x$.

Algebraic properties include closure, commutative, associative, distributive, and identity properties. These properties are defined for basic algebraic operations such as addition, subtraction, multiplication, and division. A picture explaining algebraic laws has been attached below.

Property	Operations			
	Addition	Subtraction	Multiplication	Division
Closure	$a + b \in \mathrm{R}$	$a - b \in \mathrm{R}$	$a \times b \in \mathrm{R}$	$a \div b \in \mathrm{R}$
Commutative	$a + b = b + a$	$a - b \neq b - a$	$ab = ba$	$a \div b \neq b \div a$
Associative	$a + (b + c) =$ $= (a + b) + c$	$a - (b - c) =$ $= (a - b) - c$	$a(bc) = (ab) \times c$	$a \div (b \div c) =$ $= (a \div b) \div c$
Identity	$a + 0 = 0 + a = a$	—	$a \times 1/a = 1 =$ $= 1/a \times a$	—
Distributive	$a(b + c) = ab + ac$	$a(b - c) = ab - ac$	—	—

https://www.geeksforgeeks.org/arithmetic-operations/

Task 5. ***In pairs, answer the questions.***

1. How many basic mathematical operations are there?
2. What is addition?
3. How is the addition of algebraic terms indicated?
4. When is it possible to yield a single value?
5. What is subtraction of algebraic terms?

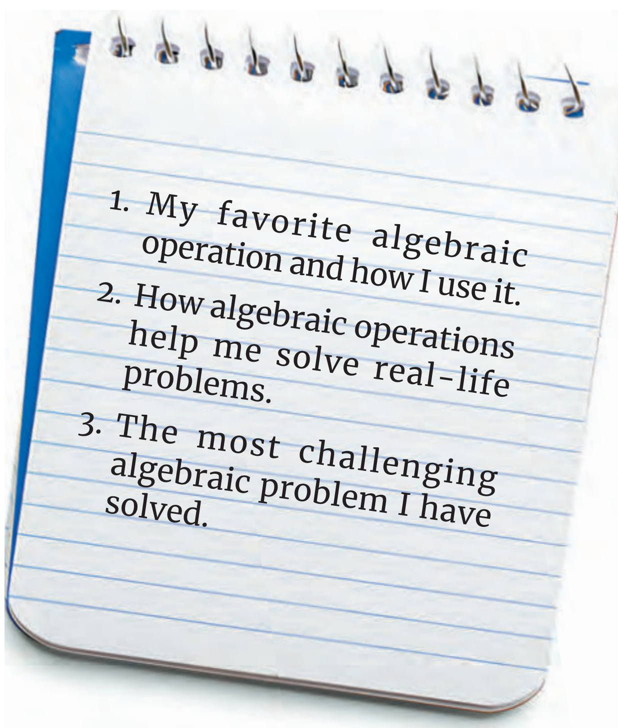
6. How is subtraction of algebraic terms indicated and when is it possible to find a difference between two terms
7. Between what terms is multiplication possible? How should numerical coefficien and variables be multiplied?
8. How is division indicated?
9. Can division be performed between like and unlike terms?
10. Concerning division, what rules should be followed for variables and numbers?
11. What properties do algebraic laws include?

Task 6. ***Make a short summary of the text (about 10—12 sentences).***

Task 7. ***Discuss the questions below in pairs.***

1. Algebraic operations are essential for solving math problems. What do you think?
2. Some people find algebraic operations difficult to understand. you agree?
3. Algebraic operations are important for everyday life. What do you think?
4. Algebraic operations can be used in different areas, such as science and engineering. Do you agree or disagree?
5. Learning algebraic operations helps develop problem-solving skills. What is your opinion?

Task 8. A. ***Write a 1-minute speech on one of the topics below.***



B. ***Present your speeches at your group's press conference. Answer your groupmates' questions.***

Task 9. ***Make a report (presentation) to speak about algebraic operations.***

VIETA'S FORMULA

Task 1. ***Have a look at the images below. Which of them depict Vieta's formula?***

$$x^2 + px + q = 0$$
$$x_1 + x_2 = -p$$
$$x_1 \times x_2 = q$$

$$e^{i\pi} + 1 = 0$$

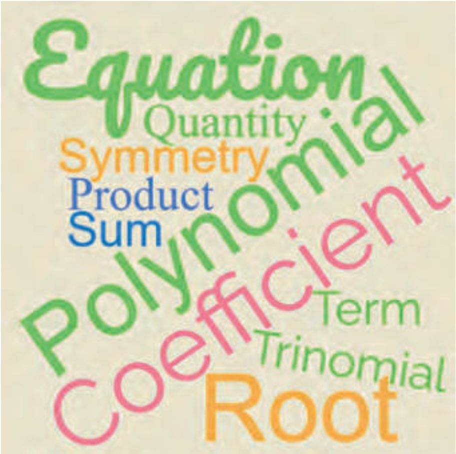
$$\int_a^b f(x)dx \approx \frac{h}{3}\left(f(x_0) + 4\sum_{i=1}^{n} f(x_{2i-1}) + 2\sum_{i=1}^{n-1} f(x_{2i}) + f(x_{2n}) \right)$$

Task 2. ***Discuss the questions below.***

1. How often do you encounter polynomials in your daily life?
2. Have you ever used formulas to solve mathematical problems?
3. Do you find it easier to solve equations by factoring or by usi g formulas?
4. Are you comfortable with complex numbers in mathematics?
5. Can you think of any real-life applications where Vieta's formula could be useful?

Task 3. A. ***Match the words with their definitions.***

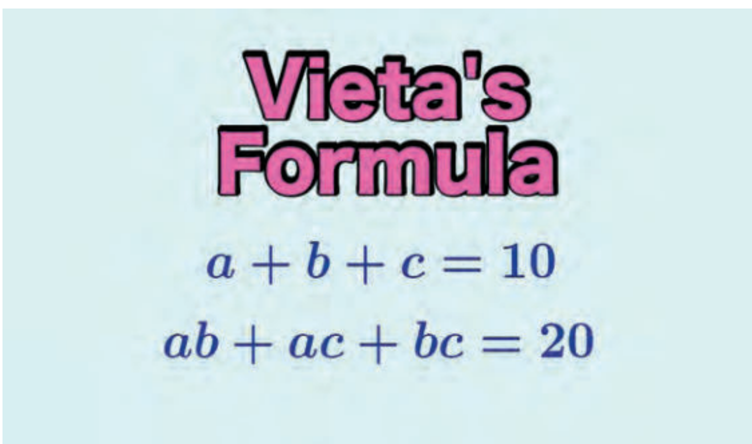
1. A value that appears in front of and multiplies another value.
2. A number or variable, or the result of adding or subtracting two or more numbers or variables.
3. The quality of having two parts that match exactly, either when one half is like an image of the other half in a mirror, or when one part can take the place of another if it is turned 90° or 180°.
4. The result you get when two or more numbers are multiplied together.
5. Another number that, when multiplied by itself one or more times, reaches that number.
6. The amount or number of something, or a particular amount or number.
7. The whole number or amount when two or more numbers or amounts have been added together.
8. A mathematical statement with three numbers or variables.
9. A mathematical statement in which you show that two amounts are equal using mathematical symbols.
10. A number or symbol used in a mathematical series or calculation.



B. ***Fill in the gaps with the words from the task above.***

1. Another exercise is completing the square in a ... .
2. We use the same rule to compute the ... of the equation.
3. In $2x + 4y = 7$, 2 is the ... of x.
4. The explicit forms of these ... equations have been found.
5. In the ... $3x - 3 = 15$, $x = 6$.
6. The ... of 5 and 3 is 15.
7. The square ... of 8 is 64, and the cube root of 64 is 4.
8. The following representation theorem states the correspondence between double symmetries and ordinary ... .
9. The ... of 14 and 6 is 20.
10. For our present purpose it is enough to ask what ... the numerical solution is supposed to approximate, at least to a first-order approximatio .

Task 4. ***What are the key facts about Vieta's formula?***
A. ***Read the facts about Vieta's formula.***
B. ***Watch the video and tick the facts mentioned in the video.***
C. ***Prolong the fact list with the information from the video.***



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1. Vieta's formula can be used to find the sum and product of a polynomial's roots.
2. It was discovered by French mathematician François Viète in the 16th century.
3. Vieta's formula can be represented algebraically as $\Pi r = \alpha$, where r is the product of the roots and α is the leading coefficien
4. This formula is also known as Vieta's theorem or Vieta's relations.
5. It can be applied to both real and imaginary roots, making it useful in solving complex equations.
6. Vieta's formula helped lay the foundation for the study of symmetric polynomials.
7. It can be used to prove the fundamental theorem of algebra.
8. The formula can be extended to find the sum and product of powers of the roots.
9. Vieta's formula is often used in trigonometry and geometry to solve problems involving angles and sides of a polygon.
10. It has been used to solve various mathematical problems, including the general solution of cubic equations.

Task 5. ***Choose the best answer to the questions. Watch the video fragment 03.00—08.10 to check your ideas.***

1. According to the video, what is the coefficient o *x* to the *n* minus 2:
   a) the sum of pairs of roots;
   b) the product of the roots;
   c) the sum of the squares of the roots;
   d) the negative of the sum of the roots?
2. What does Vieta's formula allow us to do:
   a) compute the squares of the roots;
   b) determine the product of the roots;
   c) find the sum of the roots
   d) solve polynomial equations?
3. Why is it difficult to compute the roots of the polynomial in t   video:
   a) the roots are complex numbers;
   b) the roots are irrational numbers;
   c) the roots are not easily tractable;
   d) the roots are imaginary numbers?
4. What does the negative sign in Vieta's formula represent:
   a) the sum of the roots;
   b) the product of the roots;
   c) the sum of the squares of the roots;
   d) the negative of the sum of the roots?
5. What is the value of *a* squared plus *b* squared plus *c* squared:
   a) 11/5 squared;
   b) twice 7/5;
   c) the sum of the squares of the roots;
   d) the product of the roots?
6. What does Vieta's formula help us compute:
   a) the sum of the squares of the roots;
   b) the product of the roots;
   c) the sum of the roots;
   d) the coefficients of the polynomia
7. How can Vieta's formula be useful in solving polynomial equations:
   a) it helps determine the degree of the polynomial;
   b) it allows us to find the roots of the polynomial
   c) it simplifies the polynomial equation
   d) it helps compute the coefficients of the polynomia

Task 6. ***Hold a panel discussion to dig deeper in the topic.***

1. How does Vieta's formula relate to the roots of a polynomial equation?
2. Can you explain the significance of Vieta's formula in modern m thematics?
3. In what ways has Vieta's formula been applied in real-world problems or situations?

4. Could you provide an example of how Vieta's formula can be used to solve a complex polynomial equation?
5. What are some limitations or drawbacks of using Vieta's formula in solving equations?
6. How does Vieta's formula differ from other methods of finding roots, such as factoring or graphing?
7. Can you discuss any historical context surrounding the development of Vieta's formula?
8. Are there any variations or extensions of Vieta's formula that have been proposed by mathematicians?
9. How does understanding Vieta's formula contribute to a deeper understanding of algebraic concepts?
10. Do you think Vieta's formula will continue to be relevant and useful in future mathematical research and applications?

Task 7. ***Vieta's formula is a rather disputable issue. Which opinion do you support? Why?***

Jenna:

I think Vieta's formula is a lifesaver when it comes to solving complex algebraic equations. It helped me ace my math exams in high school. I remember struggling with a particularly difficul equation, but once I applied Vieta's formula, everything fell into place and I was able to solve it easily.

Simon:

I'm not a fan of Vieta's formula. It's complicated and confusing, and it just adds unnecessary steps to solving equations. I prefer traditional methods that I am more familiar with. Whenever I try to use Vieta's formula, I end up making mistakes and wasting more time than if I had just stuck to what I know.

Leah:

I can't decide whether I like or dislike Vieta's formula. On one hand, it can be really helpful in certain situations, but on the other hand, it can be a bit overwhelming to understand and apply. I guess it depends on the complexity of the equation and my mood that day.

Mike:

I absolutely love Vieta's formula! I remember how excited I was when I first learned about it in math class. It made me feel like a genius and it has saved me countless times since then. In fact, I even wrote a song about it just for fun.

Task 8. ***Make a report (presentation) to speak about Vieta's formula.***

EXPLORING THE TOPIC

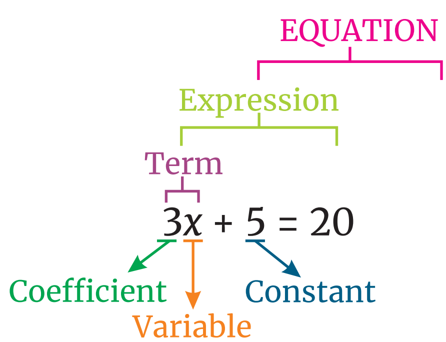
ALGEBRAIC EQUATIONS

Task 1. ***Discuss the questions below.***

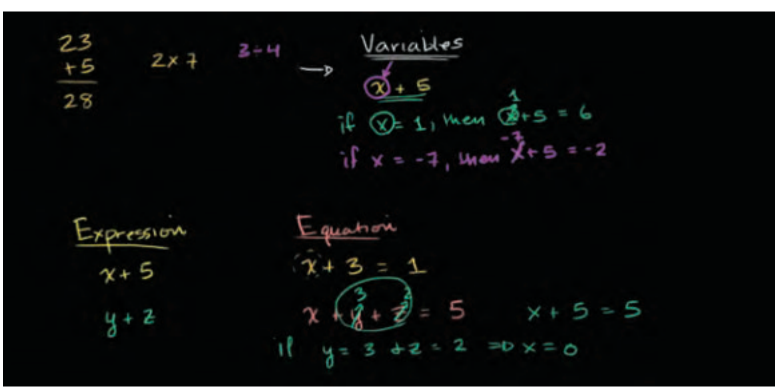
1. How do you feel about solving equations?
2. Have you ever used variables in a math problem before?
3. Do you find it easier to solve equations with one unknown or multiple unknowns?
4. What strategies do you use to simplify complex expressions?
5. Can you think of any real-life situations where equations and variables are used?

Task 2. ***Do you know the basic algebraic terms? Match the words below with their definitions.***

Expression	a) a symbol or letter that represents a quantity or value that can change or vary in mathematical equations or computer programming
Equation	b) a statement that shows the equality between two expressions or quantities, typically represented by an equal sign (=), indicating that both sides have the same value
Variable	c) a combination of numbers, symbols, and operators that represents a mathematical calculation or relationship between quantities



Task 3. ***What is the difference between variable, expression and equation? Give examples from the video.***



https://www.youtube.com/watch?v=vDqOoI-4Z6M

Task 4. ***Have a look at the title of the text. What do you think you can learn from it? What do you know about Algebraic Equations? What would you like to know? Fill in columns 1 and 2 in the grid below. Then read the part of the text assigned to you and fill in columns 3 and 4.***

I know ... (What do you already know about algebraic equations?)	*I want to know ...* (Write down any new questions you have.)	*I have learned ...* (State 3 main ideas you have learned from the text.)	*I know ... because ...* (Record 1 supporting fact or detail for each of your main ideas in the previous column.)

ALGEBRAIC EQUATIONS

(1) ... The general form of an algebraic equation is $P = 0$ or $P = Q$, where P and Q are polynomials. Algebraic equations that contain only one variable are known as univariate equations and those which contain more than one variable are known as multivariate equations. An algebraic equation will always be balanced. This means that the right-hand side of the equation will be equal to the left-hand side.

1. ______________________________

The degree of an algebraic equation is the highest power present in the variables of the equation. Algebraic equations can be classi ed according to their degree as follows:

Linear equations. (2) ... For example, $ax + b = 0$, where x is the variable, and a and b are constants.

Quadratic equations. (3) ... They contain variables with power 2. Quadratic equations will produce two possible solutions for x that satisfy the equation.

Cubic equations. (4) ... They contain variables with power 3.

Standard Form of
a Linear Equation

Coefficient of x Coefficient of y

$$Ax + By = C$$

Variable Constant

Standard Form of
a Quadratic Expression

Coefficient of x^2 Coefficient of x

$$ax^2 + bx + c = 0$$

Constant

2. ______________________________

The basic properties of algebra that you need to keep in mind when solving algebraic equations are:

• *Commutative property of addition:* changing the order of the numbers being added does not change the sum

$$a + b = b + a.$$

• *Commutative property of multiplication:* changing the order of the numbers being multiplied does not change the product

$$a \times b = b \times a.$$

• *Associative property of addition:* (5) ...

$$a + (b + c) = (a + b) + c.$$

• *Associative property of multiplication:* changing the grouping of the numbers being multiplied does not change the product

$$a \times (b \times c) = (a \times b) \times c.$$

• *Distributive property:* if you multiply the sum of two or more numbers by another number, you will get the same result as multiplying each term in the sum individually by the number and then adding the products together

$$a \times (b + c) = a \times b + a \times c.$$

Reciprocal. You can find the reciprocal of a number by swapping the numerator and the denominator

Reciprocal of $a = \frac{1}{a}$.

• *Additive identity:* (6) ...

$$a + 0 = 0 + a = a.$$

• *Multiplicative identity:* if you multiply any number by 1, you will get the same number as a result

$$a \times 1 = 1 \times a = a.$$

• *Additive inverse:* (7) ...

$$a + (-a) = 0.$$

• *Multiplicative inverse:* if you multiply a number by its reciprocal, you will get 1 as a result

$$a \times \frac{1}{a} = 1.$$

3. ______________________________

(8) ... If an algebraic equation has two variables then two equations will be required to find the solution. Thus, it can be said that the number of equations required to solve an algebraic equation will be equal to the number of variables present in the equation

To solve linear algebraic equations, you should follow the following steps:

Step 1. (9) ...

Step 2. Add or subtract to isolate the variable on one side of the equation.

Step 3. Multiply or divide to obtain the value of the unknown variable.

A quadratic algebraic equation can be solved by using identities, factorizing, long division, splitting the middle term, completing the square, applying the quadratic formula, and using graphs. A quadratic equation will always have a maximum of two roots.

The most e ective way of solving higher-order algebraic polynomials in one variable is by using the long division method. (10) ...

https://www.cuemath.com/algebra/algebraic-equations/

Task 5. A. ***Make a summary of your part of the text and present it to your partner.***

B. ***Listen to your partner's summary and fill in columns 3 and 4 of the chart in Task 1.***

Task 6. ***Give the headings to the part of the text you have read.***

A. Solving Algebraic Equations.
B. Basic Properties Of Algebra.
C. Types Of Algebraic Equations.

Task 7. ***Read the text once again and fill in the gaps with sentences from the list below. There are 2 extra sentences.***

1. Cubic equations are represented in a generic form as $ax^3 + bx^2 + cx + d = 0$, where x is the variable, and a, b, c and d are constants.
2. Linear equations are used to represent problems where the degree of the variables (i.e. x, y or z) is one.
3. Each side of the equation must be simplified by removing parentheses and combining terms
4. Adding a number and its inverse (same number with opposite sign) gives 0 (zero) as a result.
5. This decomposes the higher-order polynomial into polynomials of a lower degree thus, making it easier to find the solutions
6. An algebraic equation is a mathematical statement that contains two equated algebraic expressions.
7. Algebraic equations form the basis of all simple and complex formulas used in mathematics.
8. Changing the grouping of the numbers being added does not change the sum.
9. Quadratic equations are generically represented $ax^2 + bx + c = 0$, where x is the variable, and a, b and c are constants.
10. If you add 0 (zero) to any number, you will get the same number as a result.
11. Algebraic inequality statements are solved in the same manner as equations.
12. There are many different methods that are available for solving algebraic equations depending upon the degree.

Task 8. ***In pairs, answer the questions below.***

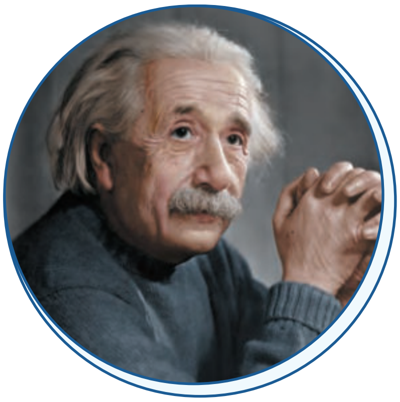
1. What is an algebraic equation?
2. What is the general form of an algebraic equation?
3. How can algebraic equations be classified
4. How are cubic equations represented?
5. What are the basic properties of algebra when solving algebraic equations?

6. What steps should you follow when solving linear algebraic equations?
7. How can quadratic algebraic equations be solved?
8. What is the most effective way of solving higher-order algebraic polynomials in one variable?

Task 9. ***Comment on the following quotations.***

The universe is governed by science. But science tells us that we can't solve the equations, directly in the abstract.

Stephen Hawking



A mathematical equation stands forever.

Albert Einstein

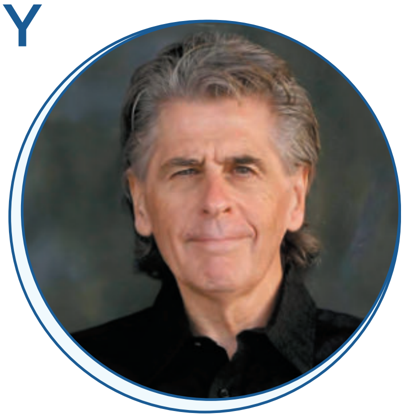
The fundamental laws necessary for tяhe mathematical treatment of a large part of physics and the whole of chemistry are thus completely known, and the difficul lies only in the fact that application of these laws leads to equations that are too complex to be solved.

Paul Dirac

Task 10. ***Write a blog post discussing the relevance of algebraic equations in everyday life. Provide examples of how this mathematical concept is used in various fields such as finance, engineering, and computer programming. Be sure to use the proper math terminology and provide enough detail for the readers to understand the concept of algebraic equations. Make it about 15—20 sentences long.***

EULER'S IDENTITY

Task 1. ***Comment on the following quotations.***



Euler's identity is a jewel of analysis, one of the most remarkable formulas in mathematics.

Keith Devlin

Euler’s identity is the mathematical equivalent of a Shakespearean sonnet.

David Bodanis

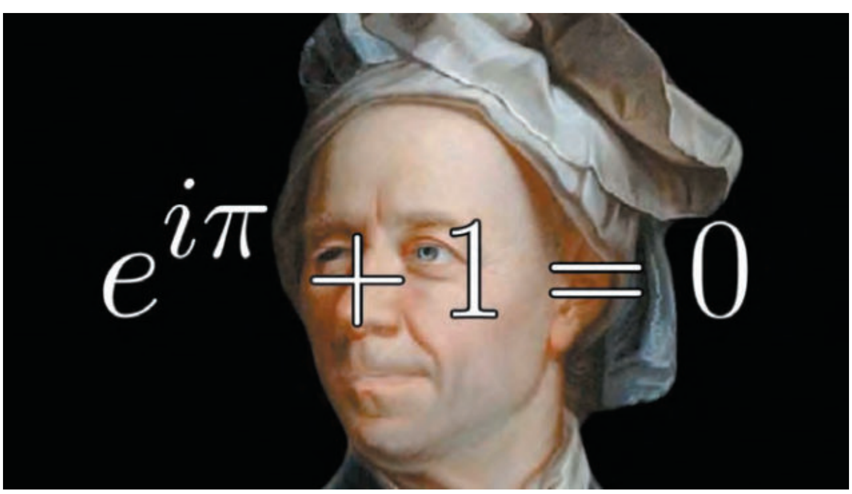
Task 2. ***Choose the right definition of an algebraic identity. Justify your choice.***

An algebraic identity is
- an equality that holds for any values of its variables;
- an inequality that holds for any values of its variables;
- an equality that holds for particular values of its variables.

Task 3. ***Have a look at the title of the video. What do you think you can learn from it? What do you know about Euler’s identity? What would you like to know? Fill in columns 1 and 2 in the grid below. Then watch the video and fill in columns 3 and 4.***

I know ... (What do you already know about Euler’s identity?)	*I want to know ...* (Write down any new questions you have.)	*I have learned ...* (State 3 main ideas you have learned from the video.)	*I know ... because ...* (Record 1 supporting fact or detail for each of your main ideas in the previous column.)

Fill in columns 3 and 4 after watching the video “Euler’s identity”.



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Task 4. A. ***Match the halves of the sentences.***

1. Mathematics is the language that allows us	a) by an infinite series of additions, multiplications, divisions.
2. Euler's identity was first expresse	b) four completely diffe ent concepts in mathematics.
3. The number "e" can actually be calculated	c) to describe the intricate way in which God created his universe.
4. In the eyes of a mathematician just because a number doesn't exist	d) doesn't mean that it cannot be useful.
5. The identity links together in one simple relationship	e) in 1748 by Leonhard Euler.

B. ***Watch the video about "Euler's identity" to check your ideas.***

Task 5. ***Arrange the sentences below in the order they appear in the video. Watch the video again if needed.***

1. "i" is the only number that, when squared, gives a negative result.
2. Leonhard Euler was born on 15th of April in 1707.
3. The beautiful thing about this equation is that it is very easy to predict what the next term will be as each time.
4. What is so amazing about this little identity is that it links together in one simple relationship four completely different concepts in mathematics
5. The problem is that when we square a number the result is always positive not negative.
6. Two formulae link most beautifully two areas of knowledge that may have seemed hitherto unrelated.
7. The trigonometric functions sine and cosine can also be calculated by infinit series.
8. Mathematics is the science of numbers.
9. It was Daniel Bernoulli who in 1727 secured Euler a post at the Imperial Russian Academy of Sciences in Saint Petersburg.
10. It was discovered by Jacob Bernoulli who came across it whilst working on the principle of compound interest.

Task 6. ***In pairs, restore the contents of the video, speaking about the following mathematical concepts linked in Euler's identity in turns.***

$$e^{i\pi} + 1 = 0$$

Euler's number i π Cosine & Sine functions

Task 7. ***Discuss the questions below in mini-groups.***

1. Are you religious? If so, do you think mathematics can describe the intricate way in which God created the universe?
2. Do you think it's important to know the history and background of mathematical concepts like Euler's number?

3. Have you ever used Euler's number or its approximation in any practical applications?
4. Have you ever encountered imaginary numbers in your studies or daily life?
5. Can you think of any other situations where imaginary numbers might be useful?
6. What do you think about mathematicians inventing numbers that don't exist?
7. Do you find it fascinating or confusing that numbers can have properties that seem contradictory?
8. Euler's Identity is often hailed as one of the most beautiful equations in mathematics. To what extent do you agree or disagree with this opinion?
9. Some people argue that Euler's Identity is too abstract and has no practical use. Is this a valid criticism?
10. Euler's Identity has been described as a "mathematical poem." What does this metaphor mean, and how does it apply to Euler's Identity?
11. Many mathematicians consider Euler's Identity to be a pinnacle achievement in mathematics. Do you think there will ever be another equation that rivals its significance
12. Euler's Identity showcases the interconnectedness of various mathematical concepts. How does understanding this equation enhance our understanding of other branches of mathematics?

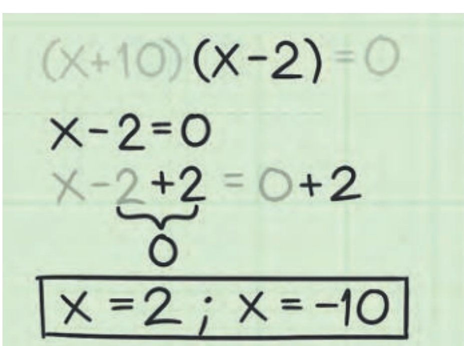
Task 8. ***Hold a group conference, sharing your experience of studying Euler's Identity. Speak on one of the points below.***

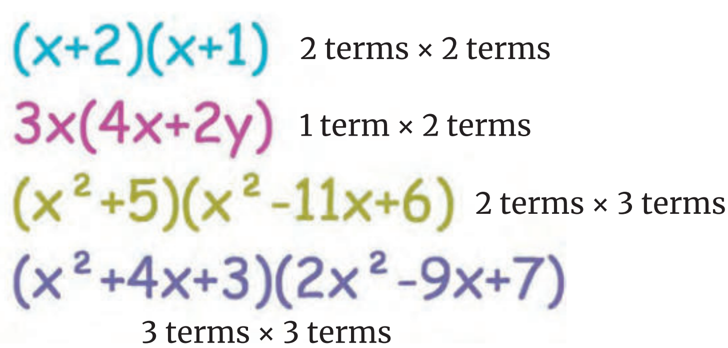
1. My fascination with Euler's Identity and how it sparked my interest in mathematics.
2. Exploring the applications of Euler's Identity in real-world problems.
3. My journey of discovering the beauty and elegance of Euler's Identity.
4. How studying Euler's Identity has deepened my understanding of complex numbers.
5. Reflecting on the impact of Euler's Identity on the field of m hematics.

Task 9. ***Write a social media post explaining the beauty of Euler's identity. Make it about 120—150 words long, using the information from the video "Euler's identity".***

POLYNOMIALS

Task 1. ***Look at the pictures. What can you see? Can you name it?***



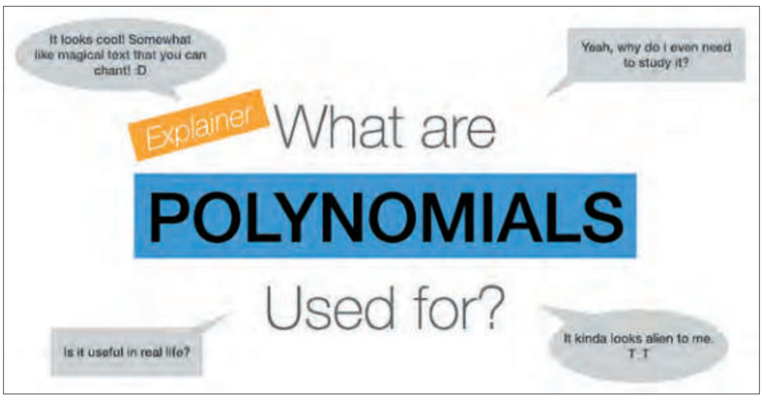


Task 2. ***Discuss the quotations about polynomials with your partner. Which one do you agree with? Why?***

1. "Polynomials are the building blocks of algebraic expressions, and understanding them is crucial for solving complex mathematical problems". *Maryam Mirzakhani*
2. "Polynomials are like puzzles waiting to be solved, with each term representing a piece that fits into the larger picture". *Terence Tao*
3. "Polynomials are the language of mathematics, allowing us to express and manipulate abstract concepts with precision and elegance". *Andrew Wiles*
4. "Polynomials are the foundation of modern cryptography, enabling secure communication and data protection in the digital age". *Whitfiel Di*

Task 3. A. ***What is the application of polynomials? Discuss your ideas with your partner.***

B. ***Compare your ideas with those given in the video.***



https://drive.google.com/file d/1-LkGnCQY4YbgbPWSFtfOk_PSVkDOU_qN/view?usp=drive_link

C. ***How many of your ideas are mentioned in the video? Have you got to know any new applications of polynomials?***

Task 4. ***Have a look at the title of the text. What do you think you can learn from it? What do you know about Polynomials? What would you like to know? Fill in columns 1 and 2 in the grid below. Then read the part of the text assigned to you and fill in columns 3 and 4.***

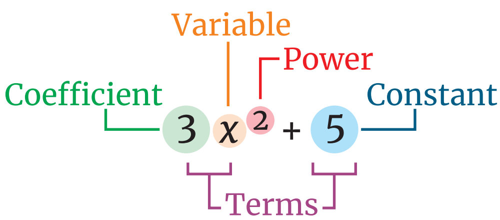
I know ... (What do you already know about polynomials?)	*I want to know ...* (Write down any new questions you have.)	*I have learned ...* (State 3 main ideas you have learned from the text.)	*I know ... because ...* (Record 1 supporting fact or detail for each of your main ideas in the previous column.)

POLYNOMIALS

1. ______________________________

Polynomials are algebraic expressions that contain indeterminates and constants. They are used to express numbers in almost every field of mathematics and are

What is a Polynomial?



considered very important in certain branches of math, such as calculus. For example, $2x + 9$ and $x^2 + 3x + 11$ are polynomials.

Polynomials are mathematical expressions made up of variables and constants by using arithmetic operations like addition, subtraction, and multiplication. They represent the relationship between variables.

(1) ... The exponents of the variables in any polynomial have to be a non-negative integer. A polynomial comprises constants and variables, but we cannot perform division operations by a variable in polynomials.

2. ______________________________

(2) ... Here, x is known as the variable. 3 which is multiplied to x^2 has a special name. We denote it by the term “coefficient” 5 is known as the constant. The power of the variable x is 2.

Below given are a few expressions that are not examples of a polynomial.

Not a Polynomial	Reason
$2x^{-2}$	Here, the exponent of variable ‘x’ is -2.
$1/(y + 2)$	This is not an example of a polynomial since division operations in a polynomial cannot be performed by a variable.
$\sqrt{(2x)}$	The exponent cannot be a fraction (here, 1/2) for a polynomial.

3. ______________________________

The highest or greatest exponent of the variable in a polynomial is known as the degree of a polynomial. The degree is used to determine the maximum number of solutions of a polynomial equation.

Example 1. A polynomial $3x^4 + 7$ has a degree equal to four.

The degree of the polynomial with more than one variable is equal to the sum of the exponents of the variables in it.

Example 2. Find the degree of the polynomial $3xy$.

In the above polynomial, the power of each variable x and y is 1. (3) ... So, we will get the degree of the given polynomial ($3xy$) as 2.

Similarly, we can find the degree of the polynomial $2x^2y^4 + 7x^2y$ by finding the degree of each term. The highest degree would be the degree of the polynomial. In this example:

- the degree of $2x^2y^4$ is $2 + 4 = 6$ and
- the degree of $7x^2y$ is $2 + 1 = 3$
- among these, 6 is the bigger number and hence it is the degree of the polynomial $2x^2y^4 + 7x^2y$.

4. ______________________________

The standard form of a polynomial refers to writing a polynomial in the descending power of the variable.

Example. Express the polynomial $5 + 2x + x^2$ in the standard form.

To express the above polynomial in standard form, we will first check the degree of the polynomial.

1. In the given polynomial, the degree is 2. Write the term containing the degree of the polynomial.

2. Now, we will check if there is a term with the exponent of variable less than 2, i.e., 1, and note it down next.

3. (4) ...

Therefore, $5 + 2x + x^2$ in standard form can be written as $x^2 + 2x + 5$.

Always remember that in the standard form of a polynomial, the terms are written in decreasing order of the power of the variable, here, x.

5. ____________________

The terms of polynomials are defined as the parts of the expression that are separated by the operators "+" or "−". For example, the polynomial expression $2x^3 - 4x^2 + 7x - 4$ consists of four terms. The terms are classified into two types: like terms and unlike terms.

Like terms in polynomials are those terms which have the same variable and same power. (5) ... Hence, if a polynomial has two variables, then all the same powers of any ONE variable will be known as like terms. Let us understand these two with the help of examples given below.

For example, $2x$ and $3x$ are like terms. Whereas, $3y^4$ and $2x^3$ are unlike terms.

6. ____________________

Polynomials can be categorized based on their degree and their power. Based on the numbers of terms, there are mainly three types of polynomials such as monomials, binomials, trinomials.

Monomial is a type of polynomial with a single term. For example, x, $-5xy$, and $6y^2$. (6) ... For example, $x + 5$, $y^2 + 5$, and $3x^3 - 7$. While a trinomial is a type of polynomial that has three terms. For example, $3x^3 + 8x - 5$, $x + y + z$, and $3x + y - 5$. However, based on the degree of the polynomial, polynomials can be classified into 5 major types.

1. Zero polynomial. It is a polynomial whose degree is equal to zero.
2. Constant polynomial. It is a polynomial with just a constant and no variables. For example, 3, 5, or 8, etc.
3. Linear polynomial. It is a polynomial with degree 1. Example: $x + y - 4$.
4. Quadratic polynomial. It is a polynomial with degree 2. Example: $2p^2 - 7$.
5. Cubic polynomial. It is a polynomial with degree 3. Example: $6m^3 - mn + n^2 - 4$.

7. ____________________

The basic algebraic operations can be performed on polynomials of different types. These four basic operations on polynomials can be given as addition of polynomials, subtraction of polynomials, multiplication of polynomials, division of polynomials.

Addition of polynomials is one of the basic operations that we use to increase or decrease the value of polynomials. Whether you wish to add numbers together or you wish to add polynomials, the basic rules remain the same. (7) ... However, when dealing with the addition of polynomials, one needs to pair up like terms and then add them up. Otherwise, all the rules of addition from numbers translate over to polynomials. Have a look at the image given here in order to understand how to add any two polynomials.

Subtraction of polynomials. (8) ... To subtract a polynomial from another, we just add the additive inverse of the polynomial that is being subtracted to the other polynomial. Another easy way to subtract polynomials is, just to change the signs of all the terms of the polynomial to be subtracted and then add the resultant terms to the other polynomial. We just have to align the given polynomials based on the like terms.

The multiplication operation on polynomials follows the general properties like commutative property, associative property, distributive property, etc. Applying these properties using the rules of exponents we can solve the multiplication of polynomials. To multiply to polynomials, we just multiply every term of one polynomial with every term of the other polynomial and then add all the results. Here is an example to multiply polynomials.

The division of polynomials is an arithmetic operation where we divide a given polynomial by another polynomial which is generally of a lesser degree in comparison to the degree of the dividend. There are two methods to divide polynomials: long division of polynomials, synthetic division.

Factorization of polynomials is the process by which we decompose a polynomial expression into the form of the product of its irreducible factors, such that the coefficient of the factors are in the same domain as that of the main polynomial. There are different techniques that can be followed for factoring polynomials, given as method of common factors, grouping method, factoring by splitting terms, factoring using algebraic identities.

Based on the complexity of the given polynomial expression, we can follow any of the above-given methods.

A polynomial equation is an equation formed with variables, exponents, and coefficient together with operations and an equal sign. (9) ... Some examples of polynomial equations are

$$x^2 + 3x + 2 = 0,$$
$$x^3 + x + 1 = 0,$$
$$x + 7 = 0, \text{ etc.}$$

Solving polynomial equations is to nd the value of the variable that satis es the equation.

The general expressions containing variables of varying degrees, coefficient positive exponents, and constants are known as ***polynomial functions***. (10) ... Here are some examples of polynomial functions:

$$f(x) = x^2 + 4,$$
$$g(x) = -2x^3 + x - 7,$$
$$h(x) = 5x^4 + x^3 + 2x^2.$$

Solving polynomials means finding the roots or zeros of the polynomials. We can apply different methods to solve a polynomial depending upon the type of the polynomial, whether it is a linear polynomial, quadratic polynomial, and so on. Let us first understand what is meant by the zero of a polynomial

https://www.cuemath.com/algebra/polynomials/

Task 5. ***Give the subheadings to the parts of the text you have read.***

A. Standard Form of Polynomials.
B. Degree of a Polynomial.
C. Types of Polynomials.
D. Polynomials.
E. Polynomial Examples.
F. Operations on Polynomials.
G. Terms of a Polynomial.

Task 6. A. ***Make a summary of your part of the text and present to your partners in groups of 3.***

B. ***Listen to your partner's summary and fill in columns 3 and 4 of the chart in Task 1.***

Task 7. ***Read the text once again and fill in the gaps with sentences from the list below. There are 2 extra sentences.***

1. As discussed above, the rules for the subtraction of polynomials are very similar to subtracting two numbers.
2. Finally, write the term with the exponent of the variable as 0, which is the constant term.
3. To avoid committing an error that tips the equation out of balance, make sure that any change on one side of the equation is reciprocated on the other side.
4. Terms that have different variables and/or different powers are known as unlike terms.
5. An algebraic expression is made up of terms, there can be one or more than one term present in the expression.
6. The general form of a polynomial equation is $P(x) = a_n x^n + \ldots + a_1 x + a_0$.
7. Let us understand this by taking an example: $3x^2 + 5$. In the given polynomial, there are certain terms that we need to understand.
8. The only difference is that when you are adding, you align the appropriate place values and carry the operation out.
9. To calculate the degree in a polynomial with more than one variable, add the powers of all the variables in a term.
10. In polynomials, the exponents of each of the variables should be a whole number.
11. In other words, a polynomial function is a function whose definition is a polynomial.
12. A binomial is a type of polynomial that has two terms.

Task 8. ***In pairs, answer the questions below.***

1. What is a polynomial?
2. What is known as the degree of polynomial?
3. How are the terms of polynomials defined
4. What are like terms in polynomials?
5. How many types of polynomials are there? What are they?

6. What operations can be performed on polynomials? Name them.
7. What is the factorization of polynomials?
8. What is a polynomial equation?
9. How do we solve a polynomial?

Task 9. ***Discuss the following cases in mini-groups.***

GROUP 1
Basic algebraic operations performed on polynomials
- Addition of polynomials
- Subtraction of polynomials
- Multiplication of polynomials
- Division of polynomials

GROUP 2
Methods for dividing polynomials
- Long division of polynomials
- Synthetic division

GROUP 3
Techniques for factoring polynomials
- Method of Common Factors
- Grouping Method
- Factoring by Splitting Terms
- Factoring Using Algebraic Identities

Task 10. ***Have a panel discussion. Share your ideas on the questions below.***

1. Some people argue that learning about polynomials is not necessary for everyday life. To what extent do you agree or disagree with this opinion?
2. The use of polynomials in data analysis has revolutionized industries such as finance and marketing. Is this a positive or negative developme t?
3. Many students struggle with understanding and applying polynomial concepts. What are some effective methods to help students improve their understanding of polynomials?
4. The concept of polynomials has been around for centuries, yet it continues to be relevant and useful in various fields. To what extent do you think polynomials will continue to play a role in the future?
5. "Polynomial" may seem like a daunting term, but it actually has many practical applications in our daily lives. Discuss some ways in which polynomials are used in everyday situations.

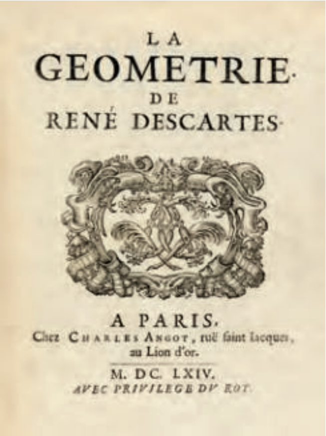
Task 11. ***Make a presentation about polynomials and their possible applications.***

PROBLEM-SOLVING

POLYNOMIALS: EASY, YET THE MOST DIFFICULT

Task 1. ***Read the introduction to the problem-solving and answer the questions in bold.***

Polynomials appear in various areas of mathematics and science. They form equations that encode problems from basic to complex sciences. Polynomial functions appear in settings ranging from chemistry and physics to economics and social science. Polynomials are also used in calculus and numerical analysis to approximate other functions. In advanced mathematics, polynomials are used to construct polynomial rings and algebraic varieties, central concepts in algebra and algebraic geometry.



The concept of polynomials has been around since ancient times, but the first recorded use of the term dates back to 1637 when René Descartes published La Géométrie.

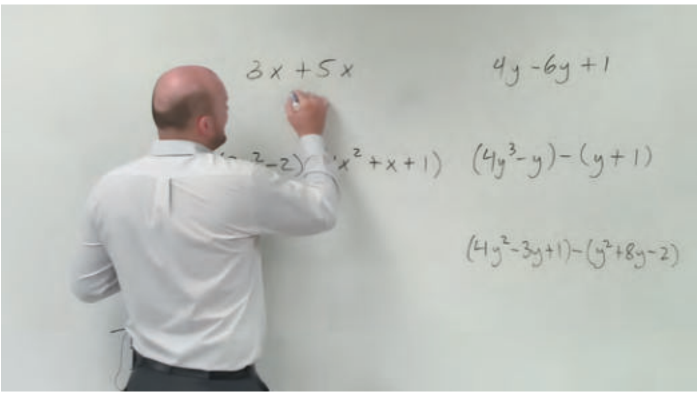
How did the concept polynomials evolve? Can you name mathematicians who have contributed to our understanding of polynomials?

Task 2. ***Discuss in small groups the questions below.***

What is the easiest math operation on polynomials? Why?
What is the most difficult math operation on polynomials? Wh

Task 3. A. ***Work in groups of three. Watch one of the videos below and fill in the table.***

Student A
Adding and subtracting polynomials



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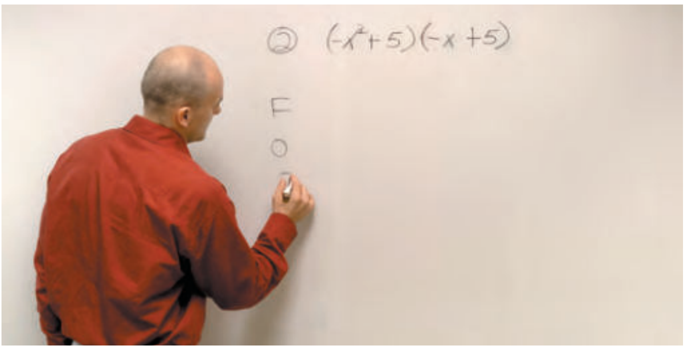
Student B
Multiplying polynomials



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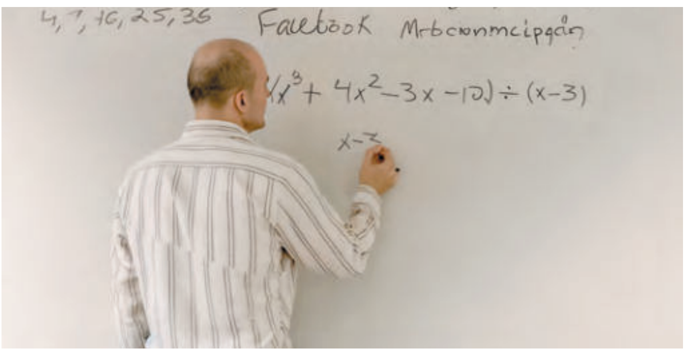
Student C
Dividing polynomials



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Type of operation on polynomials	Methods of doing operations	Examples

B. ***Share the information with your groupmates.***

C. ***Fill in the table above with the information your groupmates shared with you.***

Task 4. ***Study the table thoroughly and make your supposition on the easiest and the most difficult operations on polynomials. Support your opinion with information presented in the table and your own experience in solving polynomials. Get ready to deliver a 60 second speech presenting your ideas.***

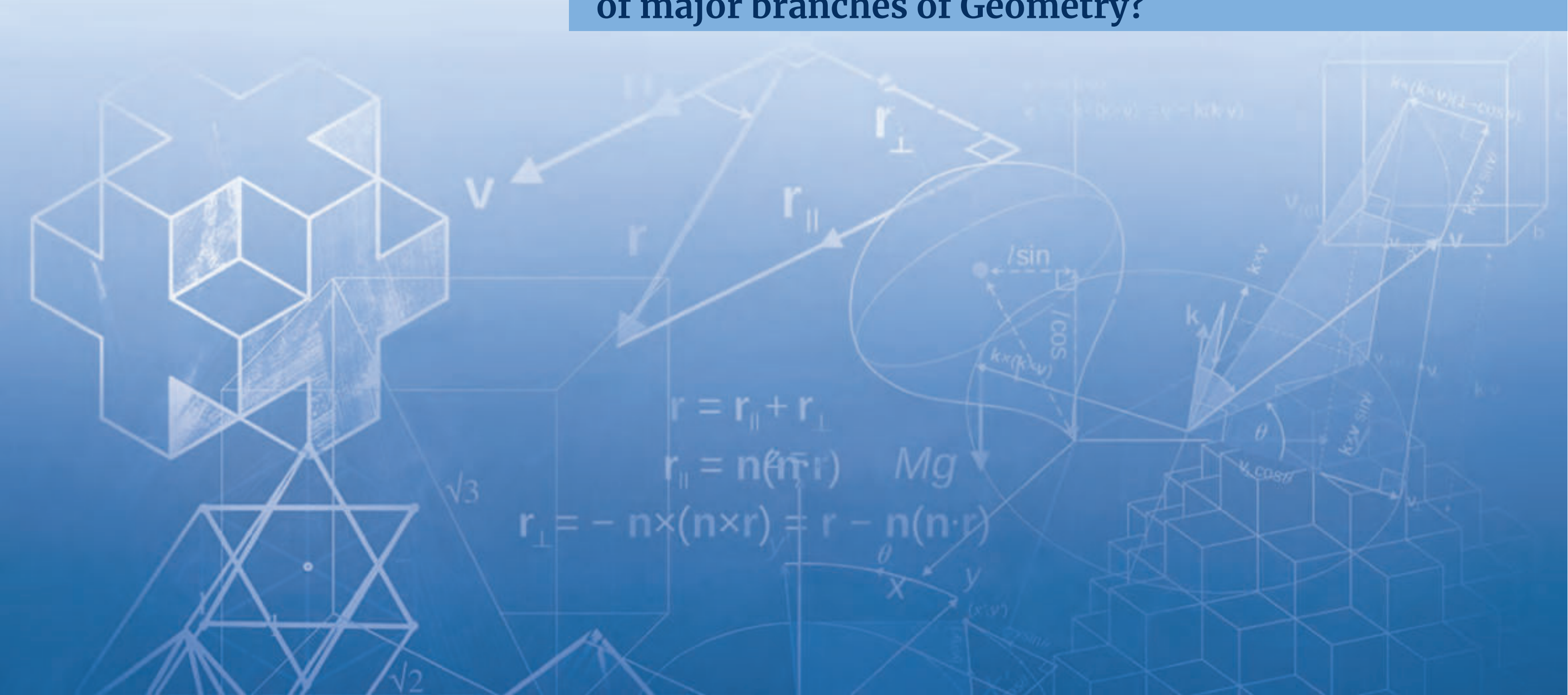
Task 5. ***Hold a class discussion, sharing your 60-second speeches.***

Task 6. ***Analyze the presentations with the view of identifying the most difficult and the easiest math operation on polynomials, based on your groupmates' learning experience and statistics.***

GEOMETRY 4 UNIT

In this unit you'll

- **examine** the history of Geometry
- **look closely at** points, lines, planes and angles
- **focus on** polygons
- **study** triangles
- **write an essay about** the importance of Euclidean Geometry
- **make a presentation about** one of the mathematicians
- **solve the problem** Geometry: What are the peculiarities of major branches of Geometry?



ACTIVE VOCABULARY

VERBS

to contribute	to mark
to convince	to obliterate
to defin	to obtain
to employ	to preserve
to extend	to quote
to generalize	to resemble
to give rise	to trisect

COLLOCATIONS

acute angle	complementary points
collinear points	point of concurrency
conic section	practical knowledge
flat surfac	precise reasoning
infinite point	reflex angl
line segment	right triangles
oblique line	theory of incommensurability
obtuse angle	three-dimensional object

ADJECTIVES

complementary	perpendicular
inaccessible	precious
intersecting	relevant
measurable	significan
particular	similar

NOUNS

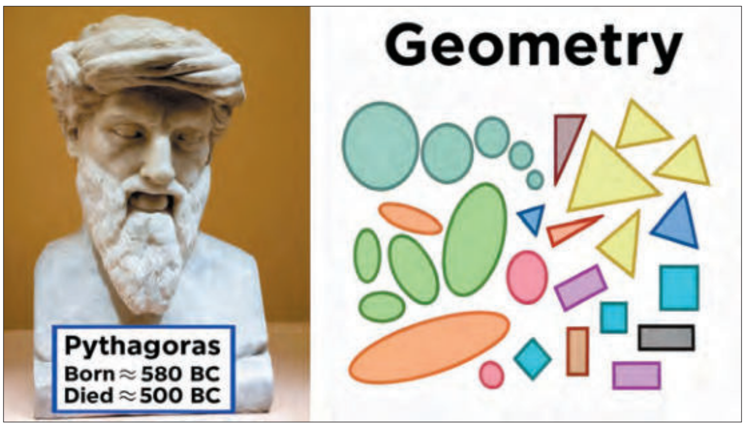
calculus	height	reference
circle	intact	rotation
dimension	length	shape
doctrine	predecessors	space
endpoint	property	sublime
excavation	protractor	value
exemplar	radian	vertex
gradian	rectangle	width

BUILDING THE CONCEPT

HISTORY OF GEOMETRY

Task 1. A. ***What is Geometry? How did Geometry evolve? In pairs make a list of the facts you can think of.***

B. ***Watch the video "Introduction to Geometry: Ancient Greece and the Pythagoreans" to check your ideas.***



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Task 2. ***Do the quiz (sometimes more than one variant is possible). Answer the questions. Watch the video again if needed.***

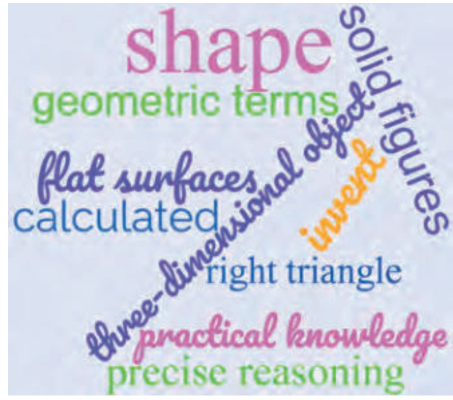
1. What civilization produced some of the most astounding leaps of intellect in the history of species?
   A. Ancient Greece.
   B. Ancient Egypt.
   C. Ancient China.
   D. Ancient Maya.
2. What fields did the Greeks revolutionize
   A. Government.
   B. Philosophy.
   C. History.
   D. Mathematics.
3. Whose work was one of the best in mathematics and influenced gr at thinkers?
   A. Socrates.
   B. Pythagoras.
   C. Plato.
   D. Euclid.

4. How was geometry derived?
   A. From sun shadows on the ground.
   B. From drawing pictures in the sand.
   C. From the low tide's lines.
   D. From the lines in the caves.
5. What shapes did great thinkers draw?
   A. Rectangles.
   B. Circles.
   C. Rombus.
   D. Triangles.
6. How was mathematics defined
   A. The study of questions that have completely objective answers.
   B. The study of questions that have completely subjective answers.
   C. The study of questions that have incompletely objective answers.
   D. The study of questions that have incompletely subjective answers.
7. To what kinds of questions does mathematics give answers?
   A. Debated questions.
   B. Questions that are inarguably true.
   C. Questions with no definite answers
   D. Questions with multiple correct answers.
8. Who are the Pythagoreans?
   A. Pythagoras' kids.
   B. The members of Pythagoras' community.
   C. Inhabitants of mount Pytha.
   D. Pythagoras' opponents.

Task 3. ***Match the words with their definitions.***

1. Shape	a) area of an object that are level and smooth, without any curves or bumps.
2. Invent	b) determined by using mathematical methods or formulas.
3. Practical knowledge	c) three-dimensional objects that have length, width, and height, such as cubes, spheres, and cylinders.
4. Solid figure	d) words used to describe shapes and their properties, such as angles, sides, and vertices.
5. Right triangle	e) using careful and accurate thinking to come to a conclusion or solve a problem.
6. Calculated	f) to create or design something new.
7. Flat surface	g) the form or outline of an object, often described as round, square, triangular, etc.
8. Precise reasoning	h) an object that has length, width, and height, and takes up space.
9. Three-dimensional object	i) a triangle with one angle measuring 90 degrees.
10. Geometric terms	j) information or skills that can be applied in real-life situations.

Task 4. ***Complete the sentences with the words from the box.***



1. Pythagoras had also spoken about the fiv … … called the mathematical solids.
2. In 1917, Einstein's novel equations of space-time had … … on the left and energy on the right.
3. The mean radius was … as the average of the maximal and minimal radius values.
4. A triangle is a … with three sides.
5. The surface of walls, oor, top of a table are examples of … … .
6. A cone is a … … or solid, which has a circular base and has a single vertex.
7. As a religious scholar and teacher, he was well known for his … and robust approaches.
8. If a … … is inscribed in a circle, then the hypotenuse is a diameter of the circle.
9. It allows students to increase their … …, primarily through laboratory work.
10. You can … your own wheel, but you might not have to.

Task 5. ***Read the text "History of Geometry" to find out more about the history of Geometry.***

HISTORY OF GEOMETRY

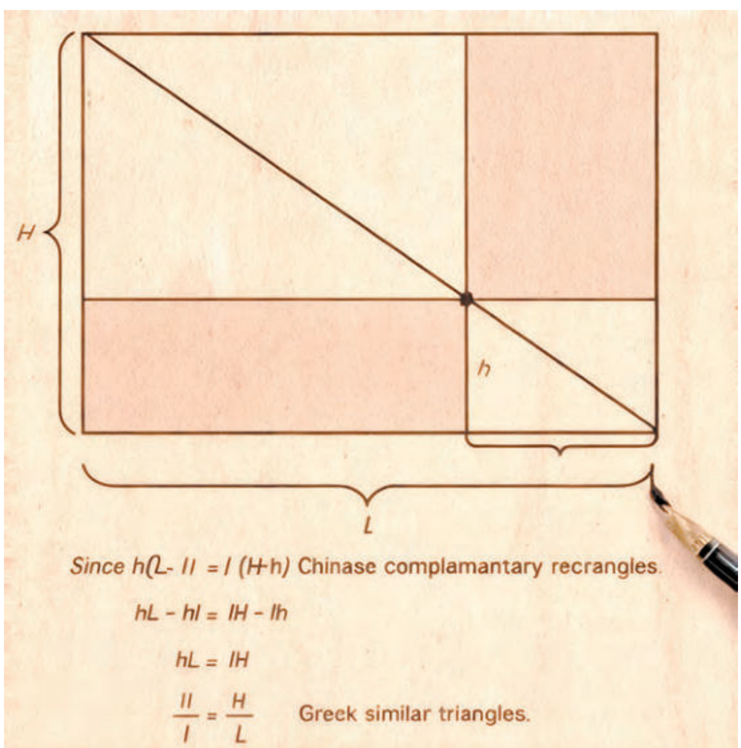
Geometry is a branch of mathematics concerned with the shape of individual objects, spatial relationships among various objects, and the properties of surrounding space. It is one of the oldest branches of mathematics, having arisen in response to such practical problems as those found in surveying, and its name is derived from Greek words meaning "Earth measurement". Eventually it was realized that geometry need not be limited to the study of flat surfaces (plane geometry) and rigid three-dimensional objects (solid geometry) but that even the most abstract thoughts and images might be represented and developed in geometric terms.

The origin of geometry lies in the concerns of everyday life. The traditional account, preserved in *Herodotus's* "*History*", credits the Egyptians with inventing surveying in order to reestablish property values after the annual flood of the Nile. Similarly, eagerness to know the volumes of solid figures derived from the need to evaluate tribute, store oil and grain, and build dams and pyramids. Even the three abstruse geometrical problems of ancient times—to double a cube, trisect an angle, and square a circle, all of which will be discussed later—probably arose from practical matters, from religious ritual, timekeeping, and construction, respectively, in pre-Greek societies of the Mediterranean. And the main subject of later Greek geometry, the theory of conic sections, owed its general importance, and perhaps also its origin, to its application to optics and astronomy.

While many ancient individuals, known and unknown, contributed to the subject, none equaled the impact of Euclid and his "*Elements*" of Geometry. It was so complete and clearly written that it literally obliterated the work of his predecessors. Greek geometry before him comes primarily from bits quoted by Plato and Aristotle and by later mathematicians and commentators. Among other precious items they preserved are some results and the general approach of *Pythagoras* (*c.* 580—*c.* 500 BCE) and his followers. *The Pythagoreans* convinced themselves that all things are, or owe their relationships to, numbers. The doctrine gave mathematics supreme importance in the investigation and understanding of the world. Plato developed a similar view, and philosophers influenced by Pythagoras or Plato often wrote ecstatically about geometry as the key to the interpretation of the universe. Thus, ancient geometry gained an association with the sublime to complement its earthy origins and its reputation as the exemplar of precise reasoning.

Ancient builders and surveyors needed to be able *to construct right angles* in the field on demand. The method employed by the Egyptians earned them the name “rope pullers” in Greece, apparently because they employed a rope for laying out their construction guidelines. One way that they could have employed a rope to construct right triangles was to mark a looped rope with knots so that, when held at the knots and pulled tight, the rope must form a right triangle. The simplest way to perform the trick is to take a rope that is 12 units long, make a knot 3 units from one end and another 5 units from the other end, and then knot the ends together to form a loop.

By ancient tradition, Thales of Miletus, who lived before Pythagoras in the 6th century BCE, invented a way to measure *inaccessible heights*, such as the Egyptian pyramids. Although none of his writings survives, Thales may well have known about a Babylonian observation that for similar triangles (triangles having the same shape but not necessarily the same size) the length of each corresponding side is increased (or decreased) by the same multiple. The ancient Chinese arrived at measures of inaccessible heights and distances by another route, using “complementary” rectangles, as seen in the next figure, which can be shown to give results equivalent to those of the Greek method involving triangles.



A Babylonian cuneiform tablet written some 3,500 years ago treats problems about dams, wells, water clocks, and excavations. It also has an exercise on circular enclosures with an implied value of $\pi = 3$. The contractor for King Solomon's swimming pool, who made a pond 10 cubits across and 30 cubits around (1 Kings 7:23), used the same value. However, the Hebrews should have taken their π from the Egyptians before crossing the Red Sea, for the Rhind papyrus (*c.* 2000 BCE; our principal source for ancient Egyptian mathematics) implies $\pi = 3.1605$.

Knowledge of the area of a circle was of practical value to the official who kept track of the pharaoh's tribute as well as to the builders of altars and swimming pools. Ahmes, the scribe who copied and annotated the Rhind papyrus (*c.* 1650 BCE), has much to say about cylindrical granaries and pyramids, whole and truncated. He could calculate their volumes, and, as appears from his taking the Egyptian *seked*, the horizontal distance associated with a vertical rise of one cubit, as the defining quantity for the pyramid's slope, he knew something about similar triangles.

https://www.britannica.com/science/geometry

Task 6. ***In pairs, answer the following questions.***

1. What is geometry?
2. Where does the origin of geometry lie?
3. What book obliterated the work of all predecessors?
4. Who convinced themselves that all things are numbers?
5. Who needed to be able to construct right angles in the field o demand?
6. What method did the Egyptians employ to construct right angles?
7. To whom was knowledge of the area of a circle of practical value?

Task 7. ***Discuss with your partner if the statements below are True or False.***

1. Geometry is the branch of mathematics concerned with the shape of individual objects, spatial relationships among various objects, and the properties of surrounding space.
2. Word "geometry" is derived from Arabic words meaning "Earth measurement."
3. The origin of geometry lies in the concerns of everyday life.
4. Eagerness to know the volumes of solid figures derived from the need to evaluate tribute, store oil and grain, and build dams and pyramids.
5. The main subject of later Algebra, the theory of conic sections, owed its general importance, and perhaps also its origin, to its application to optics and astronomy.
6. The method employed by the Egyptians earned them the name "rope pullers" in Greek, apparently because they didn't employ a rope for laying out their construction guidelines.
7. By ancient tradition, Thales of Miletus, who lived before Pythagoras in the 6th century BCE, invented a way to measure inaccessible heights, such as the Egyptian pyramids.
8. A Babylonian cuneiform tablet written some 500 years ago treats problems about dams, wells, water clocks, and excavations.

Task 8. A. ***Choose one of the topics below. Get ready to speak on it while taking part in a talk show.***

1. My personal journey of discovering the beauty and complexity of geometry.
2. How studying the history of geometry has helped me understand its relevance in modern society.
3. The impact of ancient civilizations on the development of geometric principles.
4. My favorite historical figure in the field of geometry and their contributions to the subject.
5. The role of geometry in shaping my problem-solving skills and critical thinking abilities.

B. ***Hold the talk show. Share your ideas on one of the topics above.***

Task 9. ***Make a presentation about one of the mathematicians and their contribution to Mathematics: Euclid, Pythagoras, Plato, Aristotle, Ptolemy, Archimedes.***

POINTS, LINES, PLANES AND ANGLES

Task 1. A. ***How well do you know points, lines, planes and angles?***

1. How is a point typically represented in geometry:
   a) With a dot and a capital letter;
   b) With a line and two dots;
   c) With a word and a lowercase letter;
   d) With an arrow and a number?

2. How can a line be named:
   a) by the letter that represents the line as a whole;
   b) by three points that fall on the line;
   c) by a combination of letters and numbers;
   d) by the direction it extends in?

3. What is a plane in geometry:
   a) a curved surface that extends forever;
   b) a flat surface that extends forever in all directions;
   c) a line that intersects with other lines;
   d) a point that connects multiple lines?

4. How can a plane be named:
   a) by a single letter;
   b) by two points on the plane;
   c) by three points on the plane;
   d) by the direction it extends in?

5. What is the difference between a line and a segment:
   a) a line has a definite beginning and end, while a segment extends forever;
   b) a line extends forever in both directions, while a segment has a definite beginning and end
   c) a line is curved, while a segment is straight;
   d) a line is named with a letter, while a segment is named with two points?

6. How is a ray di erent from a line or a segment:
   a) a ray has a definite beginning and end, while a line extends forever;
   b) a ray extends forever in one direction only, while a line extends in both directions;
   c) a ray is curved, while a line is straight;
   d) a ray is named with a letter, while a line is named with?

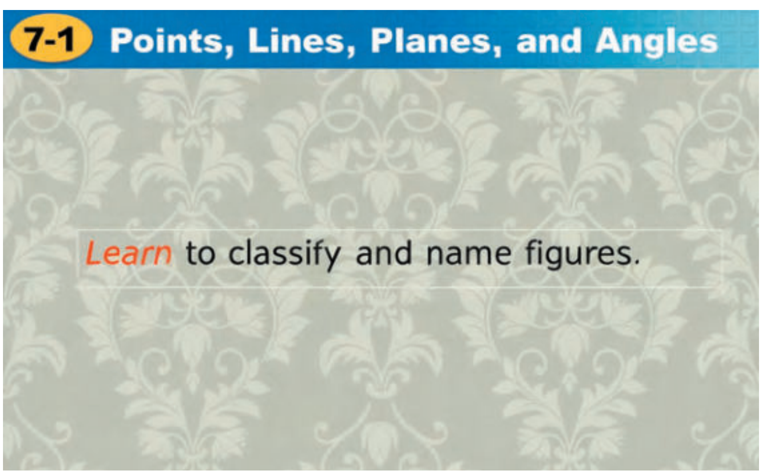
7. How is an angle formed:
   a) by two lines that intersect;
   b) by two points that connect;
   c) by two rays with a common endpoint;
   d) by two planes that overlap?

8. How can an angle be named:
   a) by the three letters that make up the angle;
   b) by the number that represents the angle;
   c) by the vertex of the angle;
   d) by any letter in the angle?

9. What does it mean when angles are supplementary:
   a) they add up to 90 degrees;
   b) they add up to 180 degrees;
   c) they add up to 270 degrees;
   d) they add up to 360 degrees?

10. What is the measure of a right angle:
   a) 45 degrees;
   b) 90 degrees;
   c) 135 degrees;
   d) 180 degrees?

B. ***Watch the video to check your ideas.***



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Task 2. ***Answer the questions. Watch the video again if needed.***

1. How can angles be named?
2. What is the measure of angle one?
3. What is the relationship between angles that form a straight line?
4. What are angles that add up to 180 degrees called?
5. How many degrees are there in a circle?
6. Define a right angle
7. Define an acute angle
8. Define an obtuse angle
9. What are complementary angles?
10. How can congruent angles be identified

Task 3. A. ***Match the words (1—10) with the options (a—j) to make mathematical collocations.***

1. Infinit	a) location
2. Intersecting	b) segment
3. Angle	c) points
4. Vertical	d) concurrency
5. Line	e) lines
6. Interior	f) measurement
7. Geometrical	g) planes
8. Capital	h) angle
9. Particular	i) shape
10. Point of	j) letters

B. ***Fill in the gaps with the appropriate word collocations from the task above.***

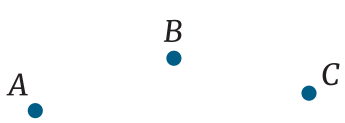
1. … … planes that are not parallel and they always intersect along a line.
2. There are two units of … …: degrees and radians.
3. In mathematics … … are often used to represent the y-axis on the coordinate plane.
4. Formally, the shortest … … between a vertex of a triangle and the opposite side.
5. A triangle has three … … .
6. Lines are called concurrent if they all intersect and the … … … of the three medians is called a centroid.
7. Every bounded … … set has at least one limited point.
8. The original manuscript has neither numerals nor … … .
9. Some of the … … examples are circle, rectangle, triangle, etc.
10. Points are used to represent a … … in diagrams and graphs.

Task 4. ***Read the text to find out more about the geometric terms.***

POINTS, LINES, PLANES AND ANGLES

Everything around us comprises geometrical shapes.

A **point** is an exact location in space. A point does not have width, length, or height and so they do not have any dimension.



Points are used to represent a particular location in diagrams and graphs. They are usually labeled in capital letters. In the diagram below, we have three points and they are labelled A, B, and C.

Lines are formed by infinite points that extend on both sides

Unlike points, lines have length and so they are one-dimensional objects, that extend on both sides infinitely

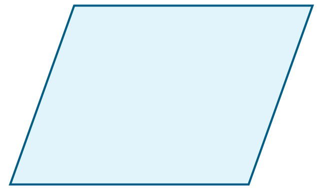
Usually, when there are no points on a line it is represented by script letters, such as r, s, and t.

A line which has a start point but no endpoint is called a **ray**.

The ray containing two points A and B, with A as the starting point is represented by $\overrightarrow{AB}$.

A line which has both a start and end points is called a **line segment**.

The segment between the points A and B is written as $\overline{AB}$.



Planes can be thought of as infinitely many intersecting lines that extend forever in all directions.

Planes can have both length and width and so they are two-dimensional objects. Planes are also represented by capital letters.

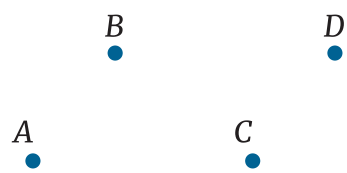
Types of points

In this subsection, we will learn about collinear, non-collinear, coplanar, non-coplanar points, and point of concurrency.

We say 3 or more points are ***collinear*** if they all lie on a straight line.

Otherwise, they are ***non-collinear***.

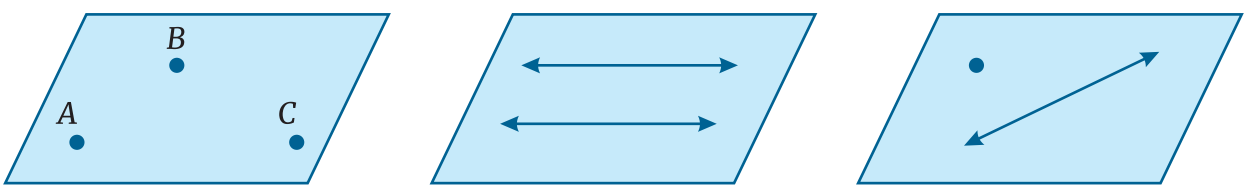
In the diagram above points A, B, C and D all lie in the same line and so they are collinear points.



Now, in the diagram above there is no line that could be drawn connecting all the four points A, B, C and D. Therefore, they are non-collinear points.

Now, non-collinear points open up the world of Geometry even more.

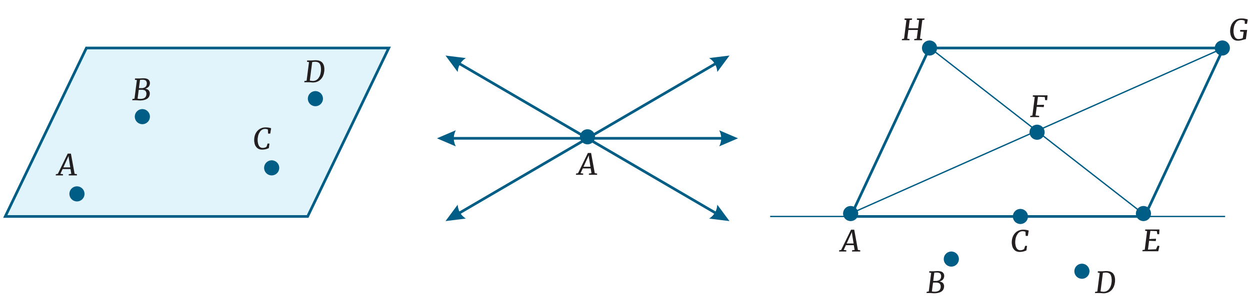
Given three non-collinear points we can draw exactly one plane which contains all of the three points. Also given a line and a point, only one plane can contain both of them. Similarly, given two parallel lines only one plane can contain all of them.



If a set of points lie on the same plane, they are called ***coplanar points***. Otherwise they ***non-coplanar points***.

If two or more lines meet at a point, it is called the ***point of concurrency***.

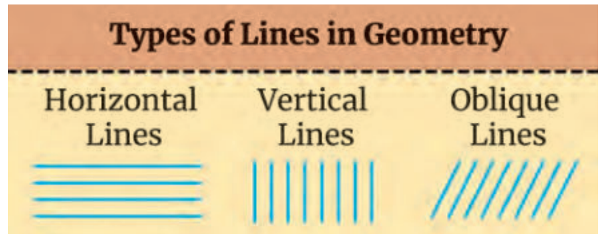
In the diagram below we have some points in 3 dimensions.



Types of lines

When lines are straight, we can categorise lines as one of the three below:

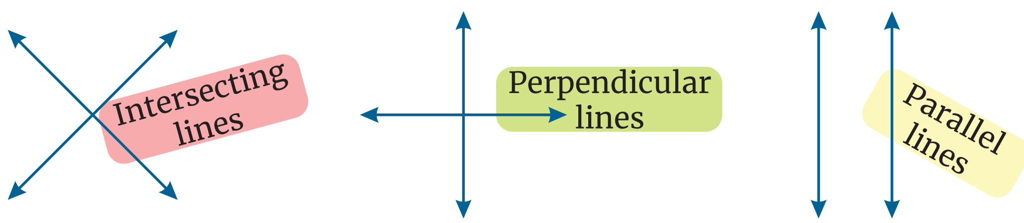
- ***horizontal lines*** go from left to right. In a cartesian diagram it runs along or parallel to the X-axis;
- ***vertical lines*** go up and down. In a cartesian diagram it runs along or parallel to the Y-axis;
- straight lines that are not vertical or horizontal are called ***oblique lines.***



When we have two lines then they either intersect or do not intersect at any point. Depending on this we have ***parallel*** and ***intersecting lines.***

We say two lines are ***parallel*** if they do not have any point of intersection.

If two lines intersect, then they are ***intersecting lines***.



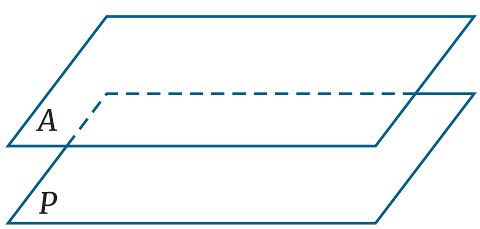
When two lines intersect they intersect at a point. In particular, if the angle between the two lines is 90°, then they are called ***perpendicular lines***.

Types of planes

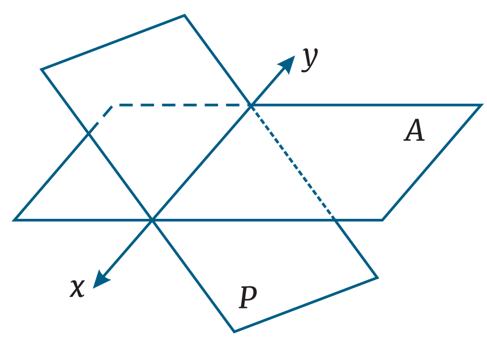
When two planes never intersect each other they are called ***parallel planes***.

Otherwise, they are called ***intersecting planes.***

Parallel planes



Intersecting planes



When two planes intersect they intersect along a line. And, similarly to lines, planes can also intersect at an angle of 90°, which are called ***perpendicular planes.***

Angles

An angle is de ned as the gure that is formed when two rays meet at a common point. Angles are denoted by $\angle$ or commonly by Greek letters. They can be either measured in degrees, radians, or gradians.

An angle is defined by two fundamentals

- The arms of the angle are the two rays that come together to form the angle.
- The vertex is the point where the two rays meet to form the angle.

There are six **types** of angles and these types are identified by their measures

- *Acute angles* are angles that are below 90° and above 0°.
- *Right angles* are angles that are exactly 90°.
- *Obtuse angles* are angles that measure above 90° and measure below 180°.
- *Straight angles* are angles that measure exactly 180°. Below is a straight line. It means that the arms of the angles are opposite to each other.
- *Refle angles* are angles that measure above 180° and measure below 360°.
- *Complete angles* are angles that measure 360°.

Protractors are used **to measure angles**. Just like most quantities, angles require a unit to be measured in. These are in degrees, radians, and gradians.

Degrees are denoted by the symbol °. An angle is said to be 1° if its rotation from the reference goes as much as $\frac{1}{360}$. By this we can establish that complete angles around a point are 360° always.

The ***radian*** measurement of angles is usually common in calculus. This is the SI unit of angles. It is denoted by 'rad'. The rotation around a point (complete angle) measures 2π radians. This means that 2π radians = 360° and π radian = 180°.

Gradian of angles is the least commonly used in mathematics. It is denoted by 'grad'. The rotation around a point (complete angle) measures 400 grad. This also means that $1\,\text{grad} = \frac{1}{400}$ of a full rotation. This also extensively means that a straight angle equals 200 grad.

https://www.splashlearn.com/math-vocabulary/geometric-shapes

Task 5. ***In pairs, answer the following questions.***

1. What is a point? How is it usually labelled?
2. What types of points are there?
3. What is a line? What types of lines are there?
4. What are planes? What can planes have?
5. What types of planes are there?
6. How is an angle defined
7. What types of angles are there?
8. How can angles be measured?
9. What units do they require to be measured in?

Task 6. ***Match the words from the box with their meanings.***

1. A device used for measuring and drawing angles.
2. A measurement of something in a particular direction, especially its height, length, or width.
3. The space between two lines or surfaces at the point at which they touch each other, measured in degrees.
4. Connected with what is happening or being discussed.
5. To make a picture of something or someone with a pencil or pen.
6. A picture that shows how two sets of information or variables are related usually by lines or curves.

7. An area of advanced mathematics in which continuously changing values are studied.
8. The point where two lines meet to form an angle or the point that is opposite the base of a shape.
9. To add to something in order to make it bigger or longer.
10. The area around everything that exists, continuing in all directions.

Task 7. ***Make a summary of the text (8—10 sentences).***

Task 8. ***Agree or disagree with the statements below. Prove your ideas.***

1. Everything around us comprises geometrical shapes.
2. A point has width, length, or height and so they do not have any dimension.
3. Planes can be thought of as infinitely many intersecting lines that extend forever in one direction.
4. If a set of points lie on the same plane, they are called collinear points.
5. If two or more lines meet at a point, it is called the point of concurrency.
6. Lines can be straight and curved.
7. We say two lines are parallel if they have a point of intersection.
8. Right angles are angles that are exactly 180°.
9. Straight angles are angles that measure exactly 90°.
10. Angles possess certain properties with regard to parallel and intersecting lines.

Task 9. ***Discuss the questions in pairs.***

1. What is the difference between a point and a line in geometry
2. Can you give an example of a real-life object that can be represented as a plane?
3. In what ways are angles used in everyday life?
4. Can you name any famous mathematicians who have contributed to our understanding of points, lines, planes, and angles?
5. How do we measure the length of a line segment?
6. Are there any shapes that only have straight lines and no curves?
7. What is the relationship between parallel lines and transversals?
8. Can you explain how to find the midpoint of a line segment using a ruler and compass?

Task 10. ***Read the passages below. Whose opinion do you support? Why?***

Julie:

I think understanding the concept of points, lines, planes, and angles is crucial for anyone in their daily lives. For example, when I need to build furniture, I need to make sure all the angles are measured correctly, otherwise, the pieces won't fit together properly. Knowing about angles helps me create sturdy and stable structures.

Mike:

To be honest, I never really paid attention to geometry class at school and I still turned out just fine. I don't see the point in learning about points, lines, planes, and angles unless you want to pursue a career in some type of math-related field. It's just not practical or applicable in my everyday life.

Aisha:

I can't decide whether points, lines, planes, and angles are important or not. On one hand, I understand how it can be useful in certain situations, but on the other hand, I feel like most people never use this knowledge outside of school. I guess it's good to have a basic understanding, just in case.

Timothy:

Oh my gosh, I absolutely love geometry! I find angles so fascinating because they show us how everything is connected and has a specific relationship with each other. It blows my mind how we can use angles to solve complex problems and create amazing structures.

Task 11. ***Be ready to deliver a lecture for students about points, lines, planes and angles.***

POINTS IN EUCLIDEAN GEOMETRY

There is no royal road to Geometry.

Euclid

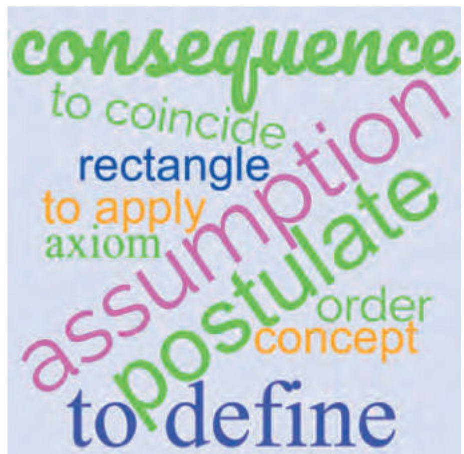
Task 1. A. ***Comment on the quotations above and answer the questions.***

1. Who is Euclid?
2. What book did he write?
3. What fundamental concepts did he explain?

B. ***Watch the video "What's the point of Geometry?" to check your ideas.***

https://drive.google.com/file d/1wAwtuNE-k9th4xfnjnUN_ hZB6VmJ_4GW/view?usp=sharing

Task 2. A. ***Match the words from the box with their meanings.***



1. To suggest a theory, idea, etc. as a basic principle from which a further idea is formed or developed.
2. Something that you accept as true without question or proof.
3. To come together in position or happen at or near the same time.
4. To make use of something or use it for a practical purpose.
5. A formal statement or principle in mathematics from which other statements can be obtained.
6. The way in which people or things are arranged, either in relation to one another or according to a particular characteristic.
7. A flat shape with four 90° angles and four sides, with opposite sides of equal length.
8. The condition of having a lasting effect; importance
9. An idea, theory, etc. about a particular subject.
10. To describe the meaning of something, esp. a word, or to explain something more clearly so that it can be understood.

B. ***Complete the sentences using the right form the words from the box.***

1. Both parallel lines and perpendicular lines do not ... with each other.
2. Put the files in alphabetical .
3. It was the Greek astronomer, Ptolemy, who ... that the Earth was at the centre of the universe.
4. The procedure ... to the 3 m focal length off-axis parabola
5. The math student teachers discovered the importance of drawing from current events to ground mathematical ... .
6. They decided to go ahead on the ... that the figures they had been given were accurate.
7. They are all straightforward ... of the definitions
8. Before I answer your question, could you ... your terms a little more?

9. The area of a ... is its height times its width.
10. Euclid's ... form the foundation of his system of geometry.

Task 3. ***Read the text to find out more about points in Euclidean Geometry.***

POINTS IN EUCLIDEAN GEOMETRY

Euclid is often considered to be the 'father' of geometry, with his fundamentals of Euclidean geometry serving as a basis for much of 'modern mathematics' understanding of geometry. Certain questions had to be answered to define these fundamental constituents of the field, such as *what is a point*, or *what is a line*?

Euclid's geometry or the Euclidean geometry is the study of Geometry based on the undefined terms such as points, lines, and planes of flat spaces. In other words, it is the study of geometrical shapes both plane shapes and solid shapes and the relationship between these shapes in terms of lines, points, and surfaces.

Euclid made five **fundamental postulates** when delving into the field of geometry. These postulates were fundamental principles of geometry that he held to be self-evident, and informed all further principles and concepts of geometry thereafter.

Postulates of Euclidean Geometry

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a str ight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as centre.
4. All right angles are congruent.
5. Given a line and a point not on that line, there exist an infinite number of lines through the given point parallel to the given line.

Euclid's axioms or common notions are the assumptions of the obvious universal truths that have not been proven. But in his book, "Elements", Euclid wrote a few axioms or common notions related to geometric shapes. Let us take a look:

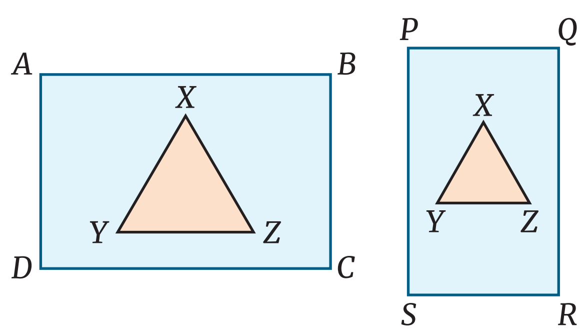
Axiom 1. Things that are equal to the same thing are equal to one another.

Suppose the area of a rectangle is equal to the area of a triangle and the area of that triangle is equal to the area of a square. After applying the first axiom, we can say that that the area of the triangle and the square are equal. For example, if $p = q$ and $q = r$, then we can say $p = r$.

Axiom 2. If equals are added to equals, the wholes are equal.

Let us look at the line segment AB, where $AP = QB$. When PQ is added to both sides, then according to axiom 2, $AP + PQ = QB + PQ$ i.e. $AQ = PB$.

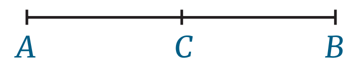
Axiom 3. If equals are subtracted from equals, the remainders are equal.



Consider rectangles $ABCD$ and $PQRS$, where the areas are equal. If the triangle XYZ is removed from both the rectangles then according to axiom 3, the areas of the remaining portions of the two triangles are equal.

Axiom 4. Things that coincide with one another are equal to one another.

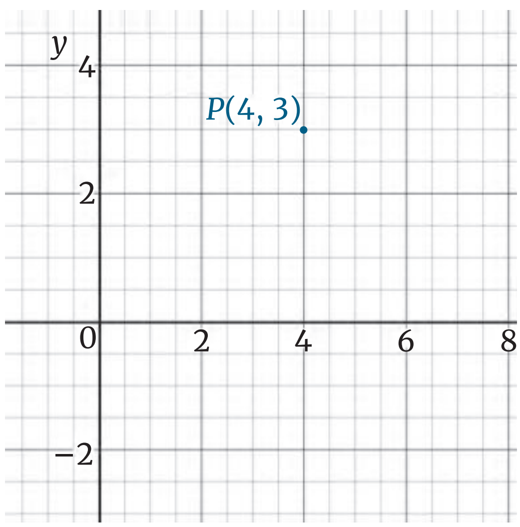
Consider line segment *AB* with C in the center. *AC* + *CB* coincides with the line segment *AB*. Thus by axiom 4, we can say that *AC* + *CB* = *AB*.



Axiom 5. The whole is greater than the part.

Using the same figure as above, *AC* is a part of *AB*. Thus according to axiom 5, we can say that *AB* > *AC*.

Axiom 6 and Axiom 7. Things that are double of the same things are equal to one another. Things that are halves of the same things are equal to one another.



Points are most often considered within the framework of Euclidean geometry, where they are one of the fundamental objects. Euclid originally defined the point vaguely, as "that which has no part". In two dimensional Euclidean space, a point is represented by an ordered pair (x, y) of numbers, where the fir t number conventionally represents the horizontal and is often denoted by x, and the second number conventionally represents the vertical and is often denoted by y.

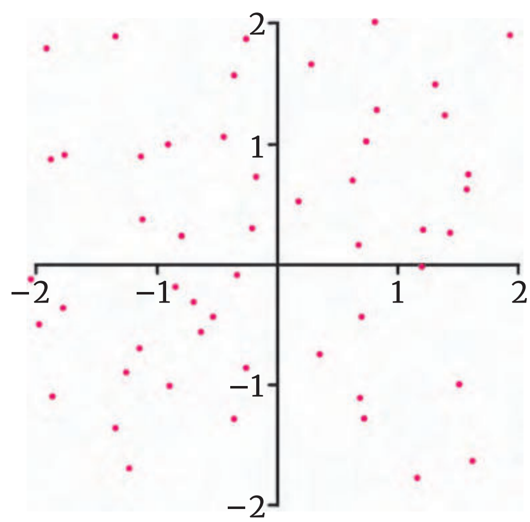
This idea is easily generalized to three dimensional Euclidean space, where a point is represented by an ordered triplet, (x, y, z) with the additional third number representing depth and often denoted by z. Further generalizations are represented by an ordered tuplet of n terms, $(a_1, a_2, \dots, a_n)$ where n is the dimension of the space in which the point is located.

Many constructs within Euclidean geometry consist of an infinite collection of points that conform to certain axioms. This is usually represented by a set of points.

A finite set of points (in red) in the Euclidean plane

As an example, a line is an infinite set of points of the for

$$L = \{(a_1, a_2, \dots, a_n) | a_1c_1 + a_2c_2 + \dots + a_nc_n = d\},$$



where c_1 hrough c_n and d are constants and n is the dimension of the space. Similar constructions exist that define the plane, line segment, and other related concepts. A line segment consisting of only a single point is called a degenerate line segment.

In addition to defining points and constructs related to points, Euclid also postulated idea about points; he claimed that any two points can be connected by a straight line. This is easily confirmed under modern developments of Euclidean geometry, and had lasting consequences at its introduction, allowing the construction of almost all the geometric concepts of the time. However, Euclid's postulation of points was neither complete nor definite, as he occasionally assumed facts about points that didn't follow directly from his axioms, such as the ordering of points on the line or the existence of specific points. In spite of this, modern developments of the system serve to remove these assumptions.

https://www.studysmarter.co.uk/explanations/math/geometry/fundamentals-of-geometry/

Task 4. ***In pairs, answer the following questions.***

1. What are the fundamentals of Geometry?
2. What main aspects does Euclid's geometry deal with?
3. What postulates of Euclidean Geometry are there?
4. What are Euclid's axioms?
5. How did Euclid originally define the point
6. How is a point represented in two dimensional Euclidean space?
7. What idea about points did Euclid claim?

Task 5. ***Discuss with your partner if the statements below are True or False.***

1. Euclid is often considered to be the 'father' of Geometry.
2. His fundamentals of Euclidean geometry serve as a basis for much of 'modern mathematics' understanding of geometry.
3. Euclid's geometry or the Euclidean geometry is the study of Geometry based on the defined terms such as points, lines, and planes of flat spa s.
4. Euclid's geometry deals with two main aspects - plane geometry and topology.
5. Euclid made three fundamental postulates when delving into the field of geometry.
6. All right angles are congruent.
7. Euclid's axioms or common notions are the assumptions of the obvious universal truths that have been proven.
8. Points are most often considered within the framework of Euclidean geometry, where they are one of the fundamental objects.
9. Many constructs within Euclidean geometry consist of an infinite collection of points that conform to certain axioms.
10. He claimed that not any two points can be connected by a straight line.

Task 6. ***Make a summary of the text (8—10 sentences).***

Task 7. ***In pairs, discuss the questions below. Google the necessary information if needed.***

1. How does Euclidean geometry differ from other types of geometry, such as non-Euclidean or projective geometry?
2. Can you explain the concept of a "point" in Euclidean geometry and how it relates to other geometric figures
3. In what ways can points be classified in Euclidean geometry, and how do these classifications affect their propertie

4. What is the significance of the distance between two points in Euclidean geometry, and how is it calculated?
5. How does the concept of a "line" intersect with the idea of a point in Euclidean geometry, and what are some examples of this intersection?
6. Can you give an example of a real-world application of Euclidean geometry involving points, lines, and distances?
7. How does the Pythagorean theorem relate to points in Euclidean geometry, and why is it considered a fundamental principle in this field
8. Are there any limitations or exceptions to the rules governing points in Euclidean geometry, and if so, how do they impact our understanding of this subject?
9. Can you discuss the role of coordinates and coordinate systems in representing points in Euclidean geometry, and how they aid in problem-solving?
10. How has the study of points in Euclidean geometry evolved over time, and what new developments have emerged in recent years?

Task 8. ***Discuss the cases below in small groups.***

1. Many people argue that Euclidean geometry is no longer relevant in today's world. To what extent do you agree or disagree with this opinion?
2. The study of Euclidean geometry can be challenging for some students. Is this a positive or negative development?
3. Some people believe that Euclidean geometry is only useful for academic purposes. Do you agree or disagree with this statement?
4. In recent years, there has been a decline in the number of students studying Euclidean geometry. What do you think are the reasons behind this trend?
5. The concept of points in Euclidean geometry has been around for centuries. To what extent has it evolved and adapted to modern times?

Task 9. ***Write an essay on the topic "The importance of Euclidean Geometry". Make it about 100—150 words long, use the information from the text above as needed.***

LAW OF SINES AND LAW OF COSINES

Task 1. ***Discuss the questions below.***

1. How do you feel about using formulas and equations in math?
2. Have you ever used the law of sines or the law of cosines in real-life situations?
3. Do you find it easier to solve math problems involving right triangles or non-right triangles?
4. When do you think it is necessary to use the law of cosines instead of the law of sines?
5. Can you think of any other situations where the law of sines or the law of cosines might be applicable?

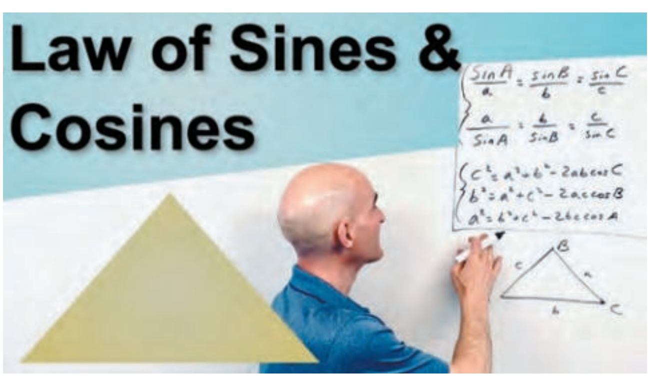
Task 2. A. ***Match the words with their definitions.***

1. The ratio of the length of the side opposite an angle less than 90° divided by the length of the hypotenuse.
2. A mathematical rule expressed in a set of numbers and letters.
3. The space between two lines or surfaces at the point at which they touch each other, measured in degrees.
4. An edge or border of something.
5. The ratio of the length of the side next to an angle less than 90°, divided by the length of the hypotenuse.
6. A comparison of two numbers calculated by dividing.
7. A proportion is also an equation that shows two ratios are equal.
8. A measurable property of an object.
9. To be a particular size.
10. A flat shape with three straight sides

B. ***Fill in the gaps with the terms from the task above.***

1. ... and cosine are written using functional notation with the abbreviations sin and cos.
2. A square has four ... .
3. Since a ... can be written as a fraction, it can also be written in any form that is equivalent to that fraction.
4. Each of sine and ... is a function of an angle, which is usually expressed in terms of radians or degrees.
5. The ... for perimeter of equilateral, isosceles and scalene triangles are a bit different
6. ... describes an amount or a number of something.
7. You can calculate the measure of an ... in a polygon if you know the shape of the polygon and the measure of its other ... or, in the case of a right triangle, if you know the measures of two of its sides.
8. Find the degree ... at which the other side of the protractor points the angle.
9. Draw a perpendicular from the vertex of the ... to its base.
10. An equation that equates two ratios is a ... .

Task 3. ***How well are you aware of the topic? Do the quiz below. Watch the video fragment 00.00—05.00 to check your ideas.***



https://drive.google.com/file d/1CnaFMe-L1cadJ2ir8Yi6X6_ iXqqLnDgQ/view?usp=sharing

1. When do we use the law of sines and the law of cosines:
   a) when we have a right triangle;
   b) when we have a triangle without a 90-degree angle;
   c) when we have a triangle with equal sides;
   d) when we have a triangle with only acute angles?
2. How many different ways are there to write the formula for the law of sines:
   a) one;
   b) two;
   c) three;
   d) four?
3. What is the purpose of using the law of sines:
   a) to find missing angles in a triangle;
   b) to find missing side lengths in a triangle;
   c) to find the area of a triangle
   d) to find the perimeter of a triangle
4. In the law of sines, which ratio is equal to the ratio of sine of angle *C* over its side opposite:
   a) sine of angle *A* over its side opposite;
   b) sine of angle *B* over its side opposite;
   c) sine of angle *A* over its side adjacent;
   d) sine of angle B over its side adjacent?
5. In the first example shown, what is the measure of angle *C*:
   a) 70 degrees;
   b) 53,7 degrees;
   c) 35 degrees;
   d) 50 degrees?
6. In the second example shown, what is the value of *x*:
   a) 10;
   b) 7,5;
   c) 35;
   d) 50?
7. When do we use the law of cosines:
   a) when we have a right triangle;
   b) when we have a triangle without a 90-degree angle;
   c) when we have a triangle with equal sides;
   d) when we have a triangle with only acute angles?
8. What is the purpose of using the law of cosines:
   a) to find missing angles in a triangle;
   b) to find missing side lengths in a triangle;
   c) to find the area of a triangle
   d) to find the perimeter of a triangle
9. When do we have to use the law of cosines:
   a) when we have three sides given;
   b) when we have three angles given;
   c) when we have two sides and one angle given;
   d) when we have two angles and one side given?

Task 4. ***Mark the sentences as True or False. Watch the video fragment 05.00—09.10 to check your ideas.***

1. The law of sines can be used when you have side-side-side information.
2. The law of cosines is used when you have side-angle-side information.
3. The law of sines can be used when you have two unknown variables.
4. The law of cosines can be used when you have three sides.
5. The law of sines can only be used in right triangles.

6. The law of cosines can be used to find missing angles
7. The law of sines can be used to find missing sides
8. The law of cosines can be used when you have side-side-side information.
9. The law of sines can be used to find missing angles
10. The law of cosines can be used when you have two unknown variables.

Task 5. ***Watch the video and put the following jumbled sentences in the logical order.***

1. What I notice first off is that I have 70 degrees and I have the side across from that 70 degree angle.
2. Some students make a little bit of arithmetic errors when they're trying to solve for angle *B* here.
3. If you use parentheses you can do all this in one step.
4. When we do the law of sines I like to use this top equation.
5. To solve angle *B* we're going to take the cosine inverse of negative point 116.
6. I'm going to take the square root because the square and the square root those are inverses.
7. What students sometimes do is they try to combine these numbers.
8. Since we're trying to solve for angle *B* let's use this middle equation where we have side *B* but not angle *B*.
9. The sine of 70 degrees over its side opposite equals the sine of angle *C* over its side opposite which is 6.
10. Notice that the capital letters are used to denote the angles and then the lowercase letters are used to denote the side lengths.

Task 6. ***Work in pairs. Interview each other using the questions below.***

Student A

1. How many different ways are there to write the formula for the law of sines?
2. What do the capital letters represent in a triangle when using the law of sines?
3. How many ratios do we need to use in order to form a proportion when using the law of sines?
4. In the first example, what is the missing quantity that we are trying to solve for?
5. What is the equation we use to solve for angle *C* in the firs example?
6. In the second example, what is the missing quantity that we are trying to solve for?
7. What is the equation we use to solve for the missing side in the second example?
8. How many formulas are there for the law of cosines?
9. When do we have to use the law of cosines instead of the law of sines?

Student B

1. Why is it not possible to solve using the Law of Sines when there are too many unknowns?
2. In example number three, what formula is used to find side *C*?
3. How is the equation *C* squared solved for side *C*?
4. In example number four, what type of triangle is given?
5. Which equation is used to solve for angle B in example number four?
6. How is the equation simplified in example number four
7. What is the value of cosine B in example number four?
8. How is angle B found using the inverse cosine function?
9. What is the approximate value of angle *B* in example number four?

Task 7. ***Hold a class discussion "The Law of Sines and Cosines". The list of questions below will be of great help to you.***

1. How does the law of sines and cosines differ from other trigon metric laws?
2. Can you explain how to use the law of sines and cosines to solve a triangle with two known sides and an unknown angle?
3. In what situations would you use the law of sines versus the law of cosines?
4. What is the relationship between the law of sines and the Pythagorean theorem?
5. How can the law of sines and cosines be applied in real-world scenarios, such as navigation or surveying?
6. Can you give an example of when the law of sines and cosines would be used in a legal case?
7. How do the law of sines and cosines relate to the concept of vectors in mathematics?
8. Are there any limitations or exceptions to the law of sines and cosines?
9. How has the understanding and application of the law of sines and cosines evolved over time?
10. Can you think of any cultural or historical significance of the law of sines and cosines?

Task 8. ***Comment on the quotations below.***

The law of sines and cosines is a powerful tool in solving triangles and understanding their relationships.

Danica McKellar

Understanding the law of sines and cosines is essential for navigating the complexities of trigonometry.

Neil deGrasse Tyson

The law of sines and cosines allows us to solve triangles and unlock the secrets of their angles and sides.

Maria Gaetana Agnesi

Trigonometry becomes much more manageable once you grasp the concepts of the law of sines and cosines.

Bill Nye

The law of sines and cosines is a fundamental principle in trigonometry that enables us to solve a wide range of problems involving triangles.

Katherine Johnson

Task 9. ***Be ready to deliver a lecture for children about Law of sines and Law of cosines.***

EXPLORING THE TOPIC

POLYGONS

Task 1. *Have a look at the title of the text. What do you think you can learn from it? What do you know about Polygons? What would you like to know? Fill in columns 1 and 2 in the grid below. Then read the part of the text assigned to you and fill in columns 3 and 4.*

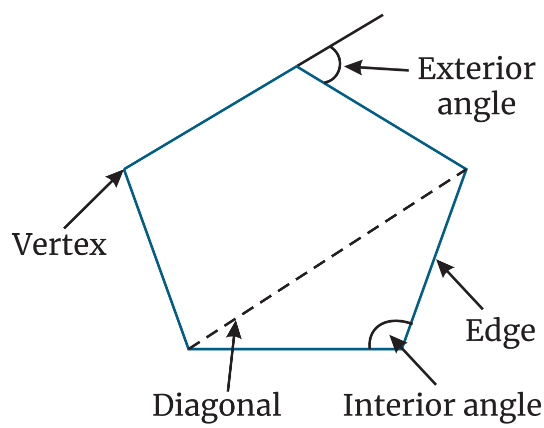
I know ... (What do you already know about polygons?)	*I want to know ...* (Write down any new questions you have.)	*I have learned ...* (State 3 main ideas you have learned from the text.)	*I know ... because ...* (Record 1 supporting fact or detail for each of your main ideas in the previous column.)

POLYGONS

1. ______________________________

A **polygon** is a 2-dimensional shape given that:
- all sides are straight;
- there are a minimum of 3 sides;
- the shape is contained (i.e. the starting point of the first side touches the ending point of the last side);
- (1) ... .

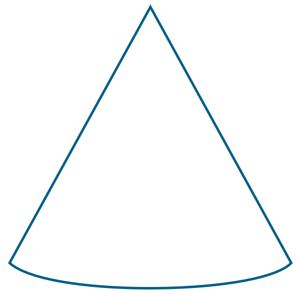
It is important to recognise these components of polygons:
- the sides, sometimes called **edges**, meet at **vertices;**
- the angles within a polygon are called the **interior angles;**
- (2) ...;
- all polygons, except triangles, have multiple diagonals.



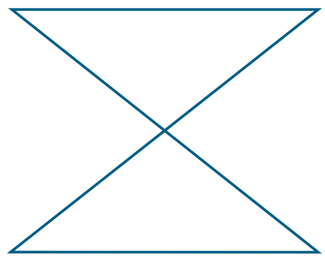
A curved shape, or a shape that contains a curve such as a semi-circle which is constructed of one straight line and one curved line, is **not a polygon**.

The following are all non-polygons.

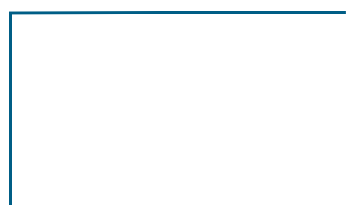
1. This is not a polygon since one side is **curved**.



2. (3) ...



3. This is not a polygon since it is not a **closed shape**.



These types are sorted by relationships of the sides of polygons or by the shape of the polygons themselves.

A polygon is **regular** when all sides and angles within the polygon are equal.

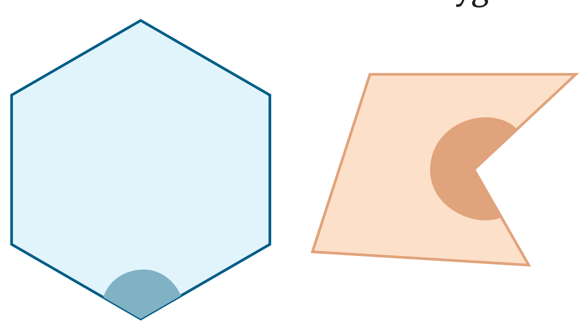
For example, a square is a regular quadrilateral shape.

A polygon is **irregular** when the sides and angles are not equal.

- A **convex** polygon is one where all of the vertices point outwards.
- (4) ... You are less likely to come across convex polygons at this stage, although they are polygons themselves

If a shape has n sides, it will also have n internal angles, and it'll be called n-*gon*.

Convex and Concave Polygon



Convex Polygon
Interior angles are less than 180 degrees

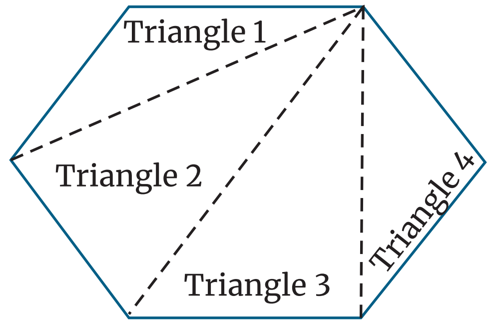
Concove Polygon
At least one of the interior angles is greater than 180 degrees

2. ____________________

We all know that a triangle contains180°and therefore the interior angles inside a triangle add up to 180°. But how do we work out how many degrees in all polygons?

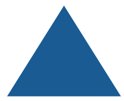
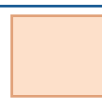
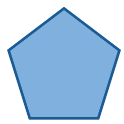
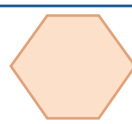
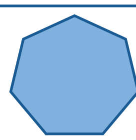
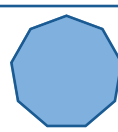
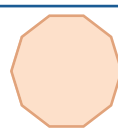
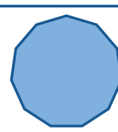
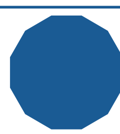
Take a quadrilateral, for example. (5) ... Since each of those triangles have interior angles adding to 180°, we now know that a quadrilateral has interior angles of 360°.

(6) ... For example, within a pentagon, you can create 3 triangles using diagonal lines. Within a hexagon, you can create 4 triangles, as demonstrated:



(7) ... Therefore, we have a simple formula for working out the number of interior angles in a polygon.

Polygons With Sides Up to 12

Polygon	Figure	Number of Sides
Triangle		3
Quadrilateral		4
Pentagon		5
Hexagon		6
Heptagon		7
Octagon		8
Nonagon		9
Decagon		10
Hendecagon		11
Dodecagon		12

Formula for interior angles: **$(n-2) \times 180$.**

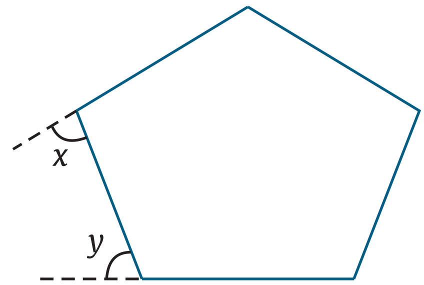
Example. What is the sum of the interior angles inside a pentagon?

$$(n-2) \times 180 = (5-2) \times 180 = 540°.$$

Taking the de nition of a regular polygon, we can now work out the interior angles within any regular polygon. (8) ... For example, a square has interior angles equal to 360/4 = 90 degrees.

The **exterior angles** are more straightforward than the interior angles. In all cases, the exterior angles sum to 360°. To calculate the exterior angle of a regular polygon, simply divide 360 by the number of sides, *n*.

Formula for exterior angles: $\mathbf{\frac{360}{n}}$.



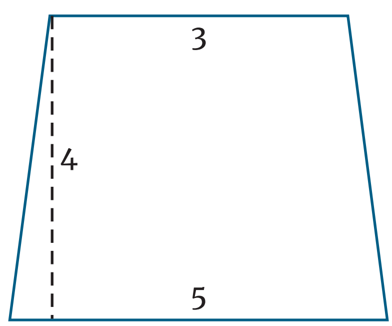
Example. The following is a regular pentagon. Find *x* and *y*.

Exterior angles of a pentagon.

(9) ...

It is helpful to be familiar with the formulae for the areas of common polygons.

Polygon	Area formula
Triangle	$\frac{1}{2}$ × base × height
Square	length²
Rectangle	Length × width
Parallelogram	Base × height
Trapezium	(10) ...
Rhombus	$\frac{1}{2}$ × (product of diagonals)



Example. Find the area of the following shape. The lengths are given in centimeters.

The formula is:

$$\frac{1}{2} \times \text{(sum of lengths of parallel sides/bases)} \times \text{height}.$$

We are given the height, 4 cm, and the lengths of the parallel sides, 3 cm and 5 cm. Plugging these into the formula we get:

$$\text{Area} = \frac{1}{2} \times (3 + 5) \times 4 = 16 \text{ cm}^2.$$

https://mathblog.com/reference/geometry/polygon/

Task 2. A. ***Make a summary of your part of the text and present to your partner.***

B. ***Listen to your partner's summary and fill in columns 3 and 4 of the chart in Task 1.***

Task 3. ***Give the headings to the part of the text you have read.***

A. Angles and Areas of Polygons.
B. A Polygon and its Types.

Task 4. ***Read the text once again and fill in the gaps with sentences from the list below. There are 2 extra sentences.***

1. The angles on the outside of the polygon are the exterior angles;
2. For a polygon of n sides, we can create $n - 2$ triangles.
3. Since all the angles need to be equal, we simply divide the number of interior angles by the number of vertices.
4. This is not a polygon since the sides cross each other.
5. 1/2 × (sum of lengths of parallel sides/bases) × height.
6. There are two ways we could go about finding these angles: using either the external or the internal angle formula.
7. A concave polygon is one in which at least one vertex points inwards.
8. Algebraic inequality statements are solved in the same manner as equations.
9. None of the sides cross each other.
10. Various operations like addition, subtraction, multiplication and division can be applied on polynomials.
11. If you divide a quadrilateral into two shapes along the diagonal, you get two triangles.
12. We can extend this logic to polygons with even more sides.

Task 5. ***In pairs, answer the questions below.***

1. What is a polygon?
2. What shapes are non-polygons?
3. When a polygon is regular?
4. What polygons are convex and what are concave?
5. What types of polygons are there?
6. What is a formula for working out the number of interior angles in a polygon?
7. What are the formulae for the areas of common polygons?

Task 6. ***Discuss the questions in pairs.***

1. Can you think of any real-life examples of polygons?
2. How would you define a regular polygon
3. Have you ever come across a concave polygon in your daily life?
4. Do you prefer working with convex or concave polygons?

Task 7. ***What do you think the video is about?***

https://drive.google.com/file/d 10EDcEaH6hRXIQ3UXshDYbx TSjIgY_sUj/view?usp=sharing

The videofilm "What's a polygon?" can be about the polygons':
- ***characteristics;***
- ***formulae;***
- ***shapes;***
- ***identities;***
- ***categories.***

Tick those themes that will be referred to in the video. Watch the film to make sure you are right.

Task 8. ***Arrange the sentences below in the order they appear in the video.***

1. It is two-dimensional that is complete it is not considered a polygon either since its edges are not straight lines but rather curved.
2. We wouldn't be able to call it a regular polygon since the line lengths aren't all the same.
3. We can actually categorize them into two categories which are regular versus irregular.
4. The concave polygon on the other hand has at least one angle on the inside of the shape that is greater than 180 degrees.
5. A polygon is a two-dimensional shape that is made up with straight line segments which means absolutely no curves.
6. And as a result of that we've got equal angles on the insides as well.
7. If we look carefully at these polygons we`ll notice that each of these sides are equal to every other side in the polygon.
8. A basketball is a three-dimensional object rather than a two-dimensional shape which is one of the criteria for a shape being a polygon.
9. So the differentiating characteristics of a convex polygon is that each angle within the polygon must be less than or equal to 180 degrees.
10. The shape for a polygon is always a closed shape.

Task 9. ***Work in pairs and:***
- ***prove that a polygon is a two-dimensional shape;***
- ***name the characteristics of a polygon;***
- ***comment on the types of polygons.***

Task 10. ***Work in pairs. Take turns presenting the information of the videofilm.***

Task 11. A. ***Get ready to speak on one of the topics below. Make a 1-minute speech.***

1. How learning about polygons in math class sparked my interest in geometry and problem-solving.
2. The challenges I faced while studying complex polygons and how it improved my critical thinking skills.
3. My experience of using polygons to create art and designs.
4. How understanding the properties of polygons has helped me in everyday life, from measuring furniture to planning a road trip.
5. The impact of learning about polygons on my overall mathematical abilities and confidence

B. ***Have a group press conference to present your speeches. Answer your groupmates' questions.***

Task 12. ***Write an essay (about 150—200 words) about polygons.***

REGULAR POLYGONS

Task 1. ***Comment on the quotations about regular polygons.***

The regularity of a polygon is a reflection of the order and harmony found in nature.

Leonardo da Vinci

Regular polygons are the building blocks of geometric beauty.

Euclid

The symmetry of regular polygons is a testament to the elegance of mathematical principles.

Pythagoras

Regular polygons are like the perfect dance partners, moving in sync with each other.

Maurits Cornelis Escher

The regularity of polygons is a visual representation of the balance and stability found in the universe.

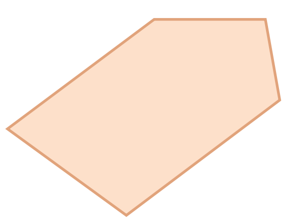
Johannes Kepler

Task 2. ***Have a look at the title of the text. What do you think you can learn from it? What do you know about Regular Polygons? What would you like to know? Fill in columns 1 and 2 in the grid below. Then read the part of the text assigned to you and fill in columns 3 and 4.***

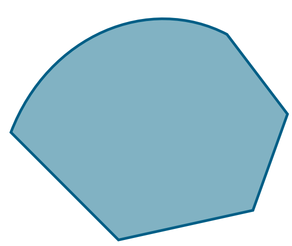
I know ... (What do you already know about regular polygons?)	*I want to know ...* (Write down any new questions you have.)	*I have learned ...* (State 3 main ideas you have learned from the text.)	*I know ... because ...* (Record 1 supporting fact or detail for each of your main ideas in the previous column.)

REGULAR POLYGONS

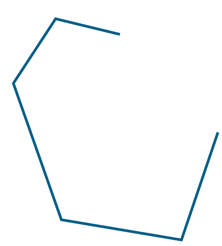
Polygons can be classi ed on a lot of di erent criteria, from the number of sides to their interior and exterior angles (1) ... .



It is a polygon as it is closed and has straight lines



It is not a polygon as it has a curved side



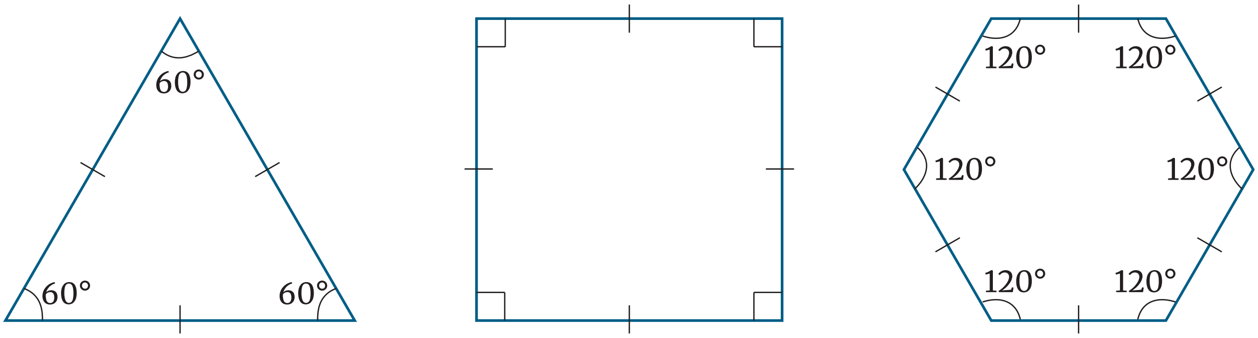
It is not a polygon as it is an open figure

If all the interior angles are less than 180° each, then the shape is classified as convex. Whereas if any one of the interior angles is greater than 180°, then the shape is concave. Convex polygons are further classified into regular or irregular polygons depending on the length of the sides and interior angles.

1. ______________________________

A **regular polygon** has sides of equal length and equal interior angles.

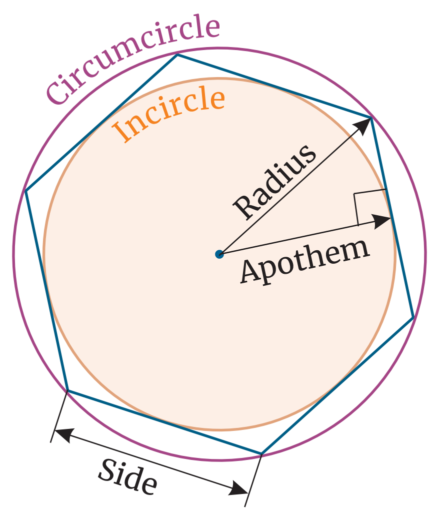
Examples of regular polygons are ***equilateral triangles, squares, rhombuses,*** and so on.



(2) ... Regular polygons are mostly convex by nature. On the other hand, concave regular polygons are sometimes star-shaped. We will be discussing the properties of regular convex polygons in detail.

There are two important circles that can be drawn on a regular polygon:

- The **circumcircle** lies outside the convex regular polygon and passes through all its vertices. The radius of the circumcircle is the distance from the center of the polygon to any of its vertex.
- The **incircle** passes through the mid-point of all the sides of the polygon and lies inside the regular polygon. The radius of the incircle is the distance between the center and a midpoint of any side (3) ... .

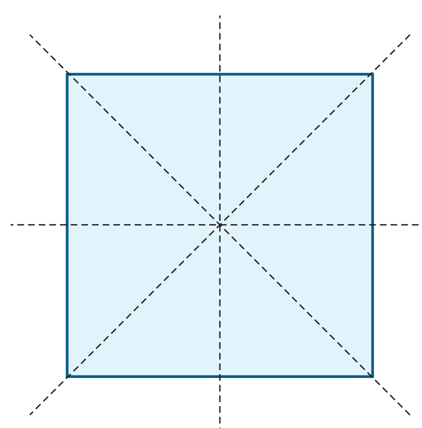


These properties of an incircle, circumcircle, and apothem can only be found in regular polygons.

So what can we do using these properties? (4) ... Any regular polygon can be broken down into triangles. Combining this with the apothem we can estimate the areas of any regular polygon of side N.

The line of symmetry can be de ned as the axis or imaginary line that passes through the center of the shape or object and divides it into identical halves. Since all the sides of a regular polygon are equal, the number of lines of symmetry = number of sides = n.

For example, a square has 4 sides. So, the number of lines of symmetry = 4.



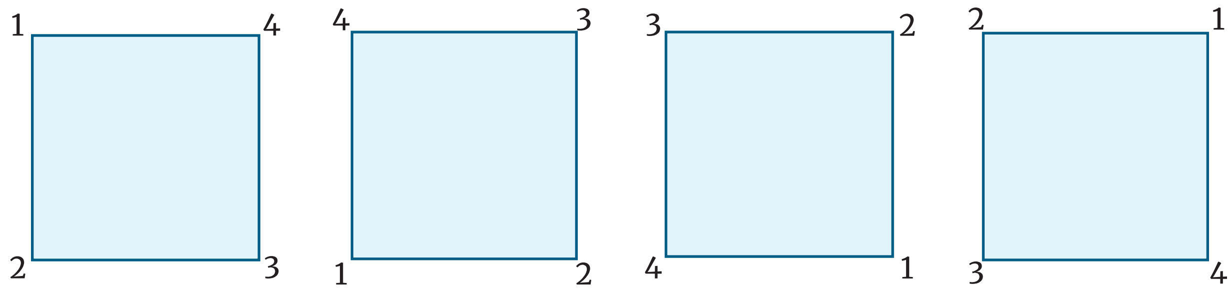
A shape has rotational symmetry when it can be rotated and still it looks the same. The order of a rotational symmetry of a regular polygon = number of sides = n.
(5) ...

For example, a square has 4 sides. So, the order of rotational symmetry = 4.

Angle of rotation = $\dfrac{360}{4} = 90°$.

This means when we rotate the square 4 times at an angle of 90°, we will get the same image each time.

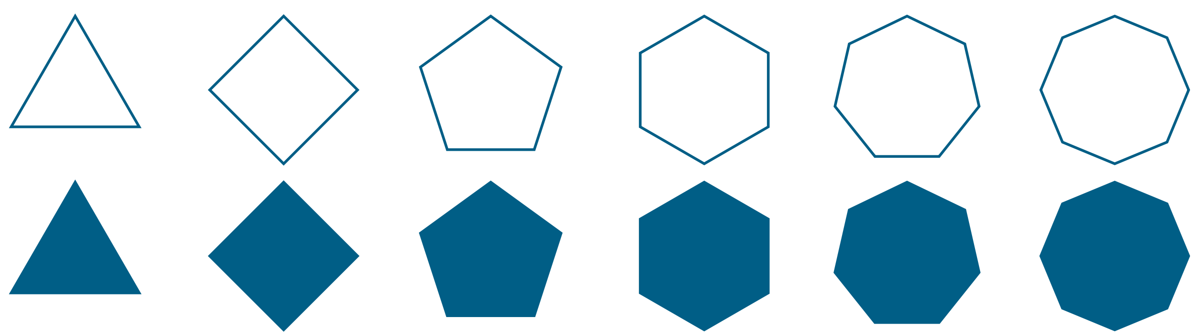
Counterclockwise Rotational Symmetries of a Square



Rotation by 0 degrees | Rotation by 90 degrees | Rotation by 180 degrees | Rotation by 270 degrees

Regular polygons with 3 sides are called equilateral triangles, 4 sides are called squares. Regular polygons with more than four sides are denoted with a 'regular' preceding the name of the polygon. (6) .... Below there are a few examples of a regular (equiangular) convex polygon.

$n = 3$ | $n = 4$ | $n = 5$ | $n = 6$ | $n = 7$ | $n = 8$

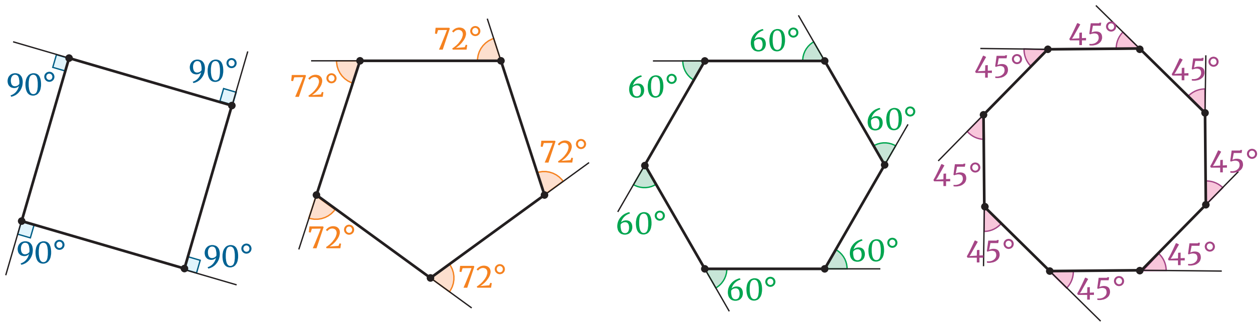


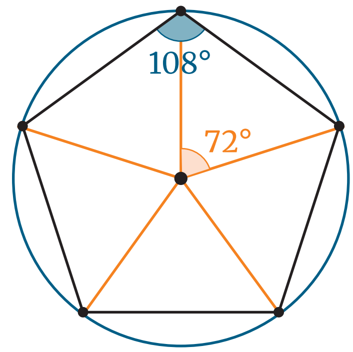
Equilateral triangle | Square | Regular pentagon | Regular hexagon | Regular heptagon | Regular octagon

2. ____________________

At any vertex of a polygon, there are 2 angles the interior and exterior. The exterior angle is obtained by the angle between an extended edge and its consecutive edge.
(7) ...

$$\varphi = \frac{360}{N}, \text{ where } N \text{ is the number of sides and } \varphi \text{ is the exterior angle.}$$

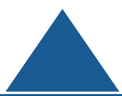
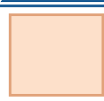
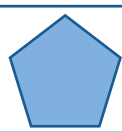
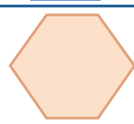
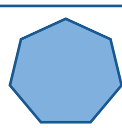
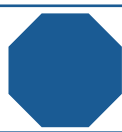
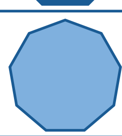
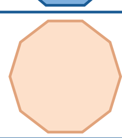




The interior angles are formed between two adjacent sides of a polygon. (8) ... For example, all triangles will have a total sum of 180°, quadrilaterals will have a sum of 360°, and so on. But what about a polygon with hundred sides.

(9) ... Using this relation we can derive a general equation that can be used to find the interior angles of any polygon by having the number of sides.

Interior Angles of Regular Polygons

Name	Sum of Interior Angles	Each Interior Angle
Equilateral Triangle 	180°	60°
Square 	360°	90°
Pentagon 	540°	108°
Hexagon 	720°	120°
Heptagon 	900°	128.57°
Octagon 	1080°	135°
Nonagon 	1260°	140°
Decagon 	1440°	144°

In a polygon with more than 3 sides, a diagonal is a line segment between any two non-consecutive points. (10) ... If a polygon has 'N' sides, then the number of diagonals is equal to:

$$number\ of\ diagonals = \frac{N(N-3)}{2}.$$

https://www.studysmarter.co.uk/explanations/math/geometry/regular-polygon/

Task 2 ***A. Make a summary of your part of the text and present to your partner.***

B. ***Listen to your partner's summary and fill in columns 3 and 4 of the chart in Task 1.***

Task 3. ***Give the headings to the part of the text you have read.***

A. Angles of Regular Polygons.
B. Regular Polygon and its Properties.

Task 4. ***Read the text once again and fill in the gaps with sentences from the list below. There are 2 extra sentences.***

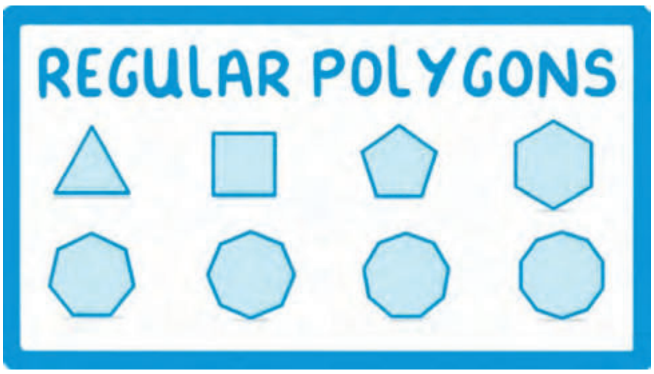
1. This distance is also called the apothem of the polygon.
2. The sum of interior angles of a polygon will depend on the number of sides that it has.
3. In a regular convex polygon, the sum of all exterior angles is always 360°.
4. The criteria that determine whether a shape is convex or concave is the magnitude of interior angles.
5. The sum of the interior and exterior angles at a vertex is always equal to 180°.
6. Unlike the concave polygons, the diagonals of a convex polygon will always lie inside the figure
7. A polygon will also have diagonals of the same length.
8. For example, a pentagon with equal sides and angles is called a regular pentagon.
9. A perimeter is the sum of all sides of a polygon.
10. One interesting application is being able to calculate the area of a regular polygon using the apothem.
11. A scalene triangle has 3 different length sides
12. Also, the angle of rotational symmetry of a regular polygon = $\frac{360}{n}$.

Task 5. ***In pairs, answer the questions below.***

1. How can a polygon be defined
2. How are convex polygons classified into
3. What does a regular polygon have?
4. What are examples of regular polygons?
5. What properties of regular polygons are there?
6. What shapes can any regular polygon be broken down into?
7. How can the line of symmetry be defined

8. A shape has rotational symmetry when it can be rotated and still it looks the same.
9. Regular polygons with 3 sides are called equilateral triangles, aren't they?
10. The interior angles are formed between two adjacent sides of a polygon, aren't they?

Task 6. ***What do you think the video "Regular polygons" is about***
- ***angles;***
- ***sides;***
- ***shapes;***
- ***numbers;***
- ***rotational symmetry?***



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Tick those themes that will be referred to in the video. Watch the film to make sure you are right.

Task 7. ***Arrange the sentences below in the order they appear in the video.***

1. And this part means that the length of all the sides and the size of all the angles are the same.
2. Now one of the nice things about regular polygons is that they all have the same number of lines of symmetry as they have sides.
3. The other part of the name is the word "regular".
4. So next we'd have a square which is a regular quadrilateral.
5. And likewise a heptagon with seven sides will have a rotational symmetry of order seven.
6. However this second shape isn't a regular polygon because its sides aren't all the same length and its angles aren't the same size.
7. They also have the same order of rotational symmetry as they have number of sides.
8. The "polygon" part of the name means a closed 2d shape.
9. Lastly the shape on the right would count as a regular polygon because its six sides are all the same length and its six angles are all the same size.
10. To be more specific we could call it an irregular quadrilateral

Task 8. ***Work in pairs and:***
- ***speak about the meaning of the part of the name "regular" and "polygon";***
- ***comment on regular and irregular polygons;***
- ***give examples of regular polygons.***

Task 9. ***Work in pairs. Take turns presenting the information of the videofilm.***

Task 10. ***Discuss the cases below in mini-groups.***

1. Some people argue that learning about polygons is not necessary for everyday life. Do you agree or disagree?
2. With the increasing use of technology, do you think traditional methods of teaching polygons in math class are becoming outdated?
3. In your opinion, should students be allowed to use calculators when learning about polygons in math class?
4. There is a debate about whether or not memorizing formulas for calculating polygon properties is important. What is your stance on this issue?

Task 11. ***Write a longread for your subscribers (150—200 words) about the class regular polygons.***

TRIANGLES

Task 1. ***Have a look at the title of the text. What do you think you can learn from it? What do you know about Triangles? What would you like to know? Fill in columns 1 and 2 in the grid below. Then read the part of the text assigned to you and fill in columns 3 and 4.***

I know ... (What do you already know about triangles?)	*I want to know ...* (Write down any new questions you have.)	*I have learned ...* (State 3 main ideas you have learned from the text.)	*I know ... because ...* (Record 1 supporting fact or detail for each of your main ideas in the previous column.)

TRIANGLES

1. ____________________

A **triangle** is a simple polygon with 3 sides and 3 interior angles. It is one of the basic shapes in geometry in which the 3 vertices are joined with each other and it is denoted by the symbol Δ.

Now, let's illustrate what we mean by a triangle. (1) ... .

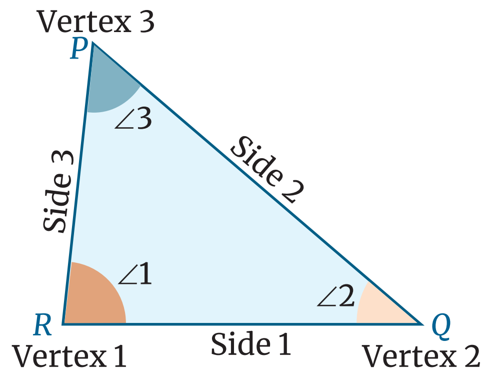
Observe the triangle *PQR* given below which shows the sides, the vertices and the interior angles of a triangle.

In the triangle given above:

- the three angles are, $\angle PQR$, $\angle QRP$, and $\angle RPQ$;
- (2) ...;
- the three vertices are *P*, *Q*, and *R*.

Note. The sum of all the angles of the triangle is equal to 180°.

All geometrical shapes have different properties related to sides and angles that help us to identify them. The important properties of a triangle are listed below.



- A triangle has three sides, three vertices, and three interior angles.
- The angle sum property of a triangle states that the sum of the three interior angles of a triangle is always 180°. Observe the triangle PQR given above in which angle P + angle Q + angle R = 180°.
- (3) ...
- As per the Pythagoras theorem, in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides i.e., ($\text{Hypotenuse}^2 = \text{Base}^2 + \text{Altitude}^2$).
- The side opposite the greater angle is the longest side.
- The exterior angle theorem of a triangle states that the exterior angle of a triangle is always equal to the sum of the interior opposite angles.

2. ____

In geometry, for every two-dimensional shape (2D shape), there are always two basic measurements that we need to nd out, i.e., the area and perimeter of that shape. Therefore, the triangle has two basic formulas which help us to determine its area and perimeter. Let us discuss the formulas in detail.

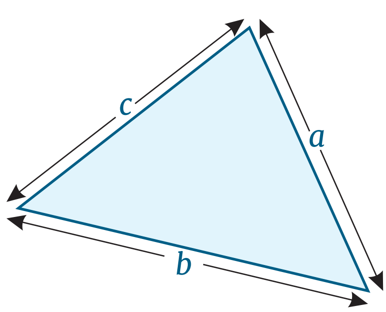
(4) ... Observe the triangle given below which shows that the perimeter of the triangle is the sum of all its sides.

The area of a triangle is the space covered by the triangle. (5) ... It is always measured in square units, as it is two-dimensional. Observe the triangle ABC given below which shows the base and height of a triangle which are used to calculate the area of a triangle.

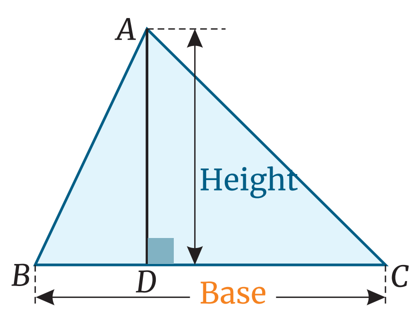
Area of $\triangle ABC = \frac{1}{2} \times BC \times AD$.

Here, BC is the base and AD is the height of the triangle.

The **median of a triangle** is a line segment that joins one vertex to the midpoint of the opposite side of a triangle. Observe the figure given below, in which AD is the median, dividing BC into two equal parts, such that, $BD = DC$.

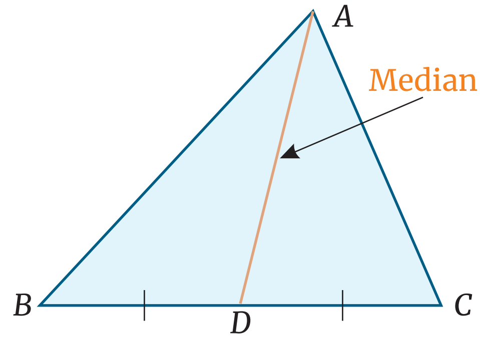


Perimeter = $a + b + c$



Area of Triangle = $\frac{1}{2}$ × Base × Height

The median of a triangle can be easily identifie with the help of the following properties.



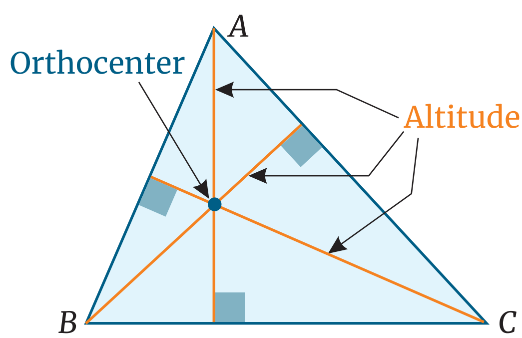
- The median of a triangle is a line segment joining the vertex of the triangle to the mid-point of its opposite side.
- It bisects the opposite side, dividing it into two equal parts.
- The median of a triangle further divides the triangle into two triangles having the same area.
- (6) ...
- Every triangle has 3 medians, one from each vertex. The point of concurrency of 3 medians forms the centroid of the triangle.
- Each median of a triangle divides the triangle into two smaller triangles that have equal areas. In fact, the 3 medians divide the triangle into 6 smaller triangles of equal area.

The **altitude of a triangle** is the perpendicular line segment drawn from the vertex of the triangle to the side opposite to it. The altitude makes a right angle with the base of the triangle that it touches. (7) ... It can be measured by calculating the distance between the vertex and its opposite side. It is to be noted that three altitudes can be drawn in every triangle from each of the vertices. Observe the following triangle and see the point where all the three altitudes of the triangle meet. This point is known as the 'Orthocenter'.

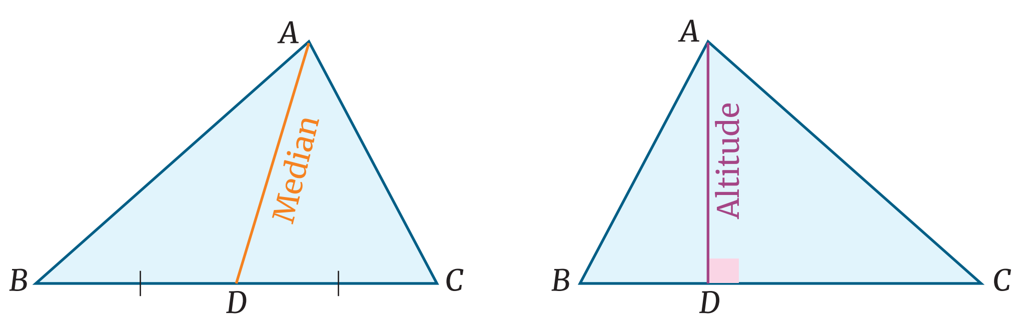
The altitudes of various types of triangles have some properties that are specifi to certain triangles. They are as follows:

- A triangle can have three altitudes.
- The altitudes can be inside or outside the triangle, depending on the type of triangle.
- The altitude makes an angle of 90° to the side opposite to it.
- (8) ...

The formula for the altitude of a triangle can be derived from the basic formula for the area of a triangle which is ***Area = 1/2 × base × height,*** where the height represents the altitude. (9) ...



We know that the median and the altitude of a triangle are line segments that join the vertex to the opposite side of a triangle. (10) ... Observe the figure and the table given below to understand the difference between the median and altitude of a triangle.



Median of a Triangle	Altitude of a Triangle
The median of a triangle is the line segment drawn from the vertex to the opposite side.	The altitude of a triangle is the perpendicular distance from the base to the opposite vertex.
It always lies inside the triangle.	It can be both outside or inside the triangle depending on the type of triangle.
It divides a triangle into two equal parts.	It does not divide the triangle into two equal parts.
It bisects the base of the triangle into two equal parts.	It does not bisect the base of the triangle.
The point where the 3 medians of a triangle meet is known as the centroid of the triangle.	The point where the 3 altitudes of the triangle meet is known as the orthocenter of that triangle.

https://www.cuemath.com/geometry/altitude-of-a-triangle/

Task 2. A. ***Make a summary of your part of the text and present to your partner.***

B. ***Listen to your partner's summary and fill in columns 3 and 4 of the chart in Task 1.***

Task 3. ***Match the headings with paragraphs.***

A. Triangle Formulas.
B. Properties of Triangles.

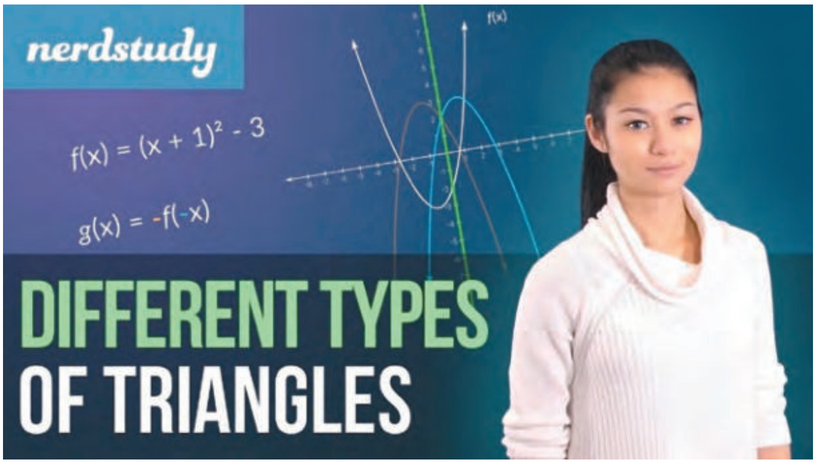
Task 4. ***Read the text once again and fill in the gaps with sentences from the list below. There are 2 extra sentences.***

1. The perimeter of a triangle is the sum of all three sides of the triangle.
2. Irrespective of the shape or size of a triangle, its three medians meet at a single point.
3. It is half the product of its base and altitude (height).
4. The point of intersection of the three altitudes of a triangle is called the orthocenter of the triangle.
5. There are numerous items around us that are square in shape.
6. Every triangle has three sides and three edges or corners which are known as vertices.
7. Using this formula, we can derive the altitude formula which will be, altitude of triangle = (2 × area)/base.
8. However, they are different from each other in many ways
9. The area and perimeter of a quadrilateral depend on its shape and size, and can be calculated using various formulas.
10. The Triangle inequality theorem states that the sum of the length of the two sides of a triangle is greater than the third side.
11. The three sides are side *PQ*, side *QR*, and side *RP*.
12. It is commonly referred to as the height of a triangle and is denoted by the letter '*h*'.

Task 5. ***In pairs, answer the questions below.***

1. What is a triangle?
2. What properties of a triangle are there?
3. What two basic measurements for every two-dimensional shape are there?
4. What is the area of a triangle? How is it measured?
5. What is the median of a triangle?
6. What properties of the median of a triangle are there?
7. What is the altitude of a triangle?
8. How is the altitude of a triangle commonly referred to? How is it denoted?
9. How can three altitudes be drawn in a triangle?
10. What is the difference between the median and altitude of a tri ngle?

Task 6. ***What do you think the video "Different types of triangles" is about***
- ***values;***
- ***types of triangles;***
- ***degrees;***
- ***sides;***
- ***length?***



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Tick those themes that will be referred to in the video. Watch the video to make sure you are right.

Task 7. ***Arrange the sentences below in the order they appear in the video.***

1. We can denote that by putting a hash mark here and then two hash marks here and then three over here to differentiate each side
2. However it's the largest angle within a triangle is less than 90 degrees then we would instead call this type of triangle and acute triangle.
3. Scalene triangle has three sides where each of the side are different in links from each other.
4. Right triangle contains an angle equivalent to 36 degrees in one of its corners.
5. If any one of the angles in a triangle where exactly 90 degrees then we would call this type of triangle by default of right angle triangle or for short a right triangle.

6. It’s the largest angle within a triangle is greater than 90 degrees then we would, call this type of triangle an obtuse triangle.
7. The isosceles has two sides that are the same in length which makes two of the opposite angles equal to each other as well.
8. We know that a triangle is made up of 180 degrees on the inside.
9. Since an equilateral triangle has all equal sides it also must have equal angles that are always 60 degrees each.
10. We can also say that the three angles in a scalene triangle are all different

Task 8. ***Work in pairs and:***
- ***name each type of triangles;***
- ***explain the differences between these types.***

Task 9. ***In mini-groups discuss the cases below.***

1. Triangles are often seen as the strongest and most stable shape. To what extent do you agree or disagree with this opinion?
2. Some people argue that equilateral triangles are the most aesthetically pleasing shape. Is this a valid argument?
3. The use of triangles in art and design has been prevalent throughout history. To what extent do you think triangles enhance the visual appeal of an artwork?
4. Many ancient civilizations, such as the Egyptians and Mayans, incorporated triangles into their architecture and artwork. Why do you think this shape was significant to them
5. Triangles have been used as symbols in many different contexts, from religion to mathematics. Do you think there is a universal meaning behind the triangle or does it hold different meanings for different culture

Task 10. ***Write an essay (not more than 200 words) about the different types of triangles.***

PYTHAGORAS THEOREM

Task 1. A. ***Comment on the quotation below. Do you support its message?***

B. ***What do you know about Pythagoras? What is his most significant discovery?***

The soul of man is divided into three parts, intelligence, reason, and passion. Intelligence and passion are possessed by other animals, but reason by man alone.

Pythagoras

C. ***In pairs, share your ideas on the topics below.***

1. My experience learning about the Pythagorean theorem and its practical applications in real life.
2. How Pythagoras Theorem has been a fundamental concept in my understanding of geometry.
3. How learning about Pythagoras Theorem has improved my critical thinking and problem-solving skills.
4. My personal journey of discovering the beauty and elegance of Pythagoras Theorem.

Task 2. ***Have a look at the title of the text. What do you think you can learn from it? What do you know about Pythagoras theorem? What would you like to know? Fill in columns 1 and 2 in the grid below. Then read the part of the text assigned to you and fill in columns 3 and 4.***

I know ... (What do you already know about Pythagoras theorem?)	*I want to know ...* (Write down any new questions you have.)	*I have learned ...* (State 3 main ideas you have learned from the text)	*I know ... because ...* (Record 1 supporting fact or detail for each of your main ideas in the previous column)

1. ____

Pythagoras theorem is also famously known as the Pythagorean theorem, explains the relationship between the three sides of a right-angled triangle and is the most proven theorem in mathematics. There are well over 371 proofs for the Pythagorean theorem, one of which is given by American President, James Abram Garfield (who was elected America's president in 1831), while discussing with a member of Congress he came up with the idea of this proof. (1) ...

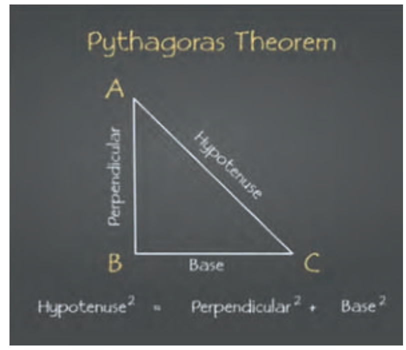
Pythagoras theorem provides us with the relationship between the sides in a right-angled triangle (2) ... The two legs meet at a 90° angle, and the hypotenuse is the longest side of the right triangle and is the side opposite the right angle. Observing any right-angled triangle, one of the angles of a right-angled triangle is always 90°, and the square of the hypotenuse is equal to the sum of the squares of the perpendicular and base of the triangle.

(3) ... It is named after the ancient Greek mathematician and philosopher Pythagoras of Samos, who lived during the 6th century BC. But the roots of this theorem go to ancient cultures, as it is very likely that Babylonians and Indians used this theorem well before Pythagoras, but its widespread use came into existence after Pythagoras stated it. One of the other reasons this theorem is known as Pythagoras or Pythagorean Theorem is because the disciples of Pythagoras spread knowledge and philosophy of Pythagoras after his death as well.

2. ____

Pythagoras theorem formula is $AC^2 = AB^2 + BC^2$, where AB is the perpendicular side, BC is the base, and AC is the hypotenuse side. (4) ...

The Pythagoras theorem formula or the Pythagorean theorem formula states that in a ΔABC, the square of the hypotenuse (AC^2) is equal to the sum of the squares of the sides ($AB^2 + BC^2$). Here, AB is the perpendicular of the triangle, and BC is the base. (5) ... .



Let's see the derivation of Pythagoras Theorem Formula. Consider a right-angled triangle having sides A, B, and C. Here, AC is the longest side (hypotenuse), and AB and BC are the legs of the triangle. Draw a perpendicular line BD at AC as shown in the figure below,

In ΔABD and ΔACB,

$\angle A = \angle A$,

$\angle ADB = \angle ABC$ (90°).

Therefore, we can say $\Delta ABD \sim \Delta ACB$.

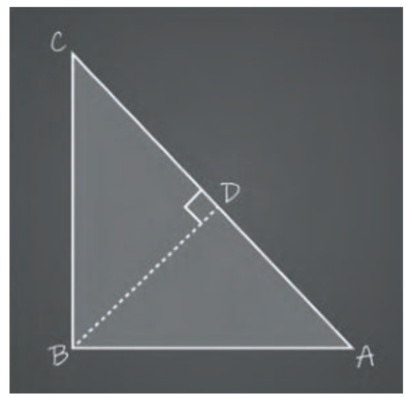
Similarly, $\Delta BDC \sim \Delta ACB$.

Hence, $AD/AB = AB/AC$.

$$AB^2 = AD \times AC \rightarrow (1)$$

And, $CD/BC = BC/AC$

$$BC^2 = CD \times AC \rightarrow (2)$$

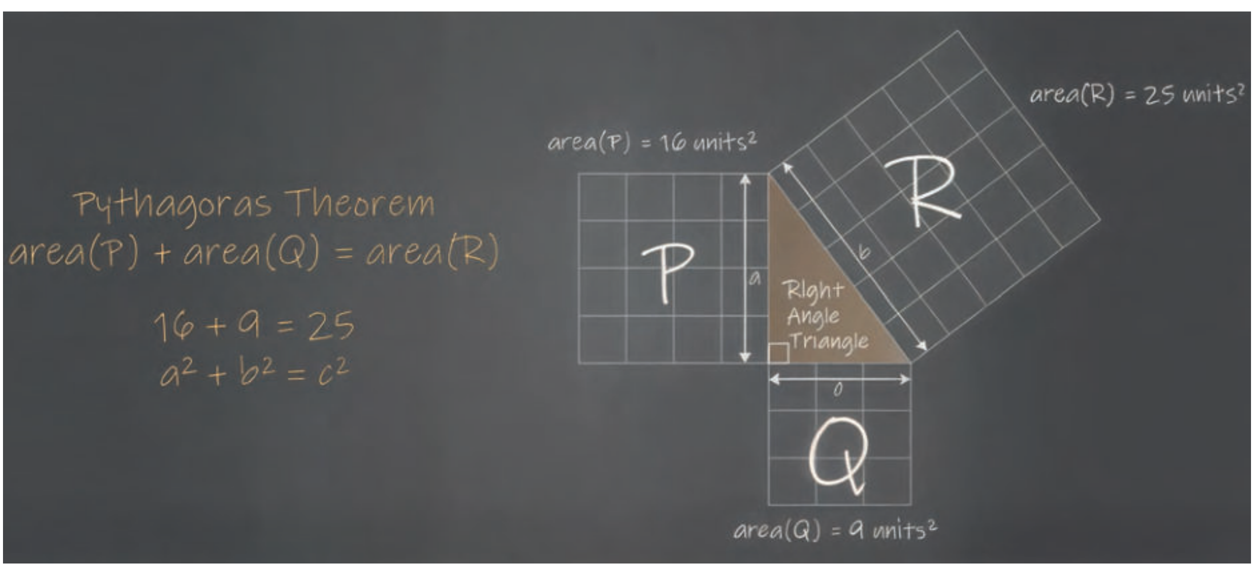


Adding equations (1) and (2),

$$AB^2 + BC^2 = AC \times AD + AC \times CD$$
$$AB^2 + BC^2 = AC\,(AD + CD)$$
$$AB^2 + BC^2 = AC \times AC$$
$$AB^2 + BC^2 = AC^2$$

Also, $AC^2 = AB^2 + BC^2$.

Hence proved.



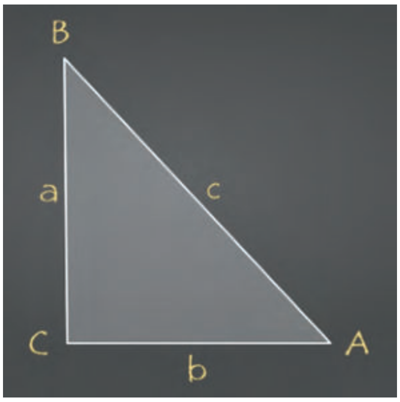
Let's see the traditional way to prove Pythagoras theorem formula which says that the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the two shorter sides. (6) ... In the example below, the area formed by side 3 units (c) and 4 units (a) is equal to the area formed by side 5 units (b).

3. ____

The converse of the Pythagoras theorem is very similar to Pythagoras theorem. To understand this theorem, you should think from the reverse of Pythagoras theorem.

(7) ...

Let's see the formula and proof of converse Pythagoras Theorem. (8) ... According to the statement, we need to prove that the condition occurs, then the triangle should be the right triangle. For that, we need to prove that the opposite angle of the longest side should be 90° if there is a triangle with lengths a, b, and c.



We assume that it satis es $c^2 = a^2 + b^2$, and by looking into the diagram, we can tell that $\angle C = 90°$, but to prove it, we require another triangle ΔEGF, such as $AC = EG = b$ and $BC = FG = a$.

In ΔEGF, by Pythagoras Theorem:

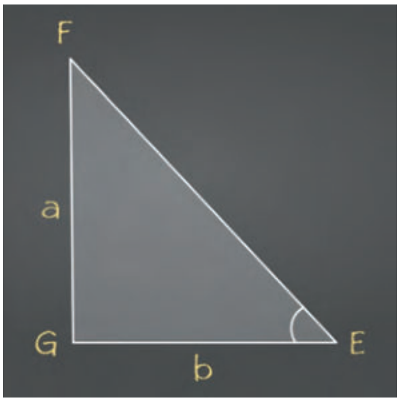
$$\Rightarrow EF^2 = EG^2 + FG^2 = b^2 + a^2 \rightarrow (1)$$

In ΔABC, by Pythagoras Theorem:

$$\Rightarrow AB^2 = AC^2 + BC^2 = b^2 + a^2 \rightarrow (2)$$

From equation (1) and (2), we have;

$$\Rightarrow EF^2 = AB^2$$
$$\Rightarrow EF = AB$$
$$\Rightarrow \Delta\, ACB \cong \Delta EGF$$
$$\Rightarrow \angle G \text{ is right angle.}$$



Thus, ΔEGF is a right triangle. (9) ...

The real-life application of Pythagoras theorem can be seen in day-to-day life and in various elds. Below are some of the applications:

For Navigation, the Pythagoras theorem is calculated in order to find the shortest distance, etc.

(10) ...

In engineering fields, Pythagoras is used to calculate the unknown dimension, say, the diameter of a particular sector.

https://www.geeksforgeeks.org/pythagoras-theorem/

Task 3. A. ***Make a summary of your part of the text and present to your partner.***

B. ***Listen to your partner's summary and fill in columns 3 and 4 of the chart in Task 1.***

Task 4. ***Match the headings (A—C) with paragraphs 1—3.***

A. Converse of Pythagoras Theorem.
B. Pythagoras Theorem Formula.
C. The history of Pythagoras Theorem.

Task 5. ***Read the text once again and fill in the gaps with sentences from the list below. There are 2 extra sentences.***

1. Just like the rectangle, the angles are 90 degrees each. The figure below is a square.
2. In order to recognize faces in security cameras, Pythagoras theorem is required to understand the distance of the person from the camera.
3. A right triangle consists of two legs and a hypotenuse.
4. Hence, we can say that the converse of the Pythagorean theorem also holds.
5. The three sides of the right-angled triangle are called the Pythagoras Triplets.
6. Other than James A. Garfield, Einstien also gave proof for Pythagoras' theorem.
7. The explanation for Pythagoras theorem can also be seen in a way that the square formed by the hypotenuse side is equal to the sum of the squares formed by the perpendicular side and base side.
8. The Pythagoras equation is applied to any triangle that has one of its angles equal to 90°.
9. If the square of the length of the longest side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.
10. The history of the Pythagoras Theorem goes back to the ancient Babylon and Egypt eras.
11. The formula will be the same as it is just converse of the Pythagoras theorem.
12. The sum of any 2 angles next to each other is 180 degrees.

Task 6. ***In pairs, answer the questions below.***

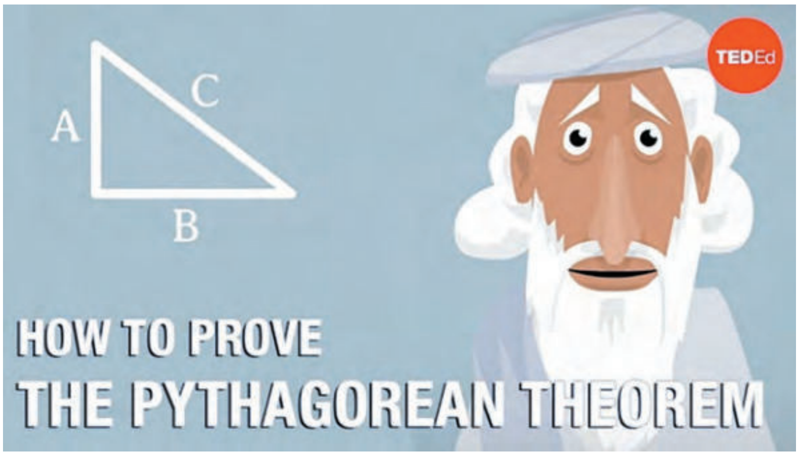
1. What does the Pythagoras theorem explain?
2. What is the Pythagoras theorem formula?
3. What does the Pythagoras theorem state?
4. How to prove Pythagoras Theorem formula traditionally?
5. What is the converse of the Pythagoras theorem?
6. Where can the Pythagoras theorem be applied?

Task 7. ***Discuss the questions below in pairs.***

1. How do you think the Pythagorean theorem has impacted our daily lives and the world around us?
2. Can you think of any practical applications of the Pythagorean theorem in fields other than math and geometry?
3. How do you think the Pythagorean theorem has influenced the development of other mathematical concepts and theories?
4. Have you ever come across a real-life situation where the Pythagorean theorem was applicable?
5. Do you think there could be any potential limitations or exceptions to the Pythagorean theorem?

Task 8. ***What do you think the video "How Many Ways Are There to Prove the Pythagorean Theorem?" is about:***

- ***history;***
- ***proofs;***
- ***application;***
- ***rules?***



https://drive.google.com/file d/185VodA7kzZlQrlUIvn4hkTL COAVr9TYr/view?usp=sharing

Tick those themes that will be referred to in the video. Watch the video to make sure you are right.

Task 9. ***Arrange the sentences below in the order they appear in the video.***

1. This statement is one of the most fundamental rules of geometry and the basis for practical applications.
2. Another proof comes from a Greek mathematician Euclid.
3. Proofs use existing mathematical rules and logic to demonstrate that a theorem must hold true all the time.
4. Each blue outlined square contains the pieces exactly of one dark and one light gray square proving the Pythagorean theorem again.
5. The Pythagorean theorem has more than 350 proofs and counting ranging from brilliant to obscure.
6. They came up with elegant proofs for the famous Pythagorean theorem.
7. Some historians speculate that ancient Egyptian surveyors used one such set of numbers.
8. Here's one that uses tessellation, a repeating geometric pattern for a more visual proof.
9. Arrange them so that their hypotenuses form a titled square.
10. The theorem is named for Pythagoras, a Greek philosopher and mathematician in the 6th century B.C.

Task 10. ***Work in pairs and:***

- ***comment on the history of the Pythagoras theorem;***
- ***name proofs of the Pythagoras theorem;***
- ***prove that theorem can be applied in different spheres.***

Task 11. ***Work in pairs. Take turns presenting the information of the video.***

Task 12. ***In mini-groups discuss the cases below.***

1. Some people argue that Pythagoras Theorem is overrated and not as useful as it is made out to be. To what extent do you agree or disagree with this opinion?
2. In today's world, there are many tools and calculators that can easily calculate the length of the hypotenuse without using Pythagoras Theorem. Is this a positive or negative development?
3. Pythagoras Theorem is often seen as a basic concept in mathematics, but its applications go beyond just finding the length of a triangle's side. Discuss some advanced uses of Pythagoras Theorem.
4. Many ancient civilizations, such as the Egyptians and Babylonians, had knowledge of Pythagoras Theorem before Pythagoras himself. To what extent do you think he deserves credit for this discovery?
5. Some people believe that the use of Pythagoras Theorem in education should be replaced with more practical and relevant mathematical concepts. Do you agree or disagree?

Task 13. ***Write an essay (not more than 200 words) about the importance of the Pythagoras theorem.***

CIRCLE

Task 1. ***Have a look at the title of the text. What do you think you can learn from it? What do you know about circle? What would you like to know? Fill in columns 1 and 2 in the grid below. Then read the part of the text assigned to you and fill in columns 3 and 4.***

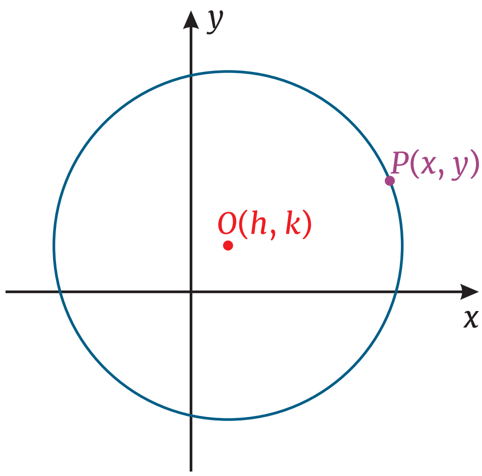
I know ... (What do you already know about circle?)	*I want to know ...* (Write down any new questions you have.)	*I have learned ...* (State 3 main ideas you have learned from the text.)	*I know ... because ...* (Record 1 supporting fact or detail for each of your main ideas in the previous column.)

CIRCLE

1. ______________________________

(1) ...

A **circle** is a two-dimensional figure formed by a set of points that are at a fixed distance (radius) from a fixed point (center) on the plane



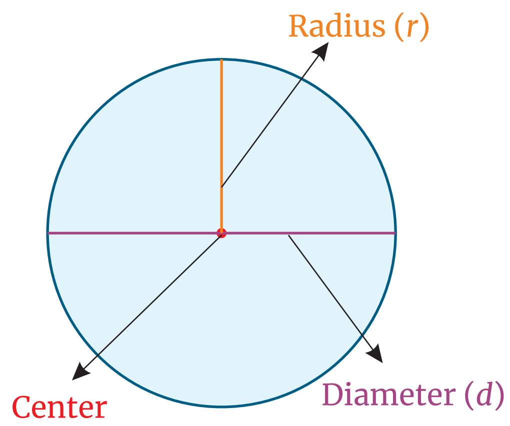
A circle is a part of the conic section, such as a parabola, ellipse, and hyperbola. (2) ... Although a circle is mainly known for its symmetrical properties, it is important to remember its definition as a part of the conic section

The fixed point is called the origin or center of the circle and the fixed distance of the points from the origin is called the radius. Observe the following figure to see the basic parts of a circle, the center, the radius, and the diameter of a circle.

There are many parts of a circle that we should know to understand its properties. A few important parts of a circle are given below.

Circumference is also referred to as the perimeter of a circle and can be defined as the length of the boundary of the circle.

Radius is the distance from the center of a circle to any point on its boundary. A circle has an infinite number of radii



(3) ... We should note that there can be multiple ***diameters*** in the circle, but they should:

- Pass through the center.
- Be straight lines.
- Touch the boundary of the circle at two distinct points which lie opposite to each other.

A chord is any line segment touching the circle at two different points on its boundary. The longest chord in a circle is its diameter which passes through the center and divides it into two equal parts.

A tangent is a line that touches the circle at a unique point and lies outside the circle.

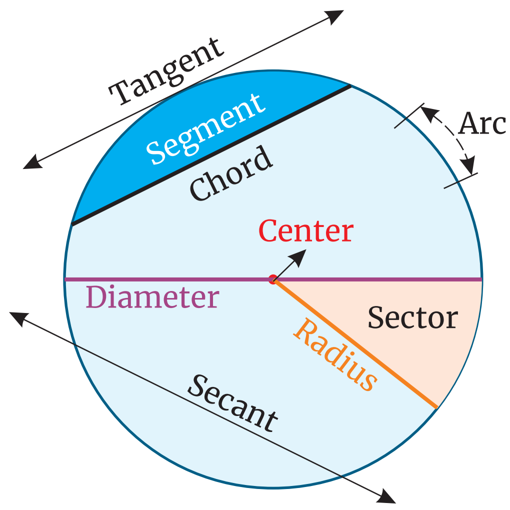
A line that intersects two points on an arc/circumference of a circle is called ***the secant***.

Arc of a circle is referred to as a curve which is a part or portion of its circumference.

Segment in a Circle. The area enclosed by the chord and the corresponding arc in a circle is called a segment. (4) ...

The sector of a circle is defined as the area enclosed by two radii and the corresponding arc in a circle. There are two types of sectors — minor sector, and major sector.

Observe the picture which shows all the important parts of a circle.



Let us move ahead and learn about some interesting properties of circles that make them different from other geometric shapes. Here is a list of properties of a circle.

- A circle is a closed 2D shape that has one curved face.
- Two circles can be called congruent if they have the same radius.
- (5) ...
- The perpendicular bisector of a chord passes through the center of the circle.
- When two circles intersect, the line connecting the intersecting points will be perpendicular to the line connecting their center points.
- Tangents drawn at the endpoints of the diameter are parallel to each other.

2. ______________________________

Earlier, we saw the farmer going around his field and wanted to measure the distance he had covered. It is nothing but the circumference of a circle.

The circumference of a circle is the distance around a circle. It is just another word for the ***perimeter*** of a circle.

If you draw a circle and trace it from one point and stop at the same point after one round, the distance you have outlined is the circumference of that circle.

To find the circumference of a circle, the concept of π is esse tial.

Every circle one can draw, at its core, has one property in common. This property or characteristic is what gives rise to π.

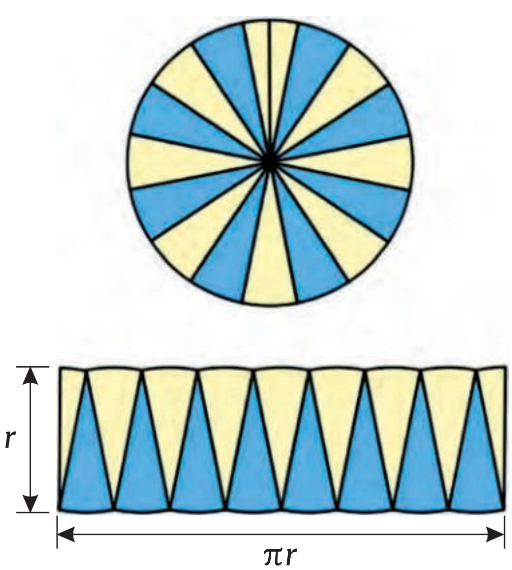
The ratio of the circumference of a circle to the diameter of the circle is known as π.

The radius and circumference of a circle have the following relationship:

$\pi = \frac{C}{2r}$ where C denotes the circumference of the circle and r is its radius. We recall that the diameter is twice the radius. This is how, from the definition of π, we get the formula for the circumference of a circle: $C = 2\pi r$.

π is an irrational number, approximately given by 3.14159265... and it never ends. But for the convenience of calculations, it is approximated to 3.14 or as the fraction $\frac{22}{7}$.

To help the farmer estimate how many pesticides and crops he will need for his field, we will discuss the area of a circle. (6 ...



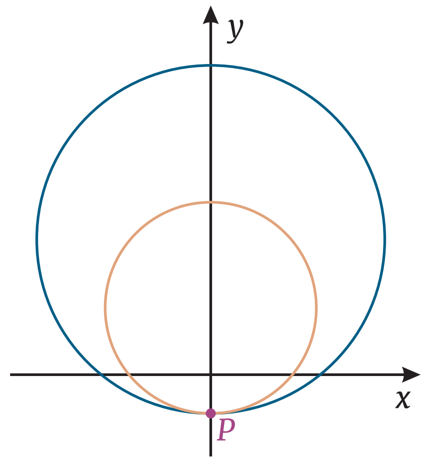
The area of a circle can be derived by cutting the circle into small pieces as follows.

If we break the circle into little triangular pieces (like that of a pizza slice) and put them together so that a rectangle is formed, it may not look like an exact rectangle. But if we cut the circle into thin enough slices, we can approximate it to a rectangle.

Observe that we have divided the slices into two equal parts and color them blue and yellow to differentiate them. Hence, the length of the rectangle formed will be half of the circumference of the circle which will be $\pi \times r$. And the breadth will be the size of the slice, which is equal to the circle's radius, r.

We did this because we have the formula to calculate the area of a rectangle: the length times the breadth. Thus, we have $A = (\pi \times r) \times r$

$$A = \pi r^2.$$



Verbally, the area of a circle with radius r is equal to π times the radius squared. Hence, the units of area are cm^2, m^2 or (any unit of length)^2.

Circles are of various types, which are uniquely related to each other. Such circles are classified into three types, as follows:

(7) ... But a unique way of them crossing will be when they intersect at one and only one point. Such circles are known as ***tangent circles***.

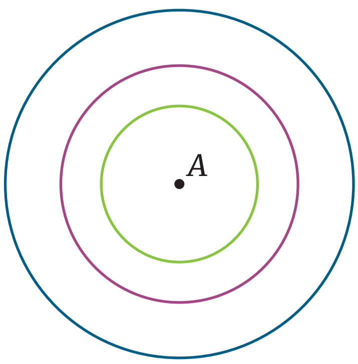
As seen in the above diagram, two circles intersect at a single point, making them tangent circles.

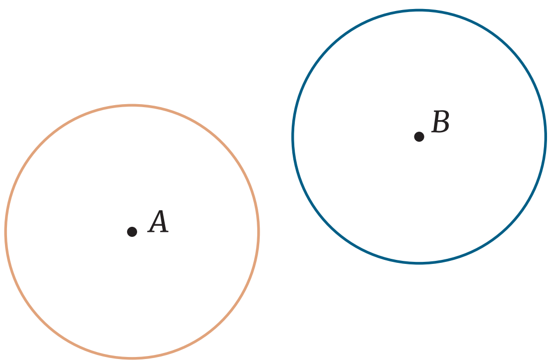
The word 'concentric' means 'having one center,' which leads to the definition

(8) ... Unlike Tangent Circles, where the circles intersect at one and only one point, concentric circles have the unique property of never crossing with one another. Concentric circles look like this:

(9) ...

Draw a circle and duplicate it, you get two ***congruent circles.***





(10) ... There is not much to be said about congruent circles apart from the fact that they are identical and need not exist at a specific location on a Cartesian plane. Here is a diagram of what two identical circles look like:

https://www.studysmarter.co.uk/explanations/math/pure-maths/circles/

Task 2. A. ***Make a summary of your part of the text and present to your partner.***

B. ***Listen to your partner's summary and fill in columns 3 and 4 of the chart in Task 1.***

Task 3. ***Match the headings with paragraphs.***

A. Circumference, Area and Types of Circles.
B. Circle and its Properties.

Task 4. ***Read the text once again and fill in the gaps with sentences from the list below. There are 2 extra sentences.***

1. Imagine an upright cone; if it is sliced in such a way that it is parallel to its base, then the cross-section formed is a circle.
2. The area of a kite is half the product of the diagonals.
3. We say that two circles are congruent if they are the same in every single way, i.e., they are identical.
4. Imagine two circles need not be congruent and can intersect in infinitely many ways.
5. In terms of the equation of these circles, since the center remains the same, the equations only differ in terms of the radius
6. Equal chords are always equidistant from the center of the circle.
7. Two or more circles that share the same center are called concentric circles.
8. We encounter many types of shapes in nature; the most symmetrical shape we can imagine is a circle.
9. There are two types of segments — minor segment and major segment.
10. Just one pair of the opposite sides are parallel.
11. The area of a circle is the region occupied by a circle in a two-dimensional plane.
12. A diameter is a straight line passing through the center that connects two points on the boundary of the circle.

Task 5. ***In pairs, answer the questions below.***

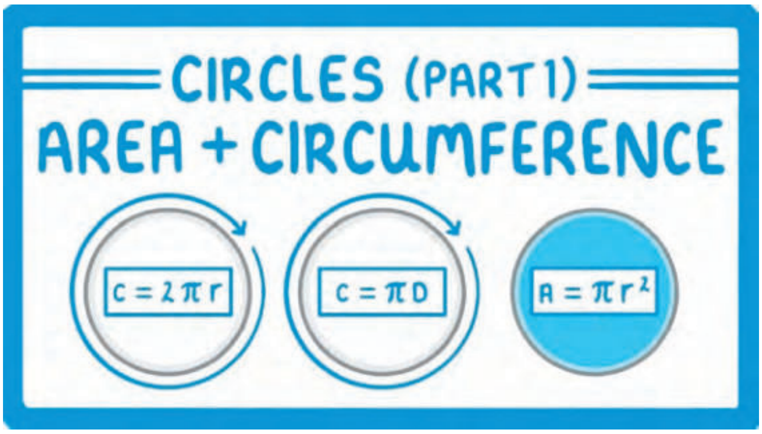
1. What is a circle?
2. What are a few important parts of a circle?
3. What properties of a circle are there?
4. What is a circumference of a circle?
5. How can the area of a circle be derived?
6. What types of circles are there?
7. What circles are called tangent circles?
8. What unique property do concentric circles have?
9. What circles are called congruent circles?

Task 6. ***Share your experience on one of the cases below with the groupmates.***

1. The role of geometry in my everyday life, specifically when calculating the area and circumference of circular objects.
2. How learning about the area and circumference of a circle helped me appreciate the beauty of math.
3. My experience using the formula for finding the area and circumference of a circle to solve real-world problems.

Task 7. ***What do you think the video "How to find the Area and Circumference of a Circle" is about:***

- ***area of a circle;***
- ***radius;***
- ***circumference;***
- ***diameter?***



https://drive.google.com/file d/1ucs3PonKsDOWbBYQHA7f4pA-9UF3OY-O/view?usp=sharing

Tick those themes that will be referred to in the video. Watch the film to make sure you are right.

Task 8. ***Arrange the sentences below in the order they appear in the video.***

1. The important thing to notice here is that these two equations for circumference both mean exactly the same thing.
2. Radius is a straight line that passes from the center of the circle to the circumference.
3. If you have a calculator you can just use the π button and do pi times 16.
4. When it comes to actually calculating the area or circumference of a circle all we have to do is use the formulas.
5. Then for the circumference we can use either equation because we now have the radius and the diameter.
6. The circumference of a circle is the curved line that makes up the outside boundary of the circle.
7. So if the radius is four centimeters then the diameter is two times four centimeters so eight centimeters.
8. We can tell from the diagram that the radius of the circle is four centimeters.
9. The diameter is a straight line that passes from one side of a circle to the other side and importantly passes through the center.

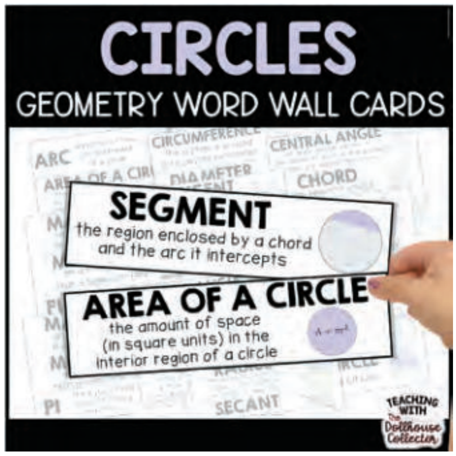
Task 9. ***Work in pairs and:***
- ***give the definitions to: circumference, diameter, radius***
- ***find: the area of a circle, circumference of a circle***

Task 10. ***In mini-groups discuss the cases below.***

1. Many people believe that the study of mathematics is only useful for those pursuing careers in science or engineering. To what extent do you agree or disagree?
2. The concept of π has fascinated mathematicians and philosophers for centuries. Is this obsession with π justified or unnecessary
3. In recent years, there has been a push to include more practical, real-world applications in math education. Should the study of the area and circumference of a circle be emphasized more in schools?

4. Some argue that memorizing formulas, such as the one for finding the area and circumference of a circle, is not necessary in today's age of technology. Do you agree or disagree?
5. The use of technology, such as calculators and computer programs, has made it easier to calculate the area and circumference of a circle. Is this a positive or negative development?

Task 11. ***Write an internet forum entry (150—200 words) about the circles.***



PROBLEM-SOVING

WHAT ARE THE PECULIARITIES OF MAJOR BRANCHES OF GEOMETRY?

Task 1. ***Let's delve into the major branches of geometry to gain a better understanding of this captivating field.***

Geometry is the branch of mathematics that explores the properties and relationships of shapes, sizes, and spaces. It's a discipline that has fascinated mathematicians, scientists, and thinkers for millennia. At its core, geometry helps us make sense of the world around us by studying the fundamental building blocks of our universe: points, lines, angles, and shapes.

So, what are the peculiarities of major branches of geometry?
DIG DEEPER TO FIND THE ANSWERS TO THE QUESTION ABOVE!

Task 2. ***Read one of the texts below and fill in the table below.***

Branches of geometry	Peculiarities	Fields of applications

Task 3. ***Share the information with your groupmates.***

Task 4. ***Fill in the table above with the information your groupmates shared with you.***

Task 5. ***Study the table thoroughly and answer the topical question***

WHAT ARE THE PECULIARITIES OF MAJOR BRANCHES OF GEOMETRY?

Discuss it in pairs.

Task 6. ***Make an educational poster "The importance of each branch of geometry", depicting peculiarities of major branches of Geometry.***

MAJOR BRANCHES OF GEOMETRY

Text 1

Euclidean geometry is the study of plane and solid figures on the basis of axioms and theorems employed by the Greek mathematician Euclid (c. 300 BCE). In its rough outline, Euclidean geometry is the plane and solid geometry commonly taught in secondary schools. Indeed, until the second half of the 19th century, when non-Euclidean geometries attracted the attention of mathematicians, geometry meant Euclidean geometry. In Euclid's great work, the Elements, the only tools employed for geometrical constructions were the ruler and the compass — a restriction retained in elementary Euclidean geometry to this day.

In its rigorous deductive organization, the Elements remained the very model of scientific exposition until the end of the 19th century, when the German mathematician David Hilbert wrote his famous "Foundations of Geometry" (1899). The modern version of Euclidean geometry is the theory of Euclidean (coordinate) spaces of multiple dimensions, where distance is measured by a suitable generalization of the Pythagorean theorem.

Euclid realized that a rigorous development of geometry must start with the foundations. Hence, he began "Elements" with some undefined terms, such as "a point is that which has no part" and "a line is a length without breadth." Proceeding from these terms, he defined further ideas such as angles, circles, triangles, and various other polygons and figures. For example, an angle was defined as the inclination of two straight lines, and a circle was a plane figure consisting of all points that have a fixed distance (radius) from a given centre

As a basis for further logical deductions, Euclid proposed five common notions, such as "things equal to the same thing are equal", and ve unprovable but intuitive principles known variously as postulates or axioms. Stated in modern terms, the *axioms* are as follows.

1. Given two points, there is a straight line that joins them.
2. A straight line segment can be prolonged indefinitely
3. A circle can be constructed when a point for its centre and a distance for its radius are given.
4. All right angles are equal.
5. If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely will meet on that side on which the angles are less than the two right angles.

Hilbert refined axioms (1) and (5) as follows

1. For any two different points, (a) there exists a line containing these two points, and (b) this line is unique.

5. For any line *L* and point *p* not on *L*, (a) there exists a line through p not meeting L, and (b) this line is unique.

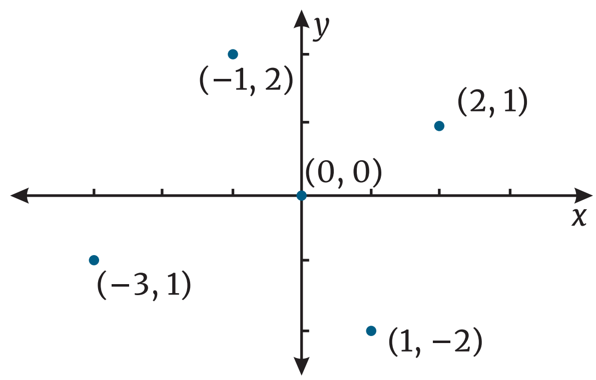
The fifth axiom became known as the "parallel postulate," since it provided a basis for the uniqueness of parallel lines. All five axioms provided the basis for numerous provable statements, or theorems, on which Euclid built his geometry.

Text 2

Analytic geometry is also called coordinate geometry, mathematical subject in which algebraic symbolism and methods are used to represent and solve problems in geometry. Analytic geometry was initiated by the French mathematician René Descartes (1596—1650), who introduced rectangular coordinates to locate points and to enable lines and curves to be represented with algebraic equations. The importance of analytic geometry is that it establishes a correspondence between geometric curves and algebraic equations.

This correspondence makes it possible to reformulate problems in geometry as equivalent problems in algebra, and vice versa; the methods of either subject can then be used to solve problems in the other. For example, computers create animations for display in games and films by manipulating algebraic equati ns.

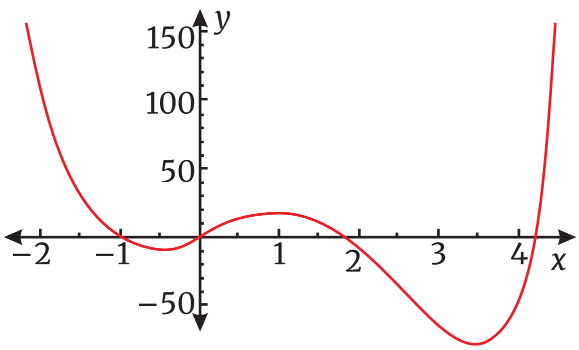
Apollonius of Perga (*c.* 262—190 BC), known by his contemporaries as the "Great Geometer," foreshadowed the development of analytic geometry by more than 1,800 years with his book *Conics*. He define a conic as the intersection of a cone and a plane. Using Euclid's results on similar triangles and on secants of circles, he found a relation satisfied by the distances from any point *P* of a conic to two perpendicular lines, the major axis of the conic and the tangent at an endpoint of the axis. These distances correspond to coordinates of *P*, and the relation between these coordinates corresponds to a quadratic equation of the conic. Apollonius used this relation to deduce fundamental properties of conics.



Several points are labeled in a two-dimensional graph, known as the Cartesian plane. Note that each point has two coordinates, the first number (*x* value) indicates its distance from the *y*-axis — positive values to the right and negative values to the left — and the second number (*y* value) gives its distance from the *x*-axis — positive values upward and negative values downward.

At the end of the 16th century, the French mathematician François Viète introduced the first systematic algebraic notation, using letters to represent known and unknown numerical quantities, and he developed powerful general methods for working with algebraic expressions and solving algebraic equations. With the power of algebraic notation, mathematicians were no longer completely dependent upon geometric figures and geometric intuition to solve problems

Descartes and Fermat independently founded analytic geometry in the 1630s by adapting Viète's algebra to the study of geometric loci. They moved decisively beyond Viète by using letters to represent distances that are variable instead of fixed Descartes used equations to study curves defined geometrically, and he stressed the need to consider general algebraic curves — graphs of polynomial equations in *x* and *y* of all degrees. He demonstrated his method on a classical problem: finding all points *P* such that the product of the distances from *P* to certain lines equals the product of the distances to other lines.



The figure shows part of the graph of the polynomial equation $y = 3x^4 - 16x^3 + 6x^2 + 24x + 1$. Note that the same scale need not be used for the x- and y-axis.

Fermat emphasized that any relation between x and y coordinates determines a curve. Fermat indicated that any quadratic equation in x and y can be put into the standard form of one of the conic sections.

Text 3

Analytic geometry had its greatest impact on mathematics via **calculus**. Without access to the power of analytic geometry, classical Greek mathematicians such as Archimedes (*c.* 285—212/211 BC) solved special cases of the basic problems of calculus: finding tangents and extreme points (differential calculus) and arc lengths, areas, and volumes (integral calculus). Renaissance mathematicians were led back to these problems by the needs of astronomy, optics, navigation, warfare, and commerce. They naturally sought to use the power of algebra to define and analyze a growing range of curves.

Fermat developed an algebraic algorithm for finding the tangent to an algebraic curve at a point by nding a line that has a double intersection with the curve at the point — in essence, inventing differential calculus. Descartes introduced a similar but more complicated algorithm using a circle. Fermat computed areas under the curves $y = ax^k$ for all rational numbers $k \neq -1$ by summing areas of inscribed and circumscribed rectangles. (*See* exhaustion, method of.) For the rest of the 17th century, the groundwork for calculus was continued by many mathematicians, including the Frenchman Gilles Personne de Roberval, the Italian Bonaventura Cavalieri, and the Britons James Gregory, John Wallis, and Isaac Barrow.

Newton and the German Gottfried Leibniz revolutionized mathematics at the end of the 17th century by independently demonstrating the power of calculus. Both men used coordinates to develop notations that expressed the ideas of calculus in full generality and led naturally to differentiation rules and the fundamental theorem of calculus (connecting differential and integral calculus)

Newton demonstrated the importance of analytic methods in geometry, apart from their role in calculus, when he asserted that any cubic — or, algebraic curve of degree three — has one of four standard equations, $xy^2 + ey = ax^3 + bx^2 + cx + d$, $xy = ax^3 + bx^2 + cx + d$, $y^2 = ax^3 + bx^2 + cx + d$, $y = ax^3 + bx^2 + cx + d$, for suitable coordinate axes. The Scottish mathematician James Stirling proved this assertion in 1717, possibly with Newton's aid. Newton divided cubics into 72 species, a total later corrected to 78.

Newton also showed how to express an algebraic curve near the origin in terms of the fractional power series $y = a_1x^{1/k} + a_2x^{2/k} + \ldots$ for a positive integer k. Mathematicians have since used this technique to study algebraic curves of all degrees.

Text 4

Projective geometry is a branch of mathematics that deals with the relationships between geometric figures and the images, or mappings that result from projecting them onto another surface.

Projective geometry has its origins in the early Italian Renaissance, particularly in the architectural drawings of Filippo Brunelleschi (1377—1446) and Leon Battista Alberti (1404—1472), who invented the method of perspective drawing. By this method, the eye of the painter is connected to points on the landscape (the horizontal reality plane, *RP*) by so-called sight lines. The intersection of these sight lines with the vertical picture plane (*PP*) generates the drawing. Thus, the reality plane is projected onto the picture plane, hence the name *projective* geometry.

Although some isolated properties concerning projections were known in antiquity, particularly in the study of optics, it was not until the 17th century that mathematicians returned to the subject. The French mathematicians Girard Desargues (1591—1661) and Blaise Pascal (1623—1662) took the first significan steps by examining what properties of figures were preserved (or invariant) under perspective mappings. The subject's real importance, however, became clear only after 1800 in the works of several other French mathematicians, notably Jean-Victor Poncelet (1788—1867). In general, by ignoring geometric measurements such as distances and angles, projective geometry enables a clearer understanding of some more generic properties of geometric objects. Such insights have since been incorporated in many more advanced areas of mathematics.

Text 5

The German mathematician Carl Friedrich Gauss (1777—1855), in connection with practical problems of surveying and geodesy, initiated the field of **differential geometry**. Using differential calculus, he characterized the intrinsic properties of curves and surfaces. For instance, he showed that the intrinsic curvature of a cylinder is the same as that of a plane, as can be seen by cutting a cylinder along its axis and flattening, but not the same as that of a sphere, which cannot be flattened without distortion.

Diffe ential geometry is a branch of mathematics that studies the geometry of curves, surfaces, and manifolds (the higher-dimensional analogs of surfaces). The discipline owes its name to its use of ideas and techniques from differential calculus, though the modern subject often uses algebraic and purely geometric techniques instead. Although basic definitions, notations, and analytic descriptions vary widely, the following geometric questions prevail: How does one measure the curvature of a curve within a surface (intrinsic) versus within the encompassing space (extrinsic)? How can the curvature of a surface be measured? What is the shortest path within a surface between two points on the surface? How is the shortest path on a surface related to the concept of a straight line?

While curves had been studied since antiquity, the discovery of calculus in the 17th century opened up the study of more complicated plane curves — such as those produced by the French mathematician René Descartes (1596—1650) with his "compass". In particular, integral calculus led to general solutions of the ancient problems of finding the arc length of plane curves and the area of plane figures. This in turn opened the stage to the investigation of curves and surfaces in space — an investigation that was the start of differential geometry

Text 6

Beginning in the 19th century, various mathematicians substituted alternatives to Euclid's parallel postulate, which, in its modern form, reads, "given a line and a point not on the line, it is possible to draw exactly one line through the given point parallel to the line." They hoped to show that the alternatives were logically impossible. Instead, they discovered that consistent **non-Euclidean geometries** exist.

Non-Euclidean geometry is literally any geometry that is not the same as Euclidean geometry. Although the term is frequently used to refer only to hyperbolic geometry, common usage includes those few geometries (hyperbolic and spherical) that differ from but are very close to Euclidean geometry

The non-Euclidean geometries developed along two different historical threads. The first thread started with the search to understand the movement of stars and planets in the apparently hemispherical sky. For example, Euclid wrote about spherical geometry in his astronomical work *Phaenomena*. In addition to looking to the heavens, the ancients attempted to understand the shape of the Earth and to use this understanding to solve problems in navigation over long distances (and later for large-scale surveying). These activities are aspects of spherical geometry.

Text 7

Topology is concerned with the properties of a geometric object that are preserved under continuous deformations, such as stretching, twisting, crumpling, and bending; that is, without closing holes, opening holes, tearing, gluing, or passing through itself.

A topological space is a set endowed with a structure, called a *topology*, which allows defining continuous deformation of subspaces, and, more generally, all kinds of continuity. Euclidean spaces, and, more generally, metric spaces are examples of a topological space, as any distance or metric defines a topology. The deformations that are considered in topology are homeomorphisms and homotopies. A property that is invariant under such deformations is a topological property. The following are basic examples of topological properties: the dimension, which allows distinguishing between a line and a surface; compactness, which allows distinguishing between a line and a circle; connectedness, which allows distinguishing a circle from two non-intersecting circles.

The ideas underlying topology go back to Gottfried Leibniz, who in the 17th century envisioned the *geometria situs* and *analysis situs*. Leonhard Euler's Seven Bridges of Königsberg problem and polyhedron formula are arguably the field's firs theorems. The term *topology* was introduced by Johann Benedict Listing in the 19th century; although, it was not until the first decades of the 20th century that the idea of a topological space was developed.

The following are some of the subfields of topology

1. ***General topology*** normally considers local properties of spaces, and is closely related to analysis. It generalizes the concept of continuity to define topological spaces, in which limits of sequences can be considered. Sometimes distances can be defined in these spaces, in which case they are called metric spaces; sometimes no concept of distance makes sense.

2. ***Combinatorial topology*** considers the global properties of spaces, built up from a network of vertices, edges, and faces. This is the oldest branch of topology, and dates back to Euler. It has been shown that topologically equivalent spaces have the same numerical invariant, which we now call the Euler characteristic. This is the number $(V - E + F)$, where V, E, and F are the number of vertices, edges, and faces

of an object. For example, a tetrahedron and a cube are topologically equivalent to a sphere, and any “triangulation” of a sphere will have an Euler characteristic of 2.

3. ***Algebraic topology*** also considers the global properties of spaces, and uses algebraic objects such as groups and rings to answer topological questions. Algebraic topology converts a topological problem into an algebraic problem that is hopefully easier to solve. For example, a group called a homology group can be associated to each space, and the torus and the Klein bottle can be distinguished from each other because they have different homology groups

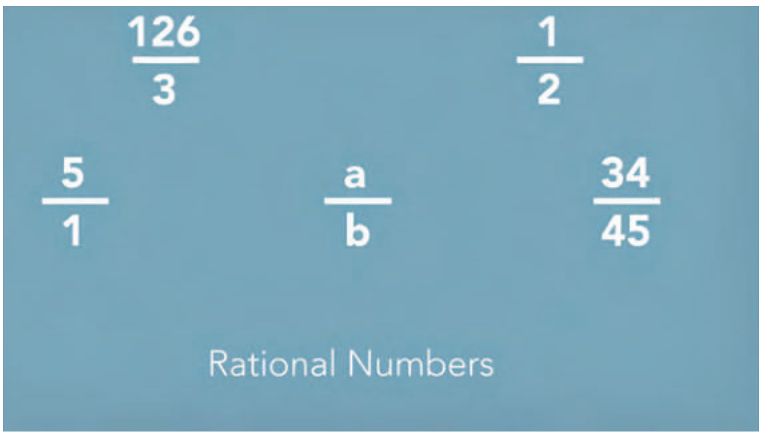
4. ***Differentia topology*** considers spaces with some kind of smoothness associated to each point. In this case, the square and the circle would not be smoothly (or differentiably) equivalent to each other. Differential topology is useful for studying properties of vector fields, such as a magnetic or electric fields.Topology is used in many branches of mathematics, such as differentiable equations, dynamical systems, knot theory, and Riemann surfaces in complex analysis. It is also used in string theory in physics, and for describing the space-time structure of universe.

https://www.britannica.com/science/Euclidean-geometry

Appendix I
BUILDING AUTONOMY

RATIONAL AND IRRATIONAL NUMBERS

Task 1. ***Have a look at the title of the video. What do you think you can learn from it? What irrational numbers do you know? What would you like to know about them?***



https://www.youtube.com/watch?v=sbGjr_awePE

Fill in columns 1and 2 in the grid below. Then watch the video and fill in columns 3 and 4.

I know ... (What do you already know about rational numbers?)	*I want to know ...* (Write down any new questions you have.)	*I have learned ...* (State 3 main ideas you have learned from the video.)	*I know ... because ...* (Record 1 supporting fact or detail for each of your main ideas in the previous column.)

Task 2. ***Watch the video and put the headings of the episodes logically in accordance with the video content.***

1. Mathematics revolutionized by Hippasus.
2. The reverence of the Pythagorean mathematicians to numbers.
3. Proof by contradiction.
4. Hippasus' "crime".
5. Decimals and ratios as ways to express numbers.
6. The number that violated the harmonious rule.
7. The famous irrational number.

Task 3. A. ***Watch the video and find the names for.***

1. A group of mathematicians, who had a religious reverence for numbers.
2. The building blocks of the Universe according to the Pythagorean mathematicians' belief.
3. The number that was not supposed to exist.
4. The number, produced by two times any number.
5. The number that represents the ratio of a circle`s circumference to its diameter.

B. ***Watch the video and find the facts that prove that.***

1. Hippasus' was punished for discovering irrational numbers.
2. $\sqrt{2}$ is an irrational number.
3. The Pythagorean worldview was not true.
4. It is possible to plot some irrational numbers on the number line.
5. It is important to explore the impossible, whatever the myths may say.

Task 4. ***In pairs, discuss the following problems.***

1. You have seen the proof of how the square root of 2 is irrational. In a similar way, is it possible to prove that the square root of 3 is irrational? Prove your ideas.
2. The diagonal of a square with sides equal to 1 unit is irrational. Are the diagonals of all squares irrational? Give arguments that confirm your an wer.
3. How can we plot $\sqrt{17}$ on the number line?
4. The diagonal of a square with sides equal to 1 unit is irrational. Are the diagonals of all squares irrational?

Task 5. ***Do you agree with the statements below? Discuss them with your partner.***

1. Irrational numbers are called so, because they exist outside the realm of rationality.
2. We could do without irrational numbers in mathematics and still have a functioning system.
3. Imaginary numbers are just constructs of our imagination.
4. Irrational numbers are completely random and arbitrary.

Task 6. A. ***What do the digits in this picture stand for?***

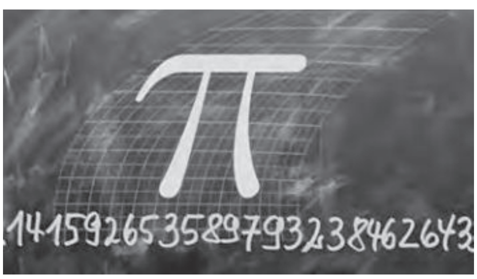
B. ***Read the text and choose the best summary.***

1. Pi, a never-ending irrational number approximated as 3.14, cannot be expressed as a finite fraction. Unlike patterns like 1/3 or 0.3333, Pi's decimal places continue infinitely. To date, it has been calculated to over 22 trillion digits, an astonishing feat achieved by a computer in 105 days. There was even an attempt to round Pi by law, but it failed due to the recognition that a mathematical constant should not be altered through legislation.
2. Pi, an ancient concept going back 4,000 years, refers to the ratio of a circle's circumference to its diameter. The term "Pi" originates from the Greek letter π, and mathematicians did not adopt it until the 18th century. Prior to that, various cumbersome descriptors were used. It is truly remarkable how this ratio remains consistent regardless of the circle's size.
3. Pi, an irrational yet incredibly useful mathematical concept, has a fascinating history. It is employed in stress testing computers and pushing the limits of human memory. Notably, March 14th is celebrated as International Pi Day, coinciding with Albert Einstein's birthday.

FASCINATING FACTS ABOUT THE IRRATIONAL YET WONDERFUL PI

π is both amazing and highly misunderstood. Not only is it irrational but it is also one of the most useful mathematical concepts in the world.

On the surface, it is a very simple concept, but when you dig a little deeper you soon realize there is something very special about this seemingly innocuous number.



It also happens to have a long and very interesting history. Whilst the concept of it has been known for thousands of years, it was only fairly recently that the world agreed on a name for it.

It is used to stress test computers and also test the very limits of human memory. Chances are you might use it every day too.

1. ______________________________

Whilst this might sound a little random at first, it's actually a funny joke — but only if you write numbers in the American format. March the 14th is, after all, 3/14 — the first 3 numbers of π or 3.14

This also happens to be Albert Einstein's birthday too. Coincidence? Yes, of course, it is. It was first 'observed' back in 1988 when Larry Shaw of San Francisco's Exploratorium Science Museum started the trend. Its popularity has grown exponentially since then, and in 2009 the U.S. Congress passed a resolution to make it official But it would have to wait until 2010 to get it's own officia Google Doodle — lucky π!

2. ______________________________

The very concept of π is very old indeed. It was first described around 4,000 years ago and had been figured out by the ancient Egyptians and Babylonians.

Just like today, they had figured out the ratio of the circumference of any circle to its diameter to equal, roughly, 3.14. The size of the circle was irrelevant, you always got the same answer.

They were even integrating it into various calculations.

The very term "Pi" also comes from the Greek letter of the same pronunciation, π. But it wasn't until the 18th Century that this became common practice amongst mathematicians.

Before that, there was no agreed name for it. Many descriptors were often a literal mouthful with examples like "the quantity which when the diameter is multiplied by it, yields the circumference" being used.

Hardly efficient, to say the leas

3. ______________________________

Whilst the approximate value of π is 3.14, being an irrational number, the number of decimal places is, in fact, never-ending. This means it can't actually easily be expressed as a proper fraction with a definite end

It also doesn't end with a repeating pattern like 1/3, or 0.33333 reoccurring. π, at least as far as thus far calculated, doesn't appear to have a certain number of finite decimal places.

For this reason, it just keeps going and going ad infinitum. To date, π has been calculated to over 22 Trillion digits — which is frankly an incredible achievement in its own right.

This took a computer, with 24 hard drives, over 105 days to achieve working nonstop. Imagine how long that would take a human being!

4. ______________________________

Since π is never-ending, you might think we should all agree to cut it o somewhere. Surely, you might think, it might just be easier to call it just 3.14 or, say, 3.2?

This is precisely what an Indiana doctor tried to do in 1897. He argued that the world should just round it up for any, and all, future calculations that required it.

Dr. Edwin Goodwin actually proposed a bill to state legislature. He also copyrighted his idea and planned to charge royalties for anyone who used it in the future.

The bill was debated with the majority of delegates agreeing that any attempt to use the force of law to change a mathematical constant was inappropriate. It failed to pass.

5. ______________________________

π is not just a play thing for mathematicians. It is used every single day by scientists, engineers, and mathematicians for important calculations.

In fact, it is quite likely that if you deal with volumes, areas, etc. regularly there will a point where you use it yourself. In a lot of cases, these calculations can make or break a project or product.

For example, Pi is a vital component in your car or smartphones GPS to calculate your exact location on the Earth.

6. ______________________________

Whilst it might be a fun party trick to reel off as many of Pi's decimals from memory as possible, the more you use in calculations, the better.

For small scale things, like the volume of your coffee cup, it might seem overkill, but as you scale up the size of something the more decimals you use the more accurate your calculations.

For example, if you were to just use the first 9 digits of Pi to calculate the Earth's circumference, your result would be pretty close to reality. For every 25,000 miles (40,234 km) or so, the use of 9 digits would only give you an error of around 1/4 of an inch (0.635 cm).

That's pretty impressive when you think about it.

7. ______________________________

Pi is an incredibly long number, as we have seen. But have you ever wondered what the record for the longest recital of these by a human being?

As it happens the record was set in March of 2015 by one Rajveer Meena at VIT University in Vellore, India. According to the Guinness Book of World Records, Meena was able to recite Pi to an amazing 70,000 decimals places.

The feat took him more than 9 hours to complete. To help him focus (and not cheat), Meena, 21 years old at the time, was completely blindfolded too.

Task 7. ***Read the text again and fill in the gaps with the subtitles below:***

A. The record for reciting Pi is to 70,000 decimal places.
B. Pi is not just a mathematical curiosity.
C. The more decimals, the better, apparently.
D. Pi is a very old concept.
E. Pi is never-ending.
F. March 14th is international Pi day.
G. There was a time that Pi was rounded up by law.

Task 8. ***In small groups, discuss the following question and support your arguments:***

1. Square roots of non-perfect squares (which are irrational numbers) can be plotted on a number line. But can Pi be plotted on a number line? Explain, why.
2. Is π a naturally occurring constant in the physical world, or is it simply a mathematical abstraction with no direct connection to realty?
3. Can we ever truly calculate the exact value of π or will it always remain an approximation due to its infinite decimal representation
4. Is π a truly fundamental concept in mathematics, or are there alternative mathematical frameworks that could lead to different values o π?
5. Why does π receive so much attention and fascination?

Task 9. ***Make up your own video-lesson about rational numbers.***

SPECIAL QUADRILATERALS

https://drive.google.com/file d/1JrZmXN7_O4_To182Epf1Sd-4lr7Q6DQ8/view?usp=drive_link

Task 1. ***Have a look at the title of the video. What do you think you can learn from it? What do you know about Euler's identity? What would you like to know? Fill in columns 1 and 2 in the grid below.***

I know ... (What do you already know about special quadrilaterals?)	*I want to know ...* (Write down any new questions you have.)	*I have learned ...* (State 3 main ideas you have learned from the video.)	*I know ... because ...* (Record 1 supporting fact or detail for each of your main ideas in the previous column.)

Fill in columns 3 and 4 after watching the video "Special quadrilaterals and their properties".

Task 2. A. ***Match the halves of the sentences.***

1. I only emphasize this because specific types of quadrilaterals
2. While the parallelogram appears unique for having both pairs of opposite sides
3. The rhombus just happens to have more properties
4. Also unlike a parallelogram diagonals now bisect their angles and because opposite angles are congruent
5. I like to think of the square as the most specific type of quadrilatera

a) parallel as opposed to just one pair or no pairs at all.
b) than that of the parallelogram.
c) and that it is loaded with many features because it inherits everything before it.
d) have specific properties
e) you will get two quartets of congruent angles thanks to the diagonals.

B. ***Watch the video about "Special quadrilaterals and their properties" to check your ideas.***

Task 3. ***Arrange the sentences below in the order they appear in the video. Watch the video again if needed.***

1. The diagonals are not necessarily congruent to one another but when they intersect they create sets of congruent.
2. They both have acquired the parallelogram straights but then have taken on different forms all together
3. There is nothing else very unique besides one pair of sides being parallel diagonals don't bisect each other or their angles.
4. The rhombus is just one way that you can mutate a parallelogram into a specifi looking shape.
5. A quadrilateral with diagonals bisecting each other can be a rectangle rhombus or square but at the very least it must be a parallelogram.
6. The first of the quadrilaterals I will focus on is the parallel gram.
7. Don't forget that so keep in mind that trapezoids are in their own family and do not fit in the same family as something like the square or its elders this includes the rhombus rectangle and parallelogram.
8. No matter what kind of quadrilateral I make with all four sides congruent the rhombus will always have both pairs of opposite sides parallel.
9. A square by definition is a quadrilateral with all right angles and all congruent sides.
10. If you may call the side lengths congruent to one another you have constructed what is called a rhombus.

Task 4. ***Choose the correct answer (A—D) for the questions below (1—15).***

1. What is the interior angle sum of a quadrilateral?
   A. 180 degrees.
   B. 270 degrees.
   C. 360 degrees.
   D. 450 degrees.

2. Which property does the parallelogram have?
   A. All four sides congruent.
   B. All right angles.
   C. Both pairs of opposite sides parallel.
   D. Diagonals bisect each other.

3. What is the name of a quadrilateral with all four sides congruent?
   A. parallelogram.
   B. Rhombus.
   C. Rectangle.
   D. Square.

4. What happens to the diagonals in a rhombus?
   A. they become parallel.
   B. They become perpendicular.
   C. They become congruent.
   D. They become longer.

5. What is the main difference between a rectangle and a parallel gram?
   A. Opposite sides are parallel.
   B. All angles are right angles.
   C. Diagonals bisect each other.
   D. All sides are congruent.

6. How many pairs of congruent angles are there in a rhombus?
   A. One.
   B. Two.
   C. Three.
   D. Four.

7. Which statement is true about special quadrilaterals?
   A. They have uniform angles and diagonals.
   B. They have congruent opposite sides.
   C. They have parallel lines and congruent angles.
   D. They have more than four sides.

8. What can a quadrilateral still be called even if it has more specific properties
   A. Parallelogram.
   B. Rhombus.
   C. Quadrilateral.
   D. Trapezoid.

9. Which shape is NOT a type of rhombus or rectangle?
   A. Square.
   B. Parallelogram.
   C. Trapezoid.
   D. Quadrilateral.

10. What is the defining characteristic of a square
   A. All sides are congruent.
   B. All angles are right angles.
   C. Opposite sides are parallel.
   D. Diagonals bisect each other.

11. What is the relationship between a rectangle and a parallelogram?
   A. A rectangle is a type of parallelogram.
   B. A rectangle is a type of rhombus.
   C. A rectangle is a type of trapezoid.
   D. A rectangle is a type of square.

12. What is the defining characteristic of a trapezoid
   A. All sides are congruent.
   B. All angles are right angles.
   C. One pair of sides is parallel.
   D. Diagonals bisect each other.

13. What is the difference between a trapezoid and an isosceles tr pezoid?
    A. An isosceles trapezoid has congruent angles.
    B. An isosceles trapezoid has congruent sides.
    C. An isosceles trapezoid has perpendicular diagonals.
    D. An isosceles trapezoid has all right angles.

14. Which statement is true about base angles in a trapezoid?
    A. Base angles are congruent.
    B. Base angles are supplementary.
    C. Base angles are right angles.
    D. Base angles are parallel.

15. What type of quadrilateral must have diagonals that bisect each other?
    A. Rectangle.
    B. Rhombus.
    C. Square.
    D. Parallelogram.

Task 5. ***In pairs, restore the contents of the video, speaking about the following special quadrilaterals:***

- ***parallelogram;***
- ***rectangle;***
- ***rhombus;***
- ***square;***
- ***trapezoid;***
- ***kite.***

Task 6. ***Discuss the cases below with your partner.***

1. Some argue that understanding the properties of special quadrilaterals is not necessary in our daily lives. Is this a positive or negative development?
2. In today's society, there is a growing emphasis on STEM education. Do you think this focus on science and technology is neglecting the importance of subjects like geometry?
3. The use of special quadrilaterals in architecture has both practical and aesthetic benefits. Discuss both views and give your own opinion
4. With the rise of digital design tools, do you think the knowledge of special quadrilaterals is becoming obsolete?

Task 7. ***Write a social media post about special quadrilaterals. Make it about 120—150 words long, using the information from the video "Special quadrilaterals and their properties".***

Appendix II
BUILDING COMPETENCE

THE BELARUSIAN STATE UNIVERSITY

Task 1. ***Complete the table with derivatives.***

Noun	Verb	Adjective
	to educate	
university		
	to develop	
		scientifi
location		
		disciplinary
institution		
	to create	
		numerous
participant		

Task 2. ***Fill in the gaps with prepositions.***

1. The characteristic feature and the main trend … modern higher and further education is not only to check students' knowledge but develop their abilities and creative thinking.
2. … that time the teaching staff consisted of 14 professors and 25 associate professors.
3. The Belarusian State University contributed much … the foundation and development of the Academy of Sciences … the BSSR, the Republic's State Library and a number of large research institutions.
4. 1941 was the year … the 20th anniversary and at the same time it was the most difficu period … the history of the University.

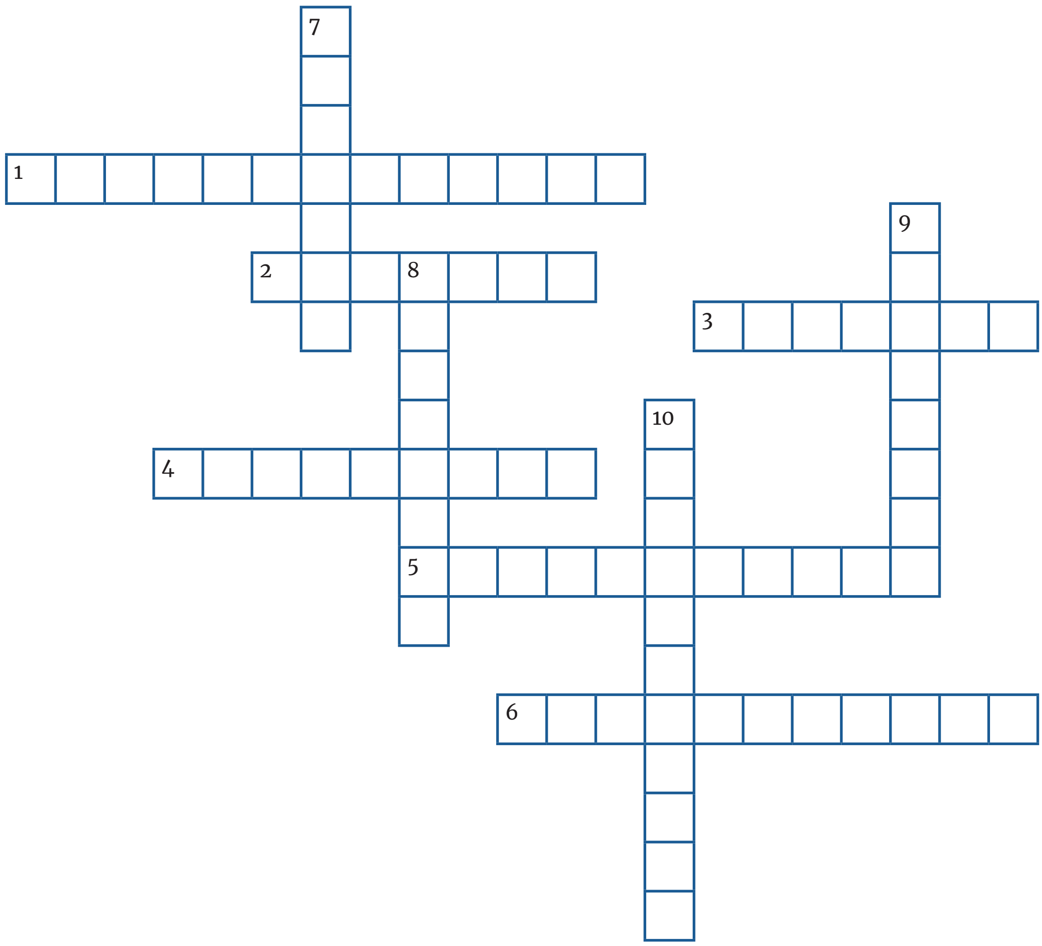
5. The evidence to this is the obelisk erected ... 1970s at the University campus ... honor of those who had perished in the battle against fascism.
6. The Research activity ... the BSU is conducted ... numerous research institutes, laboratories and University departments.
7. The University students do not only study but carry ... various research work, take part ... educational and scientific expeditions
8. The students regularly do sports ... the University sport clubs.

Task 3. ***Find the words in the fillword.***

R	U	I	S	T	A	N	D	A	R	D	P	I	J	G	B	T	L	U	B
I	H	O	Z	E	Y	T	C	Z	C	K	I	E	I	F	W	Q	N	V	Y
X	U	X	H	D	R	C	X	P	X	M	H	K	J	A	S	V	E	F	S
A	W	Q	Q	O	D	E	O	I	O	K	P	E	V	C	K	A	V	T	F
L	T	I	M	U	B	R	S	M	F	T	H	R	D	U	O	Q	C	Y	S
E	X	B	K	N	M	L	W	E	M	Z	E	F	Y	L	V	S	O	E	I
X	K	C	J	I	T	K	C	B	A	U	W	N	H	T	Y	R	L	P	P
B	T	R	J	V	D	N	T	X	D	R	N	Q	T	Y	Q	M	Q	Z	Z
D	L	A	T	E	W	G	O	V	J	L	C	I	P	I	K	V	R	Y	R
P	J	A	P	R	O	G	R	A	M	H	B	H	T	H	A	B	E	G	O
E	P	R	E	S	A	G	N	X	K	R	M	B	T	Y	L	L	N	J	I
T	D	K	Y	I	V	I	A	W	A	O	G	P	R	V	Z	I	X	C	M
D	F	U	O	T	L	F	N	A	G	X	N	R	K	R	H	H	F	V	K
Z	R	X	C	Y	Q	I	A	I	A	H	P	O	N	C	A	I	V	X	V
P	L	P	O	A	V	M	D	K	N	M	D	J	A	B	U	I	I	J	D
C	L	C	H	P	T	V	A	T	O	G	E	E	N	Q	R	L	C	T	I
G	P	R	A	N	K	I	N	G	W	C	T	C	X	P	R	C	U	E	U
K	G	R	X	M	J	L	O	K	B	B	G	T	M	V	R	P	A	E	B
Z	A	F	N	B	X	M	J	N	A	I	Y	F	W	R	F	I	J	K	C
E	S	W	E	S	X	I	U	O	Z	Z	P	G	S	S	A	P	K	F	V

EDUCATION	PROGRAM	PROJECT
TRAINING	POTENTIAL	STANDARD
COMMUNITY	FACULTY	RANKING
TEACHING	RESEARCH	UNIVERSITY

Task 4. ***Do the crossword "The BSU"***



Across

1. A student who is studying for their first degree at a college or university.
2. To give something that is needed or wanted to someone.
3. A group of people elected or chosen to make decisions or give advice on a particular subject, to represent a particular group of people, or to run a particular organization.
4. A large building at a college or university where students live.
5. An occasion or situation which makes it possible to do something that you want to do or have to do, or the possibility of doing something.
6. Something very good and difficul that you have succeeded in doing.

Down

7. A building, room, or organization that has a collection esp. of books, music, and information that can be accessed by computer for people to read, use, or borrow.
8. A type of work that you feel you are suited to doing and to which you should give all your time and energy, or the feeling that a type of work suits you in this way.
9. A place, especially including buildings, where a particular activity happens.
10. To take part in or become involved in an activity.

THE FACULTY OF MECHANICS AND MATHEMATICS

Task 1. ***Using the words below form:***

a) nouns

to study	to solve
to train	to propose
technological	to realize
to teach	to understand
scientifi	to learn
to preserve	systematic
to occupy	to found
to organize	

b) adjectives

mathematics	to create
philosophy	society
to lead	to vary
education	to apply

c) adverbs

absolute	good
new	particular
certain	theoretical
successful	

Task 2. ***Fill in the blanks using the following words.***

1. The ... of mathematicians in Belarus began in 1923 with the opening of the Faculty of Physics and Mathematics at the Belarusian State University.
2. Physics and Mathematics Department of the Faculty was organized to train teachers of mathematics, physics, as well as ... .
3. The ... was renamed and became the Faculty of Mechanics and Mathematics.
4. More than 900 ... attend their classes regularly and 300 get instructions by correspondence.
5. The educational and ... level of the faculty meets the world standards and in many respects passes ahead of them.
6. Now the Faculty includes 10 ... .
7. The students of the MMF are among the most active in the social and ... activities of the university.
8. Students also have an opportunity to ... from case studies of leading national companies.

Task 3. A. ***Form collocations using the words below. One word can be used once only.***

proud	number	functional	internet	long-standing
master's	computer	differentia	honoured	elite

scientists	history	mathematics	equations	faculties
theory	technologies	traditions	analysis	course

B. ***Use the collocations you've formed in the sentences of your own.***

NUMBERS

Task 1. ***Name:***

a) pairs of antonyms

simple	exclude
to solve	imperfect
positive	negative
to include	apparently
rational	confuse, puzzle
natural	to destroy
perfect	acquired
to suggest	refuse
to produce	irrational
actually	complex

b) pairs of synonyms

calculate	various
to consider	to learn
to perform	entire
differen	singly
to represent	present
whole	think, suppose
separately	execute, do
important	sinifican
to express	to utter
to study	compute

Task 2. ***Use the words given in capitals at the end of each line to form a word that fits in the space in the same line.***

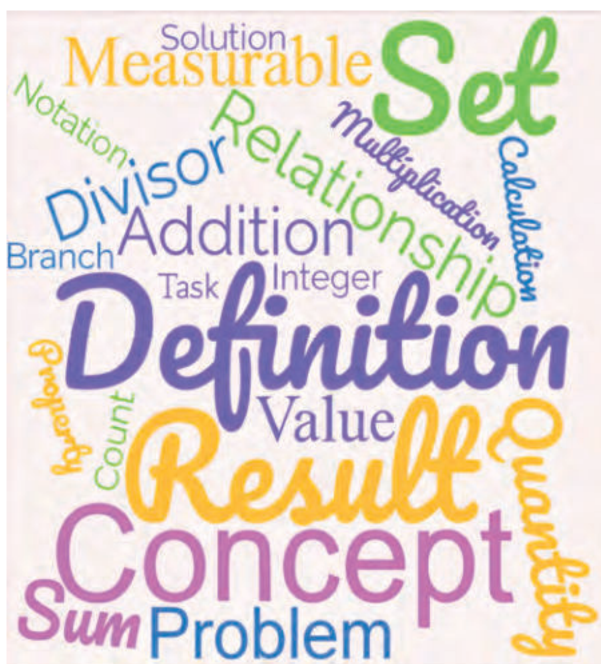
1. ... theory was born in 1896 in the work of the German mathematician F. G. Frobenius.	TO REPRESENT

2. It was the answer she ... wanted to hear.
3. Comparisons of functions could be based on suggested ... of functions.
4. Science rests on the ideals of objectivity and ... .
5. Draw a perpendicular from the vertex of the triangle to its ... .
6. In fact ... discovered the structures of the mind before the psychologists.
7. Hence, it is also ... by a system of equations using the considered operations and constants.
8. We learn ... and subtraction in first and second grade
9. This opportunity will allow people to gain ... experience.
10. The graph produced is called the frequency ... of the variable or just the ... .

LITTLE
TO CLASSIFY
RATIONAL
BASIC
MATHEMATICAL
TO DEFINE
TO ADD
VALUE
DISTRIBUTIVE

Task 3. A. ***Match these lexical items from the box with their definitions.***

1. The way in which two things are connected.
2. A group of objects with stated characteristics.
3. A statement that explains the meaning of a word or phrase.
4. An idea, theory, etc. about a particular subject.
5. Something that happens or exists because of something else.
6. The process of adding numbers or amounts together.
7. A number by which another number is divided in a calculation.
8. A number or symbol that represents an amount.
9. The process of using information you already have and adding, taking away, multiplying, or dividing numbers to judge the number or amount of something.
10. A whole number and not a fraction.
11. Able to be measured in order to discover its exact size, amount, etc.
12. To say numbers one after the other in order, or to calculate the number of people or things in a group.
13. The whole number or amount when two or more numbers or amounts have been added together.
14. A part of something larger.
15. The process of adding a number to itself a particular number of times, or a calculation in which this is done.
16. A question in mathematics that needs an answer.
17. A system of written symbols.
18. A quality that something has.
19. A piece of work to be done.
20. The answer to a problem.
21. The amount or number of something, especially that can be measured.



B. ***Complete the sentences with appropriate terms from above.***

1. The math student teachers discovered the importance of drawing from current events to ground mathematical ... .
2. When you divide 21 by 7, 7 is the ... .
3. To represent that an element is contained in a ..., we use the symbol "$\in$".
4. The ... of 13 and 8 is 21.
5. Parity is the ... of an integer of whether it is even or odd.
6. A number system defines a set of ... used to represent quantity
7. Four directly ... quantities determine the growth rate constant ab in this formula.
8. It's quality not ... that really counts.
9. Did you write things out in standard ...?
10. The ... that you made contained a few inaccuracies.
11. Twice a week the children are tested in basic mathematical skills such as ... and subtraction.
12. Algebra is a ... of mathematics in which letters are used to represent unknown quantities in mathematical expressions.
13. The numbers -5, 0, and 3 are ... .
14. We will begin by introducing a few basic ... .
15. I was good at maths and had no trouble with ... and division.
16. They face the daunting ... of rebuilding their economy.
17. A ... is a mathematical or logical connection between two variables.
18. We were given ten ... to solve.
19. I finished my exams yesterday, but I won't get the ... until Augu t.
20. They help you talk through your problems but they don't give you any ... .
21. We need to ... who's here, so we can make sure that no one's missing.

RATIONAL AND DECIMAL NUMBERS

Task 1. A. ***Complete the table with derivatives.***

Noun	Verb	Adjective
divisor		
repetition		
	to form	
		identifie
	to represent	
multiplication		
		fixe
remainder		
		respective
	to express	

B. ***Fill in the gaps choosing appropriate words from Task 1A and using them in the proper forms.***

1. The question here therefore ... an open one.
2. Continual ... assessment of student understanding through observation, student questioning, and written assignments helps teachers decide how well students are doing.
3. You need ... your priorities.
4. She ... that she had no intention of running for president.
5. When you ... two odd numbers, is the answer always odd?
6. $4xy^2$ is an ... .
7. Luckily there are ways of ... this before it becomes serious.
8. New teachers have to earn the ... of their students.
9. This statue is a ... of Socrates.
10. 10 ... by 5 is/equals 2.

Task 2. ***Find the words in the fillword.***

G	K	G	G	C	P	A	A	J	H	W	T	H	P	R	W	C	O	Q	H
B	N	Q	U	A	N	T	I	T	Y	I	I	E	B	F	A	P	P	M	P
W	R	S	W	S	A	K	J	X	C	S	K	Z	K	N	D	C	L	B	M
N	R	T	U	F	J	P	D	E	E	X	P	E	P	D	D	O	X	Z	U
U	T	E	M	B	D	J	M	E	R	E	C	U	R	R	I	N	G	T	L
B	M	R	P	W	T	T	S	Y	C	B	G	O	G	U	T	R	Z	P	T
C	L	M	B	R	E	R	E	M	A	I	N	D	E	R	I	C	W	F	I
R	K	I	C	U	E	N	A	K	W	X	M	C	D	Y	O	K	Z	D	P
D	L	N	L	H	A	S	G	H	V	R	T	A	P	O	N	V	Q	V	L
A	Y	A	S	B	I	I	E	K	E	D	N	P	L	I	T	F	N	D	I
Z	V	T	Q	J	D	I	S	N	K	N	K	H	C	S	K	F	A	B	C
Z	X	I	D	I	G	I	T	B	T	Z	D	J	R	H	W	U	E	S	A
D	S	N	K	E	I	H	E	J	F	A	Q	O	T	G	O	F	A	A	T
K	Q	G	D	I	V	I	S	I	O	N	T	K	O	R	I	N	Q	M	I
Z	K	U	P	N	G	L	I	T	B	A	D	I	N	O	A	U	V	U	O
M	F	D	G	T	V	A	P	O	R	O	V	K	O	P	O	M	S	C	N
V	N	L	O	E	X	S	I	E	H	P	W	C	U	N	H	B	V	G	T
E	C	D	L	G	H	T	M	T	O	C	G	E	W	E	Z	E	X	N	G
C	H	W	O	E	A	U	E	E	B	I	X	M	V	F	C	R	Q	F	J
W	Z	B	Z	R	N	M	I	A	M	F	R	A	C	T	I	O	N	H	K

ADDITION	RATIO	RECURRING
NUMBER	VALUE	REPRESENTATION
NUMERATOR	QUANTITY	REMAINDER
MULTIPLICATION	DIGIT	METHOD
DECIMAL	TERMINATING	DIVISION
FRACTION	SUBTRAHEND	INTEGER

Task 3. A. ***Form collocations using the words below. One word can be used once only.***

rational mixed Euler's Fibonacci
Hindu-Arabic decimal recurring infinit
resultant common

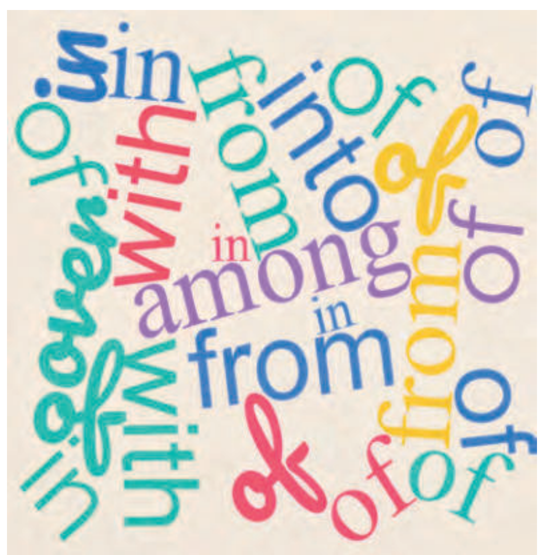
denominator digits fractions
numbers decimal fraction point
number numerals sequence

B. ***Use the collocations you've formed in the sentences of your own.***

Task 4. ***Fill in the gaps with the prepositions from the box (where necessary).***

Rational number is a number that can be represented as the quotient p/q ... two integers such that $q \neq 0$. ... addition to all the fractions, the set ... rational numbers includes all the integers, each ... which can be written as a quotient ... the integer as the numerator and 1 as the denominator. In decimal form, rational numbers are either terminating or repeating decimals. For example, 1/7 = 0.142857, where the bar ... 142857 indicates a pattern that repeats forever.

Irrational number is any real number that cannot be expressed as the quotient ... two integers — that is, p/q, where p and q are both integers. For example, there is no number ... integers and fractions that equals square root ... $\sqrt{2}$. A counterpart problem ... measurement would be to find the length ... the diagonal ... a square whose side is one unit long; there is no subdivision of the unit length that will divide evenly ... the length of the diagonal. It thus became necessary, early ... the history of mathematics, to extend the concept ... number to include irrational numbers. Irrational numbers such as π can be expressed as an infinite decimal expansion ... no regularly repeating digit or group ... digits. Together the irrational and rational numbers form the real numbers.



Real number is a quantity that can be expressed as an infinite decimal expansion. Real numbers are used ... measurements of continuously varying quantities such as size and time, ... contrast to the natural numbers 1, 2, 3, ..., arising ... counting. The word real distinguishes them ... the imaginary numbers, involving the symbol i, or square root ... $\sqrt{-1}$. Complex numbers such as $1 + i$ have both a real (1) and an imaginary (i) part. The real numbers include the positive and negative integers and the fractions made ... those integers (or rational numbers) and also the irrational numbers.

NUMBER SYSTEMS

Task 1. A. ***Using the words below form:***

a) nouns

to symbolize	structural
to represent	numerous
to operate	to introduce
to note	to fin
to systemize	

b) adjectives

infinit	decimal
digit	logic
arithmetic	to use
mathematics	algebra
to differentiat	

B. ***Complete the sentences with suitable words from Task 1A.***

1. A mathematical expression is a set of numbers and … .
2. No successful linear or … weaknesses have been reported.
3. Perhaps those mysterious mathematical … did mean something after all.
4. The mathematics used by most writers is simple … or simple number systems.
5. This … game teaches children to recognize numbers and solve simple mathematical examples.
6. An ordered ring that is not trivial is … .
7. All mathematical … are connected to physical entities.
8. Practically, sets are the simplest mathematical … because other structures with only these two dimensions seems to be way less interesting.
9. Any … multiplied by 1 is the same … .
10. Mathematical … consists of using symbols for representing operations, unspecified numbers, relations, and any other mathematical objects and assembling them into expressions and formulas.

Task 2. ***Match these lexical items from the box with their definitions.***

1. Any one of ten numbers from 0 to 9.
2. The number of times that a number is to be multiplied by itself.
3. A picture that shows how two sets of information or variables are related, usually by lines or curves.
4. To explain and describe the meaning.
5. To be the result of something, or to be something.
6. A particular way of doing something.
7. A particular method of counting, measuring, or weighing things.

8. Any whole number less than ten.
9. To do an action or piece of work.
10. To say numbers one after the other in order, or to calculate the number of people or things in a group.
11. To find an amount or number using mathematics
12. To change in form or character.
13. A group of people or things that share similar qualities.

Task 3. ***Find the words from the list below (they can be positioned horizontally, vertically or diagonally).***

I	N	W	W	P	U	N	P	K	X
V	K	P	R	O	D	U	C	T	U
S	B	O	E	W	Z	U	P	O	K
O	E	C	L	E	C	N	O	C	E
W	S	T	M	R	P	I	W	B	R
C	D	A	Y	Y	M	T	R	G	C
X	F	L	H	V	G	S	X	C	J
C	J	D	N	H	Q	U	J	E	B
Q	S	Q	N	D	I	G	I	T	S
F	S	K	C	B	I	N	A	R	Y

PRODUCT SET	BINARY UNITS DIGITS	POWER OCTAL

ALGEBRA

Task 1. A. ***Form collocations using the words below. One word can be used once only.***

unknown mathematical coordinate
quadratic abstract number
vector linear polynomial

ring mapping space theory
algebra equation geometry
expression value

B. ***Use the collocations you've formed in the sentences of your own.***

MATHEMATICAL LANGUAGE AND MATHEMATICAL SYMBOLS

Task 1. A. ***Complete the table with derivatives.***

Noun	Verb	Adjective
system		
		distinct
thought		
	operate	
equality		
	express	
		arranged
introduction		
separate		
		correct

B. ***Fill in the gaps choosing appropriate words from Task 1A and using them in the proper forms.***

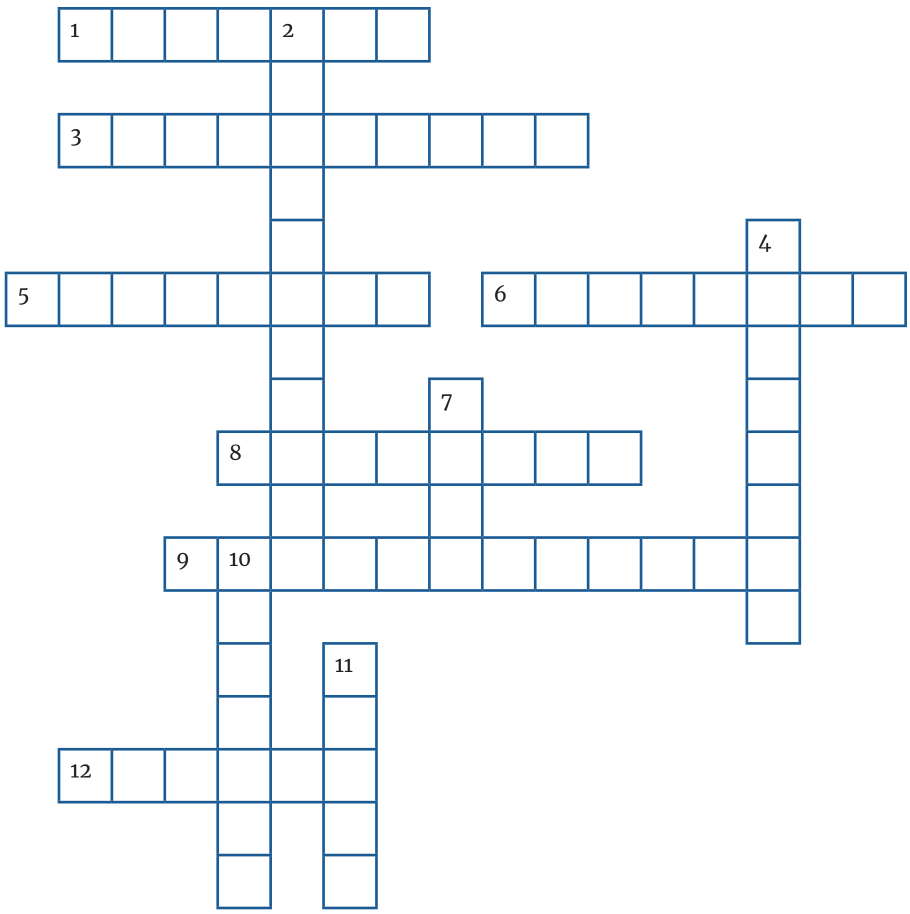
1. In first grade, math skills build upon concepts … in kindergarten, including numbers, counting, addition and subtraction, and 2-D shapes.
2. An … (or ordering) of a set of objects is called a permutation.
3. … of variables is a special method to solve some differential e uations.
4. In math, the term … number is used to refer to a number in a set that is not equal to another number.
5. It means … mathematically and to use mathematics to help in daily and working lives.
6. One educator has found that teaching students how to give a proper apology aligns well with teaching them how to … math mistakes
7. The term … means that two or more values or expressions are the same.
8. A number … is defined as a … of writing to express numbers
9. … … in math is a sentence with a minimum of two numbers or variables and at least one math operation.
10. What does it mean … on functions?

Task 2. ***Form collocations using the words below. One word can be used once only.***

logical	to express	certain	inequality
equal	mathematical	basic	to represent
	numerical	particular	

complex thoughts a value term
quantities sign sentence coefficie
mathematics ideas symbols

Task 3. ***Do the crossword.***



Across

1. Using reason.
3. A symbol or group of symbols that represent an amount.
5. Anything that can be counted or measured.
6. A particular number or amount that never changes.
8. A correct arrangement of mathematical symbols that state a complete thought.
9. Relating to mathematics.
12. A number, letter, or sign.

Down

2. A value, in mathematics, that appears in front of and multiplies another value.
4. A number, amount, or situation that can change.
7. A word or expression used in relation to a particular subject.
10. A part of mathematics in which signs and letters represent numbers.
11. The number or amount that a letter or symbol represents.

ALGEBRAIC OPERATIONS

Task 1. ***Using the words below form:***

a) nouns

to indicate	equal
to subtract	to divide
to add	to remain
to multiply	to perform
to represent	to identify

b) adjectives

algebra	to commute
differenc	association
value	to distribute
base	number

Task 2. ***Read the paragraph about special algebraic operations and fill in with the appropriate words.***

parentheses	multiplication		operations
root	number	constant	power

SPECIAL ALGEBRAIC OPERATIONS

Special ... include raising to a power, taking the square root, summation, and integral.

If you wanted to multiply a ... by itself several times, you could write $7 \times 7 \times 7 \times 7$ or $a \times a \times a$.

A shorthand way of doing that is called "raising to a power" and consists of writing the number, ..., or variable and placing a small number to the upper right hand corner, which represents how many times the ... was repeated.

$7 \times 7 \times 7 \times 7$ would then be 7^4 and is called 7 to the 4th power.

$a \times a \times a$ can be written as a^3 and is called a to the third ... or commonly called a-cubed.

a^2 is called a-squared.

Sometimes the symbol "^" is used to indicate raising to a power, especially on a computer. For example:

7^4 can be used instead of 7^4.

a^3 can be used instead of a^3.

Taking the ... of a number is somewhat abstract. For example, the square root of a number is the number that when multiplied by itself equals that number. For example, the square root of **9** is **3**, since **3** × **3** = **9**.

The symbol for square root is "$\sqrt{\ }$" or SQRT. For example:

$\sqrt{(25)} = 5$ or SQRT(25) = 5.

... is a way of grouping items, such that an operation acts on everything within the parentheses.

$\sqrt{(20 + 5)}$ means the square root operation acts on 25.

Task 3. ***Find the words from the list below (they can be positioned horizontally or vertically).***

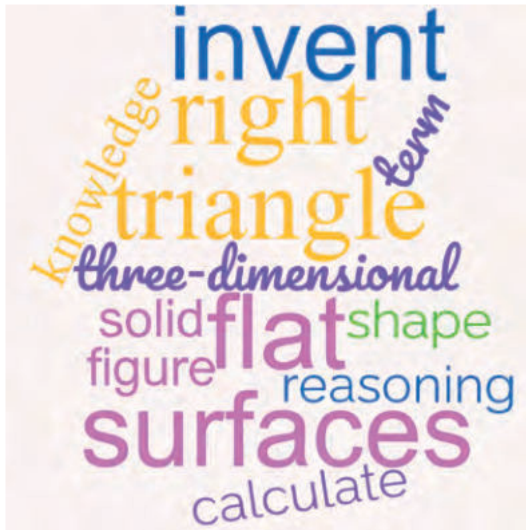
L	Z	Y	B	X	H	N	S	G	D	I	F	F	E	R	E	N	C	E	D
U	S	L	P	L	M	E	X	P	O	N	E	N	T	Z	P	M	N	B	M
J	W	E	U	T	G	D	H	E	I	Y	W	L	J	N	J	Q	H	A	N
N	O	N	S	U	B	T	R	A	C	T	I	O	N	I	S	X	T	Q	F
M	G	S	B	N	B	E	B	N	E	T	V	J	S	Q	M	Z	V	C	N
R	L	K	P	E	J	C	I	L	X	W	A	O	V	C	D	S	E	Y	L
U	M	V	M	R	F	B	P	Q	U	G	O	L	G	U	L	J	I	H	K
C	E	D	U	N	O	K	Q	M	D	Y	A	D	G	W	K	I	N	I	U
Q	T	M	L	G	F	D	T	C	C	V	I	L	R	E	M	R	D	P	J
R	H	B	T	Z	S	S	U	B	T	R	A	C	T	T	B	L	I	V	G
B	O	A	I	K	E	U	X	C	O	S	D	L	I	P	P	R	C	K	G
Z	D	D	P	X	L	M	B	L	T	W	V	I	U	Q	J	L	A	F	T
L	X	D	L	Y	A	M	P	G	J	C	O	N	V	E	H	J	T	I	D
D	G	I	I	M	W	A	C	Z	Y	G	S	X	U	I	J	O	E	Y	C
Y	Y	T	C	N	Q	T	I	O	D	K	B	R	Z	H	S	E	C	H	T
B	V	I	A	A	S	I	Q	Z	L	P	E	I	N	J	F	I	L	Z	Q
X	U	O	T	Y	L	O	O	Z	H	A	U	I	I	D	N	X	O	U	Z
S	H	N	I	F	K	N	T	E	V	N	O	Q	Y	H	X	B	W	N	O
V	L	B	O	N	I	B	B	R	T	S	Q	W	F	G	N	O	G	C	B
C	O	M	N	I	Y	I	J	A	H	C	A	O	A	I	N	S	B	G	Y

SUMMATION	ALGEBRAIC	VALUE
ADDITION	METHOD	PRODUCT
MULTIPLICATION	DIFFERENCE	INDICATE
DIVISION	SUBTRACT	EXPONENT
SUBTRACTION		LAW

HISTORY OF GEOMETRY

Task 1. A. ***Give definitions.***

1. To create something that has never been made before.
2. The forms of things in geometry that have boundaries, angles, and surfaces.



3. It is the region of 3D space bounded by a two-dimensional surface.
4. The process of thinking about something in order to make a decision.
5. It is a plane.
6. A number or symbol used in a mathematical series or calculation.
7. To find an amount or number using mathematics
8. Awareness, understanding, or information that has been obtained by experience or study, and that is either in a person's mind or possessed by people generally.
9. Triangle that has one angle of 90°.
10. Having or appearing to have height, length, and width.

B. ***Fill in the blanks using appropriate words from the box.***

parentheses	three-dimensional	
calculate	terms	flat surface
shape	right triangle	
invent	knowledge	solid figure

1. ... are identified according to the features that are unique to e ch type of solid.
2. The ... behind her conclusion is impossible to fault.
3. But I didn't ... the story — everything I told you is true.
4. The ... of a geometric series form a geometric progression, meaning that the ratio of successive terms in the series is constant.
5. He was the first to list the six distinct cases of a ... in spher cal trigonometry.
6. The solutions of the singularity equation span three ... surfaces.
7. A cube is a three-dimensional ... that has 6 faces, 8 vertices, and 12 edges. The faces of the cube are square.
8. ... the cost of 10 apples when each apple costs 0,50.
9. How will we use our increasing scientific ...
10. The ... of a solid figure are its faces, or sides as they are c mmonly called.

Task 2. ***Fill in the blanks using the following words.***

Geometry is a branch of mathematics concerned with … of space such as the distance, shape, …, and relative position of fi ures. Geometry is, along with …, one of the oldest branches of mathematics. A mathematician who works in the field of geometry is called a geometer. Until the 19th century, geometry was almost exclusively devoted to Euclidean geometry, which includes the notions of point, line, …, distance, angle, surface, and curve, as fundamental … .

properties
size
arithmetic
plane
concepts

Originally developed to model the … world, geometry has applications in almost all …, and also in art, architecture, and other activities that are related to graphics. Geometry also has … in areas of … that are apparently unrelated. For example, methods of algebraic geometry are fundamental in Wiles's … of Fermat's Last Theorem, a problem that was stated in terms of elementary arithmetic, and remained unsolved for several centuries.

physical
sciences
applications
mathematics
proof

During the 19th century several discoveries enlarged dramatically the scope of … . One of the oldest such discoveries is Carl Friedrich Gauss' Theorema Egregium ("remarkable theorem") that asserts roughly that the Gaussian curvature of a … is independent from any specific embedding in a Euclidean … . This implies that surfaces can be studied intrinsically, that is, as stand-alone spaces, and has been expanded into the … of manifolds and Riemannian geometry. Later in the 19th century, it appeared that geometries without the parallel postulate (non-Euclidean geometries) can be developed without introducing any contradiction. The geometry that underlies general … is a famous application of non-Euclidean geometry.

geometry
surface
space
theory
relativity

Since the late 19th century, the scope of geometry has been greatly expanded, and the … has been split in many subfield that depend on the underlying methods — differential geometry, algebraic geometry, computational geometry, algebraic …, discrete geometry, etc. — or on the properties of Euclidean spaces that are disregarded — projective geometry that consider only alignment of … but not distance and parallelism, affin geometry that omits the concept of angle and …, finite geometry that omits continuity, and others. This enlargement of the scope of geometry led to a change of meaning of the word "space", which originally referred to the three-dimensional space of the physical world and its model provided by Euclidean geometry; presently a geometric space, or simply a space is a mathematical … on which some geometry is defined

fiel
topology
points
distance
structure

Task 3. *Find the words (they can be positioned horizontally, vertically of diagonally).*

O	W	H	Y	X	I	P	Y	T	H	A	G	O	R	A	S	K	W	D	W
A	G	H	K	Y	Z	Y	S	K	A	Y	J	D	W	D	S	H	A	P	E
R	B	J	A	C	D	N	C	W	F	L	M	V	X	A	S	B	F	L	K
N	O	U	O	Q	O	Y	J	W	F	S	U	T	J	T	A	T	G	L	F
S	U	T	D	S	R	I	Z	W	I	D	T	N	V	O	O	N	Q	B	J
J	T	K	N	O	W	L	E	D	G	E	E	M	Q	Q	A	S	B	O	Y
V	X	S	E	E	Z	A	P	T	U	Y	W	B	J	I	R	K	E	H	Q
S	U	H	J	Q	Q	E	P	R	R	Q	M	C	R	G	G	Z	S	X	Q
U	T	D	O	D	D	E	B	A	E	Q	W	T	O	P	Q	F	E	D	C
R	C	Y	F	X	C	H	S	K	X	K	Q	P	U	W	R	D	U	Z	S
F	R	B	P	N	H	W	E	Q	S	X	B	C	E	V	H	F	C	F	R
A	J	W	O	W	L	X	D	W	I	E	A	S	Y	F	D	V	L	J	P
C	R	C	O	O	Y	J	U	L	O	Y	K	W	C	G	I	U	I	C	L
E	S	G	C	J	A	S	C	Z	R	G	B	F	D	Q	M	J	D	S	I
N	U	F	T	C	M	S	X	T	L	L	S	Q	J	T	I	Y	I	C	M
K	R	J	Y	K	C	P	E	N	W	V	A	Q	N	K	I	A	X	R	N
C	Z	R	Z	L	E	M	W	A	A	X	Q	J	F	Y	P	T	F	W	J
W	F	H	E	R	O	D	O	T	U	S	Y	Q	K	T	P	Z	R	K	R
T	O	H	P	E	G	O	E	R	P	Q	O	F	T	O	A	C	U	L	A
A	W	X	G	C	I	U	F	M	B	E	Y	F	M	H	Z	V	I	S	W

SURFACE FIGURE EUCLID GEOMETRY	HERODOTUS PYTHAGORAS TRIANGLE	CONCEPT THEORY SHAPE KNOWLEDGE

POINTS, LINES, PLANES AND ANGLES

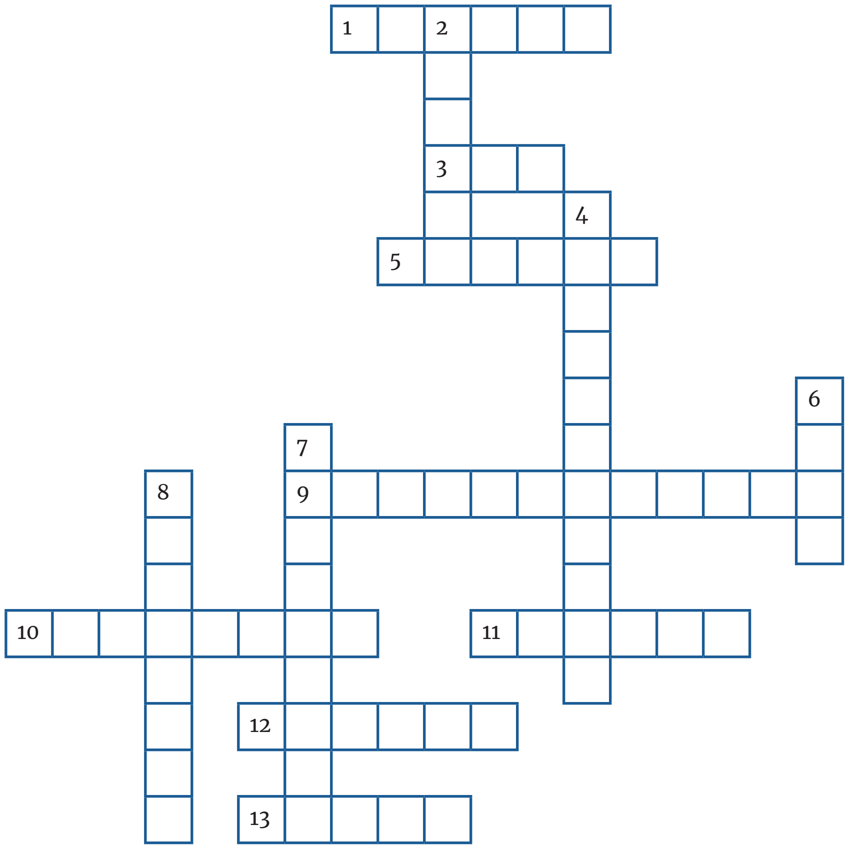
Task 1. A. *Form collocations using the following words. Make use of the text if necessary. One word can be used once only.*

particular line | point of infinit | capital Intersecting | interior vertical | Geometric angle

concurrency measurement | shape segment angles | lines | point letters | planes location

B. *Use the collocations you've formed in the sentences of your own.*

Task 2. ***Fill in the crossword puzzle.***



Across

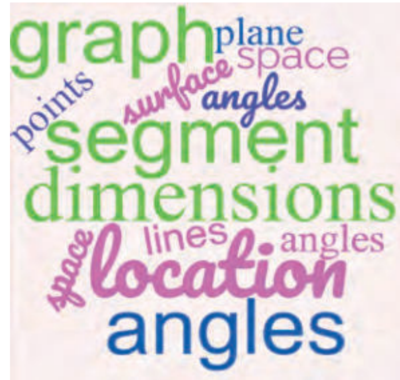
1. The measure of an angle at the centre of a circle opposite an arc that is the same length as the radius.
3. A line with a single endpoint (or point of origin) that extends infinitely in one direction.
5. The point where two lines meet to form an angle, or the point that is opposite the base of a shape.
9. A point, line, or curve common to two or more objects.
10. A point that marks the end of a line segment or a ray.
11. The measurement of something from end to end or along its longest side.
12. A geometric shape that is a combination of lines, points, or planes that form a closed boundary.
13. The space between two lines or surfaces at the point at which they touch each other, measured in degrees.

Down

2. Any of various units of measurement, especially of temperature or angles, usually shown by the symbol ° written after a number.
4. The act or process of measuring.
6. An infinitely long object with no width, depth, or curvature.
7. A measurement of something in a particular direction, especially its height, length, or width.
8. A quality that something has.

Task 3. ***Fill in the blanks using the following words.***

1. Join the … *A* and *B* together on the diagram with a straight line.
2. Please specify the … of the room.



3. Draw a pair of parallel ... .
4. The point ... problem is a fundamental topic of computational geometry.
5. The interior ... of a square are right ... or ... of 90 degrees.
6. In geometry, a three-dimensional ... is a mathematical ... in which three values are required to determine the position of a point.
7. Learn how to draw a ... of the equation.
8. You can think of a piece of paper or the surface of a wall as a part of a geometric ... .
9. A ... is a set of points consisting of two points of the line called the endpoints, and all of the points of the line between the endpoints.
10. In algebraic geometry, a ... may cross itself while, in topology and differential geometry, it may not.

POINTS IN EUCLIDEAN GEOMETRY

Task 1. ***Using the words below form:***

a) nouns

to extend to add to multiply to coincide segment	equal to complete related to assume to consider

b) adjectives

to apply to defin order fundamental basis	to realize relationship shape to introduce to deal with

Task 2. ***Fill in with the appropriate words:***

definitio	axioms	mathematician	fla	
assumptions	shapes	plane	postulates	figure

EUCLIDEAN GEOMETRY

Euclidean geometry is the study of geometrical ... (plane and solid) and ... based on different ... and theorems. It is basically introduced for ... surfaces or ... surfaces. Geometry is derived from the Greek words 'geo' which means earth and 'metrein' which means 'to measure'.

Euclidean geometry is better explained especially for the shapes of geometrical figures and planes. This part of geometry was employed by the Greek ... Euclid, who has also described it in his book, Elements. Therefore, this geometry is also called Euclid geometry.

The axioms or ... are the ... that are obvious universal truths, they are not proved. Euclid has introduced the geometry fundamentals like geometric shapes and figure in his book elements and has stated 5 main axioms or postulates. Here, we are going to discuss the ... of Euclidean geometry, its elements, axioms and five important postulates.

Task 3. ***Find the words from the list below (they can be positioned horizontally or vertically).***

B	X	P	I	D	D	H	D	B	B	J	L	M	G	V	V	J	B	Q	C
Y	G	W	Z	B	B	A	P	B	A	D	H	Z	X	X	W	S	O	D	X
G	M	D	F	D	O	H	M	O	C	F	E	D	N	M	C	Y	G	P	D
W	Y	A	M	M	H	J	T	Z	S	Y	S	F	C	D	O	G	F	K	D
V	H	Q	J	K	O	V	G	M	M	T	L	T	I	E	N	J	Q	H	L
G	T	V	K	I	A	N	W	R	F	K	U	A	B	N	S	H	E	O	C
P	J	M	F	M	O	J	F	H	G	S	M	L	A	F	E	H	Z	M	O
J	Y	U	P	E	M	O	X	U	C	V	O	J	A	B	Q	U	B	Z	I
L	I	B	I	O	M	P	C	O	N	C	E	P	T	T	U	C	Z	D	N
W	R	Z	G	A	S	S	U	M	P	T	I	O	N	M	E	C	P	B	C
B	G	C	J	X	D	L	M	K	P	Q	L	T	B	C	N	T	S	T	I
R	J	D	V	I	Y	D	L	S	K	M	B	C	P	D	C	E	T	W	D
E	A	C	L	O	L	K	I	P	Z	F	A	W	Y	X	E	A	J	I	E
C	A	S	Y	M	P	Q	G	I	J	G	E	H	J	L	Z	U	N	Q	P
T	R	T	N	O	R	D	E	R	X	Y	M	Q	E	A	U	F	G	K	N
A	B	O	S	Q	W	S	F	D	R	L	M	Y	K	Z	P	T	J	V	S
N	K	H	X	K	M	U	X	B	T	F	S	B	N	C	X	P	Z	V	U
G	Y	K	U	O	N	S	M	S	D	U	A	U	X	H	D	T	L	B	O
L	A	M	R	N	W	I	S	B	P	Y	K	A	D	C	T	N	S	Y	Z
E	G	E	X	V	Q	E	P	W	J	H	L	K	H	N	X	O	H	H	F

COINCIDE ORDER POSTULATE	APPLY CONCEPT ASSUMPTION CONSEQUENCE	DEFINE RECTANGLE AXIOM

Appendix III
BUILDING CONFIDENCE

MAKING AN ORAL PRESENTATION

TIPS FOR PREPARING YOUR PRESENTATION

Preparation is vital to ensuring the success of your presentation. A well-prepared presentation can leave an impact on your audience and assist you in achieving your objectives. Here are some tips to help you prepare effectively

1. Research Topic And Gather Information.

If you want to learn how to make a good PowerPoint presentation, research is the key. Thoroughly research your topic and collect all necessary information to support your main points. This will give you the con dence to present your information effectively and answer any questions from your audience

2. Choose Appropriate Visual Aids.

Choose appropriate visual aids, such as PowerPoint graphics, images, or videos, to enhance your presentation and support your key points. Ensure that your visual aids are professional, clear, and easy to understand.

3. Plan The Structure And Flow Of The Presentation.

To guarantee that your message is appropriately presented, prepare the structure and flow of your presentation. Start with an attention-grabbing introduction, state your key points clearly, and conclude with a memorable closing statement.

4. Rehearse The Presentation.

Rehearse your presentation to ensure that you are familiar with the material and confident in your delivery. Practice before a family member or a friend to get feedback and make changes.

5. Start With An Attention Grabber.

Start with an attention-grabbing statement, question, or story to engage your audience and capture their attention. This helps establish the tone for the remainder of your presentation and keeps your audience engaged.

6. Clearly State Key Points And Support Them With Examples.

Clearly state your key points and support them with relevant examples, data, or visual aids. This will help reinforce your message and ensure your audience understands your information.

7. Use Visual Aids Effectively To Enhance Understanding

If you want to learn how to make a presentation interesting, use visual aids, such as slides, images, or videos. It can enhance understanding and support your key points. Ensure that your visual aids are clear, easy to understand, and professional.

8. Conclude With A Summary Of Key Points And A Strong Closing Statement.

Conclude your presentation by summarizing your key points and a strong closing statement. This will assist in reinforcing your message and create a lasting effect on your audience.

TIPS FOR A SUCCESSFUL DELIVERY OF YOUR ORAL PRESENTATION

1. Clear and simple structure.

Remember that your audience will benefit most from a very clear and logical structure. Don`t overload the audience and try to use simple language.

2. Your introduction.

Some experts say this is the most important part of your presentation. In the rst few minutes you can get the audience`s attention, build rapport and create a positive impression.

3. Topic and objective.

Clearly say what the topic and objective (or purpose) of your talk is. Repeat the topic and objective at some later time.

4. Signposting.

Let the audience know at all times what you want to do and how you want to do it. This method is common in the business world — so use it!

5. Repeating new information.

Always repeat new details. This helps your audience to remember them and ensures optimal flows of information

6. Summarizing points.

At the end of each section summarize the main facts to make sure everybody is following.

7. Interaction with the audience.

Your audience usually expects direct interaction! So treat them as individuals, show them that you care about their individual needs. Engage your audience with interactive elements, such as questions, activities, or polls, to keep them interested and involved in your presentation.

8. Presenter's role.

The presenter is often considered as important as his or her topic, and the presenter's role is to make sure the presentation — even one on a dry topic — is interesting and entertaining. To achieve this goal, use your personality more and tend to be more enthusiastic.

USEFUL LANGUAGE: MAKING A PRESENTATION

Introducing yourself

Good morning/afternoon, ladies and gentlemen! Let me introduce myself. I'm/ My name is ...

It's pleasure to welcome you today.
Hello everyone, My name is ...
As you know, I'm ...
It's good/nice to see you all here.

Stating the aim / topic

Today, I'd like to talk about ...
My purpose/ subject of my presentation is to talk to you about ...
In my presentation I would like to report on ...
Today I'm going to talk about ...
What I want today is ...
In my talk I'll tell you about ...

Timing

My presentation will take about 20 minutes.
It should take about 20 minutes to cover these issues.

Questions

There will be time for questions after my presentation.
If you have any questions, feel free to interrupt me at any time.
Feel free to ask questions at any time during my talk.

Outlining the Presentation

There are three parts to my presentation.
My presentation is/will be divided into ... parts.

- **First (of all)**, I'll give / present / provide you some basic information / facts.
- **Firstly**, I'll begin / start with ... / begin / start of by explaining / showing
- **Secondly**, I'll be looking at / focus on ...
- **Next/After that**, I'll talk / speak about ...
- **Then**, I'll discuss / show / describe / demonstrate ...
- **Finally**, I'll tell you about ...

Introducing New Information

Here are some basic facts.
Here are some key facts about our company.

Saying what is coming

In this part of my presentation, I'd like to tell you about.
So, let me give you a brief overview.

Moving on to the next point

Moving on now to ...
OK/ Right. What about our distribution system?
This leads directly to the next point of my talk.
This now leads us to my next point.
Let's move on to the next point.
Let's now turn to the next issue.

Indicating the end of the section

This brings me to the end of my second/third point.
So much far.
So that's the background.

Referring back

As I mentioned before, ...
As I said earlier.
Let's go back to what we were discussing earlier.
Let me now come back to what I said earlier.

Talking about di cult issues

to identify the problem / issue
to clarify a few points
to deal with a problem / issue
to cope with
to solve the problem
to take care of

Referring to other point

in connection with / concerning
regarding
With respect / regard to
According to

Adding Ideas

In addition to
Moreover / Furthermore

Emphasizing important points

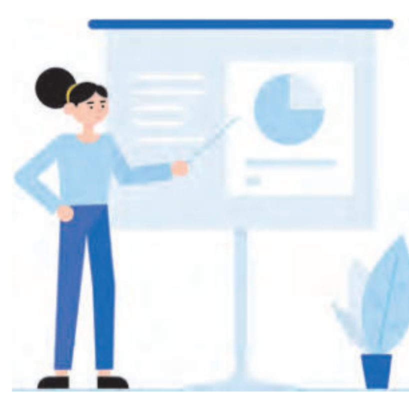
to stress / to point out
to draw / focus your attention
to emphasize
to outline / highlight
What is really important is ...
What we should do is ...
It's interesting to note that ...
I should repeat that ...

Summarizing main points / Summing up

I'd like to sum up the main point.
Let me briefly summarize what I've said so far

Ending a Presentation

Thanks very much for listening to my presentation.
Thanks for coming to my talk.
Are there any questions?



ESSAY WRITING TIPS

An essay is a common type of academic writing that you'll likely be asked to do in multiple classes. Before you start writing your essay, make sure you understand the details of the assignment so that you know how to approach the essay and what your focus should be. Once you've chosen a topic, do some research and narrow down the main argument(s) you'd like to make. From there, you'll need to write an outline and flesh out your essay, which should consist of an introduction, body, and conclusion. After your essay is drafted, spend some time revising it to ensure your writing is as strong as possible.

1. Understanding Your Assignment.

1. Read your assignment carefully. The style, structure, and focus of your essay will vary depending on the type of essay you are writing. If you've been assigned to write an essay for a class, review the assignment carefully and look for information about the nature of the essay. A few common types of essays include:

- ***The compare / contrast essay***, which focuses on analyzing the similarities and differences between 2 things, such as ideas, people, events, pl ces, or works of art.
- ***The narrative essay***, which tells a story.
- ***The argumentative essay***, in which the writer uses evidence and examples to convince the reader of their point of view.
- ***The critical or analytical essay***, which examines something (such as a text or work of art) in detail. This type of essay may attempt to answer specific questions about the subject or focus more generally on its meaning.
- ***The informative essay***, that educates the reader about a topic.

2. Check for formatting and style requirements. If you're writing an essay for a class or a publication, there may be specific formatting and style requirements you need to follow. Read your assignment carefully to make sure you understand requirements such as:

- How long your essay should be?
- Which citation style to use?
- Formatting requirements, such as margin size, line spacing, and font size and type.

3. Narrow down your topic so your essay has a clear focus. Depending on your assignment, you may already have a specifi topic you are supposed to write about, or you may simply be asked to write about a general theme or subject. If the assignment doesn't specify your topic, take some time to brainstorm. Try to pick a subject that's specific, interesting to you, and that you think will give you plenty of material to work with.

If you're doing a research-based essay, you might find some inspiration from reading through some of the major sources on the subject.

For a critical essay, you might choose to focus on a particular theme in the work you're discussing, or analyze the meaning of a specific passage

4. Ask for clarification if you don't understand the assignment. If you're not sure what you're supposed to be writing about or how to structure your essay, don't hesitate to ask! Your instructor can help clarify anything you don't understand, and they may even be able to provide examples of the type of work they're looking for.

If you're having trouble narrowing down your topic, your instructor might be able to provide guidance or inspiration.

2. Planning and Organizing Your Essay.

1. Find some reputable sources on your topic. If you're writing an academic essay or any type of essay that requires you to support your claims with evidence and examples, you'll probably need to do some research. Head to your library or go online to find up-to-date sources that provide accurate, verifiable information a out your topic.

Academic books and journals tend to be good sources of information. In addition to print sources, you may be able to find reliable information in scholarly databases such as JSTOR and Google Scholar.

You can also look for primary source documents, such as letters, eyewitness accounts, and photographs.

Always evaluate your sources critically. Even research papers by reputable academics can contain hidden biases, outdated information, and simple errors or faulty logic.

> **Tip**
> **In general, Wikipedia articles are not considered appropriate sources for academic writing. However, you may be able to find useful sources in the "References" section at the end of the article.**

2. Make notes as you do your research. As you're researching your topic, keep detailed notes about relevant information, ideas that interest you, and questions that you need to explore further. If you plan to use any of the information that you find in your paper, write down detailed citation information. This will allow you to find the information again and cite it properly

You might find it helpful to write your notes down on individual note cards or enter them into a text document on your computer so you can easily copy, paste, and rearrange them however you like.

Try organizing your notes into different categories so you can identify specific ideas you'd like to focus on. For example, if you're analyzing a short story, you might put all your notes on a particular theme or character together.

3. Choose a question to answer or an issue to address. As you do your research, you will likely find yourself narrowing your focus even further. For example, you might discover that there is a particular question you want to answer, or that there's a popular argument or theory about your topic that you'd like to try to disprove. This question or issue will form the basis for your thesis or main argument.

Create a thesis statement that summarizes your main argument. Once you've hit on a specific question or idea you'd like to address in your essay, look at your research and think about the major point or argument you'd like to make. Try to summarize your main point, in 1—2 sentences. This will be your thesis statement.

One easy way to come up with a thesis statement is to briefly answer the main question you would like to address.

Write an outline to help organize your main points. After you've created a clear thesis, briefly list the major points you will be making in your essay. You don't need to include a lot of detail — just write 1—2 sentences, or even a few words, outlining what each point or argument will be. Include sub-points addressing the evidence and examples you'll be using to back up each point.

When you write the outline, think about how you would like to organize your essay. For example, you might start with your strongest arguments and then move to the weakest ones. Or, you could begin with a general overview of the source you're analyzing and then move on to addressing the major themes, tone, and style of the work.

3. Drafting the Essay.

It does look like a piece of candy — the substance is inside (body), but we need to be enticed to eat it (introduction), and we certainly don't just leave the wrapper out and open on the table or ground. We have to wrap it up and truly finish it conclusion).

Introduction — begin broad — get more specific

Body — The "meat" of the essay. Goes into depth about your three main reasons listed in your thesis statement. One reason per paragraph.

Conclusion begin by restating your thesis, then get more broad.

Write an <u>introduction</u> to provide context. Once you've written your thesis and outline, write an introduction to your essay. This should consist of a brief, general overview of your topic, along with your thesis statement. This is the place to provide information that will help orient the reader and put the rest of your essay in context.

A strong introduction should also contain a brief transitional sentence that creates a link to the first point or argument you would like to make

Tip

Some writers find it helpful to write the introduction after they've written the rest of the essay. Once you've written out your main points, it's easier to summarize the gist of your essay in a few introductory sentences.

Present your argument(s) in detail. Working from your outline, write a series of paragraphs addressing each of the major points you'd like to make. Each paragraph should contain a topic sentence, which is like a miniature thesis — it briefly explains the main point you are trying to make with your paragraph. Follow up your topic sentence with a few concrete examples to support your point.

For example, your topic sentence might be something like, "Arthur Conan Doyle's Sherlock Holmes stories are among the many literary influences apparent in P. G. Wodehouse's Jeeves novels." You could then back this up by quoting a passage that contains a reference to Sherlock Holmes.

Try to show how the arguments in each paragraph link back to the main thesis of your essay.

3. Use transition sentences between paragraphs. Your essay will fl w better if you build connections or smooth transitions between your arguments. Try to nd logical ways to link each paragraph or topic to the one before or after.

When creating transitions, transitional phrases can be helpful. For example, use words and phrases such as "In addition," "Therefore," "Similarly," "Subsequently," or "As a result."

4. Address possible counterarguments. If you're writing an argumentative essay, get familiar with the major arguments against your point of view. You'll need to incorporate those counterarguments into your essay and present convincing evidence against them.

5. Cite your sources properly. If you plan to use someone else's ideas or information that you got from another source, you will need to credit the source of your information. This is true whether you're quoting another source directly or simply summarizing or paraphrasing their words or ideas.

The way you cite your sources will vary depending on the citation style you're using. Typically, you'll need to include the name of the author, the title and publication date of the source, and location information such as the page number on which the information appears.

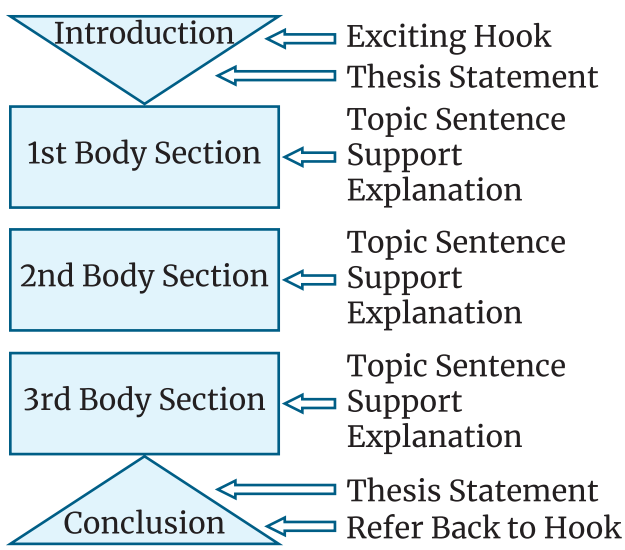
In general, you don't need to cite common knowledge. For example, if you say, "A zebra is a type of mammal," you probably won't need to cite a source.

If you've cited any sources in the essay, you'll need to include a list of works cited (or a bibliography) at the end.

6. Wrap up with a concluding paragraph. To finish off your essay, write a paragraph that briefly reiterates the main point of your essay. State how your arguments support your thesis and briefly summarize your major insights or arguments. You might also discuss questions that are still unanswered or ideas that merit further exploration.

Keep your conclusion brief. While the appropriate length will vary based on the length of the essay, it should typically be no longer than 1—2 paragraphs.

Essay Structure



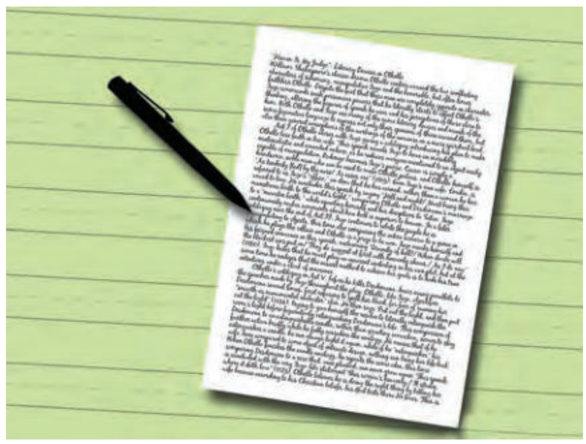
For example, if you're writing a 1,000-word essay, your conclusion should be about 4—5 sentences long.

4. Revising the Essay.

1. Take a break after finishing your first draft. After drafting your essay, it's a good idea to take some time away from it. That way, you can come back to it and look at it again with a fresh perspective. If possible, take 1—2 days before you look at it again.

If you don't have time to spend a couple of days away from your essay, at least take a few hours to relax or work on something else.

2. Read over your draft to check for obvious problems. When you're ready to work on your essay again, fi st read it over to look for any major problems. You might fi d it helpful to read the essay out loud since your ears can pick up on things your eyes might miss. If you spot anything, make a note of it, but don't try to fix it right away. Look out for issues such as:



- excessive wordiness;
- points that aren't explained enough;
- tangents or unnecessary information;
- unclear transitions or illogical organization;
- spelling, grammar, style, and formatting problems;
- inappropriate language or tone (e.g., slang or informal language in an academic essay).

3. Correct any major problems you find. Once you've read over your essay, go through and edit it. When you're done, go back over the essay again to make sure it flows well and that there aren't any problems you missed

You might have to cut material from your essay in some places and add new material to others.

You might also end up reordering some of the content of the essay if you think that helps it flow better

4. Proofread your revised essay. After editing your essay, go over it again closely to spot any minor errors, such as typos or formatting problems. There may be issues that you missed during your initial round of editing, and there could also be new typos or formatting issues if you made changes to the original draft. You may find it easier to spot these errors on a printout rather than on an electronic version of your essay.

Read over each line slowly and carefully. It may be helpful to read each sentence out loud to yourself.

Tip

If possible, have someone else check your work. When you've been looking at your writing for too long, your brain begins to fill in what it expects to see rather than what's there, making it harder for you to spot mistakes

USEFUL LANGUAGE:
WRITING ESSAYS

Opening

It is often said that...
Many people claim that...
We live in an age when many of us are...
In this day and age... / Nowadays... / These days...
... is a hotly-debated topic that often divides opinions.
... is often discussed yet rarely understood.
It goes without saying that... is one of the most important issues facing us today
The following essay takes a look at both sides of the argument.

Introducing Points

Firstly, let us take a look at ... / To start with, ...
First of all, it is worth considering...
Secondly, ... / Thirdly, ...
Furthermore, ... / In addition, ... / What is more, ... / On top of that, ...
Another point worth noting is...
Lastly ... / Finally, ... / Last but not least, ...

Presenting Ideas and Giving Examples

When it comes to (+ Noun / Gerund)
In terms of (+ Noun / Gerund)
With respect to (+ Noun / Gerund)
Not only ... but also...
According to experts, ...
Research has found that...
There are those who argue that...
For instance... / For example... / such as...

Finishing an Introduction Paragraph

In this essay, I will look at some of the arguments for
This essay will discuss different ways of
This essay outline some of the reasons why...
Let us examine both views before reaching a concrete decision.
The following essay takes a look at both sides of the argument.

Expressing Your Opinion

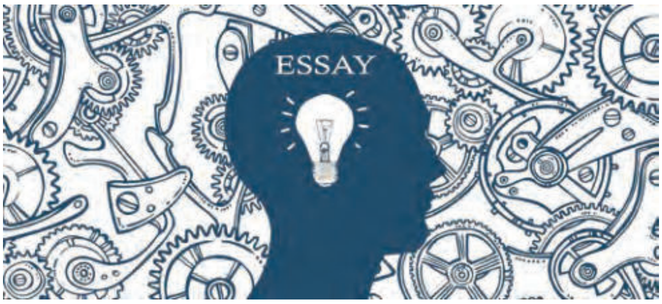
In my opinion,
I strongly agree with the idea that...
I strongly disagree with the idea that...
I strongly opine that...
I strongly believe that...
In my view...

As far as I am concerned...
It seems to me that...
However, I strongly believe that...
I oppose the view and my reasons will be explained in the following paragraphs.
I will support this view with arguments in the following paragraphs.
I personally believe that...
Thus the advantages far outweigh the disadvantages...

Listing Your Ideas

First...
First of all...
Firstly...
First and foremost...
Initially...
To begin with...
To start with...
In the first place
On the one hand...
Second(ly) ... (do not use "Second of all")
Third(ly)...
Then...
Next...
After that...

And...
Again...
Also...
Besides...
Likewise...
In addition...
Consequently...
What's more...
Furthermore...
Moreover...
Apart from that...
Finally...
Last but not the least...



Showing a Comparison

In the same way...
Likewise...
Similarly...
Like the previous point...

Similar to...
Also...
At the same time ...
Just as ...

Showing Contrast

On the other hand...
On the contrary...
However...
Nevertheless... / Nonetheless...
But...
Nonetheless / Nevertheless...

Oppositely...
Alternatively...
Unlike...
While...
Whilst...
Although...

Though...
Even though...
Despite... / In spite of...
In spite of the fact that...
Alternatively...
In contrast to this...
Then again...
On the other hand...
Despite the fact that...
Even so...
Yet...
Meanwhile...

Expressing Condition

If...
Provided that...
Because of that...
For this reason...
Unless...
Providing that...
So that...
In case...
Whether

Expressing Certainty

Certainly...
Definitely
No doubt...
Of course...
Doubtlessly...
Without any doubt...
Undoubtedly...

Adding Further Information

In addition...
And...
Moreover...
Similarly...
Furthermore...
Also...
As well as...
Besides...
Even...
Too...
What's more...
Again...
In a similar fashion...
Likewise...

Agreement & Disagreement

While writing your essay, as a writer you are required to show whether you agree & disagree or partially agree with a given statement or opinion.

Expressing Agreement

I strongly agree...
I completely agree that...
I totally agree with the given idea that...
I agree with the opinion that...
I am quite inclined to the opinion that...
I accept that...
I accept the fact that...
I am in agreement...
I consent that...

Expressing Disagreement

I disagree with the opinion that...
I strongly disagree...
I completely disagree with...
I totally disagree with the given idea that...
I disagree with the statement...
I quite oppose the opinion that...
I disapprove that...
I totally do not accept the fact that...
My own opinion contradicts...
I disagree with the group of people...
However, my opinion is different from

Expressing Partial Agreement

To some extent...
In a way...
I agree with the given statement to some extent...
Up to a point, I agree...
More or less...
So to speak...

CONDUCTING DEBATE

Engaging in friendly or formal argument is an ancient art. These days, you can match wits in a regular backyard spar, or as part of an organized debate. Whether you're debating spontaneously or as part of a team or going at it solo, it can be helpful to learn some of the popular formal and informal strategies of debate.

1. Adhere to all rules and professional standards.

While rules will vary situation to situation, many standards are common to most debates. Come dressed to play the part of a serious debater, and bring an attitude to match. Don't wear anything tight or revealing.

- Face the judge when you speak, and speak standing.
- Read full citations when you are quoting.
- If you're not sure if what you are doing is professional, ask the judge's permission. For instance, if you want to leave the room for water, ask.

- In team debates, avoid prompting your partner unless they are immediately jeopardizing your chances of winning. Try not to do it at all.
- Keep your cell phone off
- Do not curse.
- Limit jokes to those that would be appropriate in a professional setting. Don't tell jokes that are off-color or that rely on insensitive stere types.

2. Be ready to receive a topic.

In British Parliamentary, for instance, one team must debate the "affirmative stance, and the other must debate the "negative" stance. The team that agrees with the topic is called the affirmative while the team that disagrees is called the negative.

- For Policy Debate, the affirmativ team proposes a plan and the negative team argues that it should not be enacted.
- Both teams will be seated near the front of the room they are to speak in — affirmativ team (Government) on the left, negative team (Opposition) on the right.
- The chairperson or adjudicator will start the debate, and the first speaker will present their speech. The order of the speakers is generally affirmative negative, affirmative, negative, and so o

3. Define the topic simply when necessary

You might need to offer a definition of the topic before you pr eed.

- The affirmativ always gets the first and best opportunity to define the topic. To define well, try to mirror the way an average person on the street might define the topic. If your interpretation is too creative, the other team might attack it.
- The negative team is given an opportunity to refute the definition (otherwise known as challenging the definition) and offer their own, but only if the affirmative definition is unreasonable or it renders the negative's position obsolete. The firs negative speaker must refute the affirmative' definition if s/he wishes to challenge it.

4. Write your speech in the time allotted.

Keep your eye on your watch, and set a timer for a minute before your time is up so that you can look over your argument before you are done. Your allotted writing time will depend on the style of debate. For British Parliamentary, for instance, seven minutes is likely. To write e ciently, get your main points down rst, then ll in evidence, additional refutations, and any examples or anecdotes you are choosing to include.

Depending on what position you argue, you must follow certain protocol such as defining the topic or presenting a main argument

5. Support your argument.

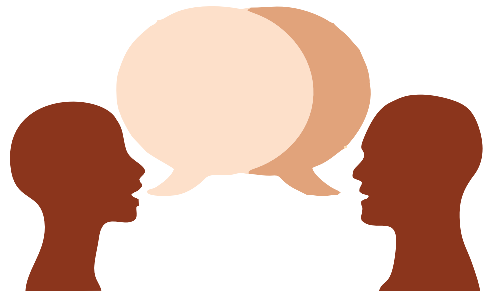
If you say "I think the death penalty should be abolished," be ready to prove why this is the best course of action. Provide supporting arguments, and give evidence for each. Make sure your supporting arguments and evidence truly relate to your stance, or your opposition may co-opt them or ask for them to be thrown out.

- Your opposing arguments might be "The death penalty is more expensive than life in prison," "the death penalty provides no opportunity for redemption," or "the death penalty makes us look bad in the international community."
- Evidence can include statistics and expert opinions.

6. Choose what to include carefully.

If you don't know it, don't debate it unless you have no other choice. If you don't know much about the topic, try to at least come up with some vague, ambiguous information so that your opponents will have a hard time refuting your contentions.

- If they don't understand it, they can't refute it. Keep in mind that the judge probably won't understand you so well either, but trying is probably better than saying, "I know nothing. I give the case to my opponents."



- Don't use rhetorical questions. Always give a clear answer to every question you ask. Leaving a question open-ended gives your opponents room to refute.
- Use religion only when appropriate. Things that are written in the Bible, Torah, Quran, etc, are not usually sound resources to use to prove your argument, as not everyone takes these sources to be the truth.

7. Present your argument with feeling.

Be passionate in your speech — a monotone voice will cause people to drift off, and they may miss the point of what you're trying to say. Speak clearly, slowly, and loudly.

- Make eye contact with whomever decides the winners of the debate. While it's okay to look at your opponents every once in a while, try to direct your argument at the judge.
- Give a layout of your argument before you make it. That way, your audience will know what to expect and your judge won't cut you off unless you run way overtime.

8. Strike a balance between presenting your team's point(s) and rebutting the opponent's point.

Since teams take turns debating, it's always possible to offer rebuttals unless you are the first affirmativ speaker. For British Parliamentary, for example, both teams might organize their debate strategy thus:

1st affirmativ

- Define the topic (optional) and present the team's main line
- Outline, in brief, what each affirmative speaker will talk abou
- Present the first half of the affirmative's argume

1st negative:

- Accept or reject the definition (optional) and present the team s main line.
- Outline, in brief, what each negative speaker will talk about.
- Offer a rebuttal of a few of the points presented by the first ffirmativ
- Present the first half of the negative's argument
- This will continue into second and third affirmative and negati arguments.

9. Rebut the main points of your opponents' argument.

When rebutting a team's argument, remember:

- Offer evidence for your rebuttal. Do not rely on vigorous assertion alone. *Show* the chairperson why the other team's argument is fundamentally flaw d; don't just tell.
- Attack the most important parts of their argument. It's not very effective if you pick bones with an obscure part of the opponent's argument. Go for the crux of their argument and pick it apart with the ruthless efficiency of a s geon.

• For instance, if they are arguing for an increase in the military budget, but they also make a casual assertion about citizens being ungrateful for what the military does, you can dismiss the latter with a calm "I beg to disagree" and focus on the problems with increasing the actual budget.

• No ad hominem attacks. An ad hominem attack is when you criticize another person instead of their ideas. Attack the idea, not the person.

10. Use up all your time (or most of it).

The more you talk, the more you'll convince the judge. Note that this means you should come up with many examples, not that you should ramble. The more the judge hears about why you are correct, the more inclined she will be to believe you.

11. Know what aspects of the debate you will be judged on, if appropriate.

For the most part, debates are judged on three main areas: matter, manner, and method.

• **Matter** is amount and relevancy of evidence. How much evidence does the speaker marshall to support his/her claims? How strongly does the evidence used support the argument?

• **Manner** is eye contact and engagement with audience. Don't stare at your cue cards! Speak clearly. Accentuate your arguments with volume, pitch and speed to highlight important parts. Use your body to emphasize your arguments: stand straight and gesture confidently. Avoid stammering, fidgeting, pacing.

• **Method** is team cohesion. How well does the entire team organize their arguments and rebuttals? How well do the individual arguments mesh together, as well as the rebuttals? How clear and consistent is the team line?

USEFUL VOCABULARY CONDUCTING A DEBATE

FORMAL SECTION PHRASES

Opening the debate

[some nice opening, e.g. quote]
Ladies and Gentlemen, welcome to this debate.
Welcome from this side of the house...
The motion for debate today is: ...

De ning the motion

Now we as today's proposition/opposition strongly believe that this is true/not true, but before we come to our actual argumentation, let us first define some important terms in this debate.

We believe that what is meant by... is... / that... are...
When we say... should... we mean that...

Presenting the teamline

We as today’s proposition / opposition have structured our case as follows:
I, as the first speaker, will be talking about
Our second speaker, ..., will elaborate on the fact that...
And our third speaker, ..., will do the rebuttal.

Rebutting arguments, rebuilding your case

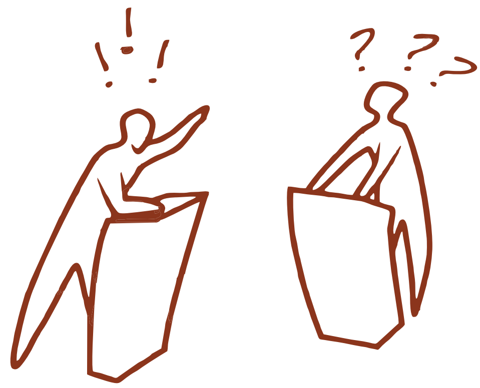
But before I come to my own arguments, let us first have a look at what ... has said

I will continue our case in a minute, but before that there are some things about the ... speech that need to be addressed.

The first prop / opposition speaker has told us ...; on the contrary...

He / She also said that ...; but in fact.

He / She was claiming that ...; but as my firs speaker already told you, ...



Introducing arguments

Let me come to my first / second / ... / next argument: [concise label of argument]

My first / ... argument is

The first / ... reason why we’re prop / opposing this motion is: explaining arguments:

Giving examples

There are many examples for this / for ..., for instance.
In fact, you can find many examples for this in real life. Jus think of...
And there are similar cases, such as ..., ...
So in this simple example we can clearly see the effect of

Summarizing & linking the argument

So as we have seen [argument label], and therefore [motion].
Now because of this ..., we have to support this motion.

Summarizing & ending your speech

So Ladies and Gentlemen, what have I told you today? Firstly..., Secondly...
[some nice closing words]
And for all of these reasons, the motion must stand / fall.

Making / rejecting / accepting / answering points of information

Point of information, Sir / Madam.

On that point.

Wouldn’t you have to agree ...? / Doesn’t what you’re saying contradict with ...? / What about the ...? / How would you explain, that ...?

No, thank you, Sir / Madam.
Declined.
Yes, please. / Go ahead.
Thank you very much, Sir / Madam, I'm going to come to this very point in my second argument in a minute.

Giving reply speeches

Ladies and Gentlemen, welcome for the last time from today's prop / opposition. It is now my pleasure to summarize this debate, take a look at what both sides have said and see what the outcome of this debate actually is.

A firs / second /... major clash was: ... Today's prop / opposition told us ...; we had to find

[some particularly nice closing words]

And for all these reasons, I beg you to prop/oppose

INFORMAL DEBATE PHRASES

When you are listening to the other side

I see your point, but I think...
Yes, I understand, but my opinion is that...
That's all very interesting, but the problem is that...
I'm afraid I can't quite agree with your point.
I think I've got your point, now let me respond to it.
We can see what you're saying. Here's my reply...

When you need to say something now

I'm sorry to interrupt, but you've misunderstood our point.
Excuse me, but that's not quite correct.
Sorry, I just have to disagree with your point.
Let me just respond to that, please.
Forgive me for interrupting, but I must respond to that.
Hold on a moment, that's not correct.
If you would allow me to add a comment here...
If you don't mind, I'd like to take issue with what you just said.

When you haven't replied yet

The other side will have to explain why ... otherwise we win that point.
We said that ... but the other side has not replied to our point.
I'd like to focus on two points that the other side has failed to address.
There are two points that we have succeeded in establishing...

I want to call your attention to an important point that our opponents have not addressed yet.

I'd like to point out that there are two issues our opponents have failed to dispute, namely...

I must stress again that our point has not been refuted by the other side.

When you give your rebuttal

The first point I would like to raise is this
Our position is the following...
Here's the main point I want to raise...
I'd like to deal with two points here. The first is
Our opponents have still not addressed the question we raised a moment ago...
The other side has failed to answer our point about...
Notice that the affirmative side has not addressed our main poi .
Let me just restate my position.
Just to be clear, here is what I mean...

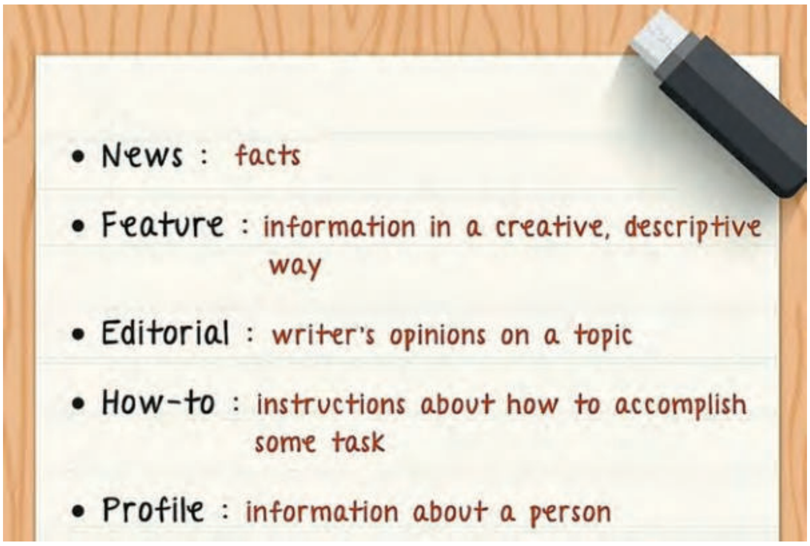
When you give concluding statements

To sum up, here are the main points our opponents have not addressed...
We pointed out that...
Our opponents have claimed that...
To recap the main points...
Let's sum up where we stand in this debate.
Let me summarize our position in this debate.
In summary, we want to point out that...
Let's see which arguments are still standing.
Let's take stock of where we are in this debate.

HOW TO WRITE AN ARTICLE

An article is a written piece aimed at reaching a large audience.

There are a multitude of different types of articles, including news stories, features, profiles, instructional articles, and so on. While each has specific qualities that are unique to its type, all articles share some common characteristics. From forming and researching your idea to writing and editing your work, writing articles can give you a chance to share compelling and important information with readers.



As you're figuring out your topic and focus, think about the type of article that will best suit the points that you want to convey. Some article types are better suited to certain topics. Some of the most common types of articles are:

- News: This type of article presents facts about something that happened recently or that will happen in the near future. It usually covers the 5 Ws and H: who, what, where, when, why and how.
- Feature: This type of article presents information in a more creative, descriptive way than a straight news article. It can be an article about a person, a phenomenon, a place, or other subject.
- Editorial: This article presents a writer's opinions on a topic or debate. It is intended to persuade the reader to think a certain way about a topic.
- How-to: This article gives clear instructions and information about how to accomplish some task.
- Profile This article presents information about a person, using information that the writer typically gathers through interviews and background research.

The basic structure of an article contains 4 parts:

- title;
- opening paragraphs (introduction);
- the main discussion (body section);
- closing paragraphs (conclusion).

A GUIDE TO CREATING A GREAT ARTICLE

An article writing involves researching the basic text structure and following specific guidelines

Let's put together a list of points you must remember to produce great content.

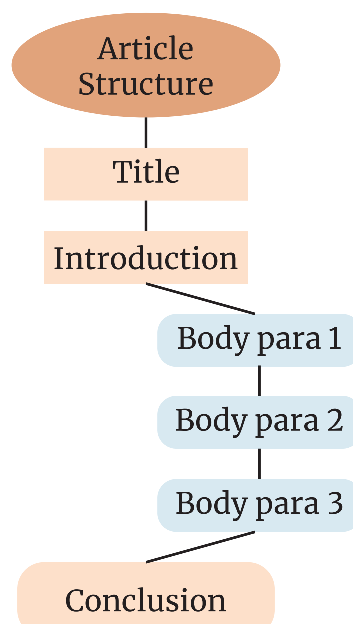
Pitch Your Ideas

One of the best article-writing tips is— always prepare a list of ideas

Don't Distract

Article writing can be a burden in case you have something else in mind. Keep working in a quiet space without TV shows, noisy kids, or shouting neighbors.

An environment matters: it might be a coffee shop nearby, an o   ce, or your living room. The longer you are in a distraction-free space, the more your mind can concentrate on completing tasks.



Understand a Topic

Research is the first thing to do before you write. You have to be fluent in any topic; otherwise, you shouldn't sign up for a project.

At this point, your brain might be flooded with a lot of information, so your main task is to filter them based on the task requirements. Attempting to follow an inspiration is not a good idea because the content will likely be chaotic.

Simplify Your Words

An article writer is still a human being. You must convey the right message to your target audience as a creator. Therefore, avoid fancy words and keep the article relatable to knowledgeable readers.

Add Bullet Points

A professional article writer knows how to present even the most extraordinary topic in a laconic way. Bullet points are helpful for a range of reasons:

- they draw attention to important information;
- they are visually appealing;
- they add variety to your article's body.

Besides, the information written in bullet points is easier to remember. One list per article can make a big difference; remember it

Edit and Proofread

Proofreading is a crucial factor to take into account. Before writing a summary of an article, make sure to read the whole piece thoroughly. Then, remove the unused sentences or add some to clarify information.

Follow a Deadline

Last but not least: meeting deadlines is essential.

Before you start working on an article, ensure you know what the piece will be about and how fluent you are in the given topic. Online article writing demands a certain level of knowledge. In addition, you have to navigate through tons of information quickly.

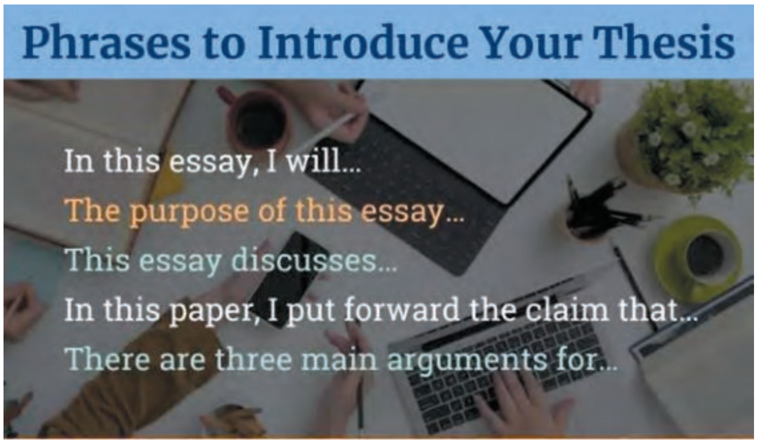
You can use time-management apps to get more organized. It is a note-taking program that sorts your important things by hashtags and will remind you about an upcoming deadline.

USEFUL VOCABULARY: WRITING AN ARTICLE

VOCABULARY TO USE IN THE ESSAY INTRODUCTION

Referencing another author or speaker

To use the words of X
According to X
As X states



Introducing the rst of a series of arguments

First
First and foremost
First of all
To begin with

VOCABULARY TO USE IN THE BODY OF THE ESSAY

Introducing a second correlating idea

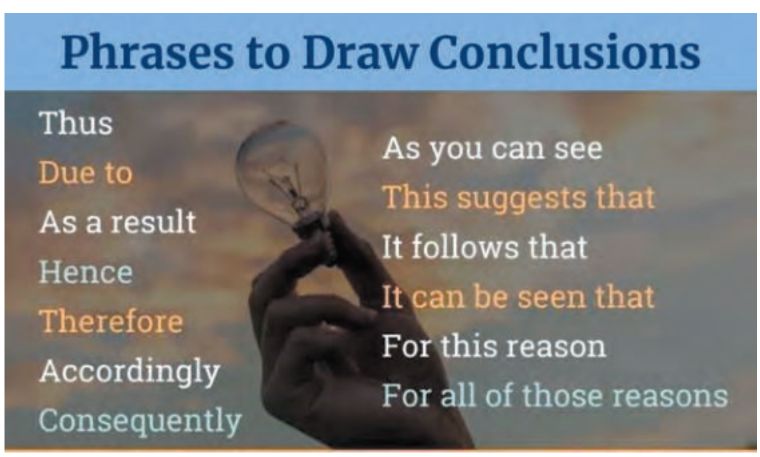
Additionally
In addition
Also
Secondly
Furthermore
Another key thing to remember
In the same way
Similarly
Likewise
Correspondingly

Restating an idea

In other words
To put it another way
That is to say
To put it more simply

Giving an example

For instance
To give an illustration of
To exemplify
To demonstrate
As evidence



Introducing a new idea

Moreover
Further
What's more
As well as
Along with
Besides
Not only ... but also
Not to mention
To say nothing of
Another key point

Comparing and contrasting ideas

On the one hand / on the other hand
Conversely
However
Alternatively
In contrast to
On the contrary
Whereas
By contrast
In comparison

Introducing a new angle that contradicts your previous idea, you can use the following phrases

Having said that
That said
Even so
Then again
Differing fro
Granted
Despite
Yet
In spite of
While
With this in mind
Provided that
Nevertheless
Nonetheless
Notwithstanding
Admittedly

Vocabulary to Use in Your Essay Conclusion

In conclusion
To summarize
To sum up
In summary
In a nutshell
In brief
In short
In essence
All in all
Given the above
As described
All things considered
Finally
Lastly

To place emphasis on a certain point or a key fact

Unquestionably
Undoubtedly
Particularly
Especially
Importantly
Singularly
Chiefl
Namely
Conclusively
It should be noted
Above all
Ultimately
On the whole

REPORT WRITING TIPS

Report writing is a great way to present information, share research findings analyze a problem, and recommend a solution. But there are a few rules you must follow if you want to get it right.

FORMAT FOR REPORT WRITING

Reports address a specific situation. In it, you need to state why it is worthy of research, while also citing other studies on the subject. You will also have to describe your research methods, evaluate the findings, and finally, make conclusions or recommendations.

Not all reports follow this exact format. Some include more headers, while others omit a few titles. Listed below are the diff rent sections of a report. You may choose to add or delete a section depending on the layout recommended to you.

But if you are unsure about what format to employ, consult your lecturer (at university) or find out what your company's recommended layout s.

1. Title page.

This should include the title of your report, your name, the date, and academic information (the name of your course and instructor).

2. Acknowledgments.

Be sure to credit any person or piece of work that assisted or provided you with information. These can be colleagues, experts, academics, and even libraries and research papers.

3. Reference (optional).

References give your readers an idea about what your report might contain.

This defines the scope and limitations of your report, as well as your writing goal and audience.

4. Abstract / Summary.

This contains the objectives (if you don't include the reference section), main findings, conclusions, and recommendations. It basically shares the most important aspects of your report in a nutshell.

5. Contents Page.

This section includes all of your steps in report writing. Such as sections and subsections, page references, as well as a list of diagrams or illustrations, and appendices.

6. Introduction.

Your introduction addresses the reason you are researching the topic, the context and goals of your research, your research methods, and a brief about the outcome.

7. Methods / Methodology / Procedure (optional — if not covered in the introduction).

This section should include how you conducted your research, including the techniques, equipment, and procedures you employed.

8. Discussion / Main body (the longest part of your report).

This section contains the analysis and interpretation of your fi dings (often linked to a current theory or previous research), which should be divided into headings and subheadings for clarity. You can also include visual information, such as diagrams, illustrations, charts, and so on, in this section.

9. Outcomes (can also go before the main body of the report).

Here, you can add your research findings (also presented in tables, etc.), but with no discussion or interpretation of them.

10. Conclusion.

Writing about results and recommendations at the end of your report is the perfect way to sum it up.

This section shares your discoveries from the finding — your deductions and the most important revelations from your research report writing. Ideas for improvement, which can also be part of the conclusion section, will be under a subheading called "Recommendations," in a number-format.

WHAT MAKES A GREAT A REPORT?

1. Use names and pronouns.

When writing about yourself and others on the scene, use names and pronouns (I, he, she). Avoid clichés like "this officer, "the above-mentioned person," and "officials. Certain people argue that using impersonal terminology ensures objectivity and accuracy, but this is not true. You have the same level of integrity whether you refer to yourself as "I" or as "me."

2. Present only one idea per sentence.

Short, simple sentences are easier to read and understand, and they save everyone's time. They are especially important when someone is reviewing your report writing in preparation for an important business meeting. Furthermore, the longer a sentence is, the more likely it is that you will make an error writing it.

In English, a short sentence and its structure usually begin with a noun, and the grammar is simple. Complicated sentences, on the other hand, necessitate confusing punctuation and invite sentence errors. So try to keep your sentences to a maximum of three commas. If a sentence contains more than three commas, it is likely to be too complicated to read with ease.

3. Be clear and specific

Strive for clarity at all times. For example, "contacted" is a broad term. Instead of using it, clarify if you paid the witness a visit, called, or sent an email. So is "residence." Is it better to use in a house, an apartment, or a mobile home? If yes, do so!

4. Use straightforward language.

"Since" is more understandable (and easier to write) than "as much as." Another example is the phrase "pertaining to" which is a fancy (and time-wasting) way of saying "about."

5. Stick to verifiable facts

Educated guesses, hunches, and other thought processes have no place in report writing. So limit yourself to the facts. In court, a statement like "He was aggressive" will not be accepted. Instead, "Jackson clenched his fists and kicked a chair, which now sits broken" is acceptable.

6. Use more paragraphs.

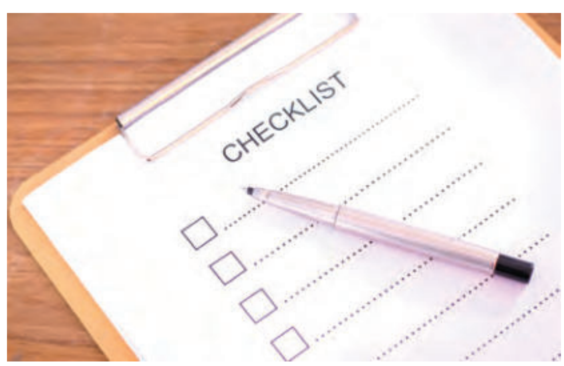
Organizing information in groups has two significant advantages- it will make your report more logical, and easier to read and understand at a later stage.

7. Have an active voice.

A common misconception is that using the passive voice ensures objectivity and accuracy. However, this is not the case, especially when you're expressing yourself with clarity. So always use an active voice — it will not only give you room for objectivity, but also make your report more readable.

8. Make use of bullet points.

Bullet points or numbered lists are a great way to present lengthy but relevant information



Bullet style is simply the way you have probably been writing shopping lists your entire life. When recording several pieces of related information, use bullet points as your format. This will make it easier for the reader to comprehend information.

USEFUL VOCABULARY FOR WRITING REPORTS

Introduction

The aim / intention / purpose of this report is to outline / present / discuss / sum up...

Further to my visit to ..., I have prepared the following report.

I have recently visited ... and have prepared the following report for your consideration.

This involved visiting / looking at / investigating ... / The data was obtained by...

In order to help make this report I asked / discussed / gave out a questionnaire...

It is based on my observations / the feedback from participants...

My findings are outline / presented below. / I outline my findings below

The report contains the relevant details concerning the problem as you required.

Introducing points

To begin with... / Let us start with...

First(ly)... / In the first place... / First of all... / The first aspect / thing to consider is...

Second(ly)... / Third(ly)...

Moreover... / Furthermore... / What is more...

Another aspect to consider... / Yet another aspect / consideration is...

Besides that... / Apart from that... / In addition to this... / On top of that...

Introducing your opinion

I think / believe that... / In my opinion... / I am of the opinion that... / It seems to me that...

Personally I believe that... / In my view... / If you ask me... / To my mind... / As far as I am concerned...

I would like to suggest / recommend... / I therefore suggest / recommend...

I (strongly) recommend... / My recommendation is to...

Introducing someone else's opinion

A few / Many / The majority / minority of people said / reported / complained...

According to ... / As ... said ... / In the words of...

It is said that ... / It is often suggested that...

Giving examples

For example / instance...

This can be shown / illustrated / demonstrated / clarified by

Let me just give you an example, ...

The picture / diagram shows / illustrates...

Comparing

One of the main / biggest / most significant / ... differences between ... and ... is...
Unlike ..., ... is ... / While / Whereas / Although ... is, ... is...
... is completely / entirely / totally different from
... is a little / slightly / somewhat / a great deal bigger / more elegant / ... than...
... is not quite / nearly as comfortable / expensive / convenient / ... as...
... is virtually / exactly the same as ... when it comes to...

Conclusion

It can be seen from the data / reactions / information above that...
All things considered, I believe that... / Taking everyone's comments into consideration...
In general / On the whole I found that...
In conclusion... / To conclude... / To sum up... / In summary...
To put the matter in a nutshell... / In a nutshell...

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