**БЕЛОРУССКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ /
BELARUSIAN STATE UNIVERSITY**

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Ректор Белорусского
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ИНТЕГРАЛЬНЫЕ ПРЕОБРАЗОВАНИЯ И ТЕОРИЯ ФУНКЦИЙ КОМПЛЕКСНОГО ПЕРЕМЕННОГО/

INTEGRAL TRANSFORMS AND COMPLEX VARIABLE FUNCTIONS

Учебная программа учреждения образования по учебной дисциплине для специальности:
The program of the educational institution of the discipline for the speciality:

Специальность: 7-06-0533-06 – Механика и математическое моделирование /
Speciality: 7-06-0533-06 – Mechanics and mathematical modeling

Профилизация: Теоретическая и прикладная механика /

Profilization: Theoretical and Applied Mechanics

2024

Учебная программа составлена на основе ОСВО 7-06-0533-06-2023, учебного плана БГУ № M54a-54-114/уч. 11.04.2023 г.

СОСТАВИТЕЛИ:
С.В. Рогозин, доцент кафедры аналитической экономики и эконометрики, кандидат физико-математических наук, доцент.

РЕЦЕНЗЕНТЫ:
Л.А.Хвощинская – доцент кафедры общей и медицинской физики МГЭИ им. А.Д. Сахарова, кандидат физико-математических наук, доцент;

С.И.Василец – доцент кафедры математики и методики преподавания математики БГПУ им. М. Танка, кандидат физико-математических наук, доцент.

РЕКОМЕНДОВАНА К УТВЕРЖДЕНИЮ:
Кафедрой теоретической и прикладной механики БГУ
(протокол № 12 от 28 мая 2024);

Научно-методический Совет БГУ
(протокол № 9 от 28.06.2024).

Заведующий кафедрой теоретической
и прикладной механики, доктор
физико-математических наук, профессор ______________ М.А.Журавков

ПОЯСНИТЕЛЬНАЯ ЗАПИСКА

Цели и задачи учебной дисциплины

Цель учебной дисциплины – сформировать углубленные понятия о возможностях использования методов теории функций комплексной переменной и интегральных преобразования для моделирования процессов в механике сплошной среды и решения соответствующих математических задач.

Задачи учебной дисциплины:

1. Изучить основные понятия и методы, применяемые в указанных разделах математики.

2. Изучить методы моделирования процессов в механике сплошной среды, основанные на использовании свойств функций комплексного переменного и основных интегральных преобразований.

Место учебной дисциплины в системе подготовки специалиста с высшим образованием.

Учебная дисциплина относится к модулю «общие математические курсы» компонента учреждения образования.

Учебная программа составлена с учетом межпредметных **связей** и программ по дисциплинам: дифференциальные уравнения в частных производных, механика сплошной среды, механика твердого тела.

Требования к компетенциям

Освоение учебной дисциплины «Интегральные преобразования и теория функций комплексного переменного» должно обеспечить формирование следующих компетенций:

СК - Применять математические методы для моделирования динамических систем.

В результате освоения учебной дисциплины студент должен:

знать: <u>структуру и алгебраические свойства множества комплексных чисел, свойства функций комплексного переменного, включая топологические, дифференциальные, интегральные, основные конструкции теории интегральных уравнений;</u>

уметь: <u>решать задачи, использующие методы функций комплексной переменной, использовать методы интегральных преобразования для решения математических задач и моделирования процессов механики сплошной среды;</u>

иметь навык: <u>владения аналитическими методами теории функций комплексной переменной, методами вычисления комплексных интегралов, методами вычисления интегральных преобразований и решения некоторых дифференциальных и интегральных уравнений с помощью этих преобразований.</u>

Структура учебной дисциплины

Дисциплина изучается в 1 семестре. Всего на изучение учебной дисциплины «Интегральные преобразования и теория функций комплексного переменного» отведено:

Всего на изучение учебной дисциплины «Интегральные преобразования и теория функций комплексного переменного» отведено: 36 часов, из них количество аудиторных часов 36 часов, в том числе лекций – 18 часов, лабораторных занятий – 18 часов.

Трудоемкость учебной дисциплины составляет 3 зачетные единицы.

Форма промежуточной аттестации – экзамен.

EXPLANATORY NOTE

Aim and tasks of the discipline

Aim of the discipline – to create advanced concepts on possibilities of use the methods of the complex variable functions and integral transforms for modelling of processes in continuum mechanics and in solving corresponding mathematical problems.

Tasks of the discipline:

1. To study basic notions and methods in the mentioned branches of mathematics.

2. To study the methods of modelling of processes in continuum mechanics based on the use of the properties of the complex variable functions and main integral transforms.

Place of the academic discipline in the system of training a specialist with higher education.

The academic discipline is part of the module «general mathematical courses» of the educational institution component.

Academic discipline is completed accounting interdisciplinary relations and is connected with the following academic discipline: "partial differential equations", "continuum mechanics", "solid mechanics".

Requirements for competences

Mastering of the academic discipline «Integral transforms and complex variable functions» should provide the formation of the following competences:

SC - Apply mathematical methods for modeling dynamic systems.

As a result of mastering the academic discipline, the student is expected to:

know: the structure and algebraic properties of complex numbers, the properties of the complex variable functions, including topological, differential and integral properties, the basic construction of the theory of integral transforms;

be able to: solve problems applying the methods of complex variable functions, use the methods of integral transforms for the solution of mathematical problems and modelling the processes of continuum mechanics;

have skills in: the analytic methods of complex variable functions, the methods of calculation of complex integrals, the methods of calculation of integral transforms and solution of certain differential and integral equations using integral transforms.

Structure of the academic discipline

The discipline is studied in the 1st semester. In total for the study of the discipline Integral transforms and complex variable functions is allocated:

36 hours, including 36 in-class hours, of them: lectures – 18 hours, practical classes – 18 hours. The labour intensity of the discipline is 3 credit units. Form of certification – exam.

CONTENT OF THE STUDY MATERIAL

Section 1 *Complex variable functions*

Topic 1.1 *Complex numbers. Planar sets. Riemann sphere.*

Topic 1.2 *Functions of complex variables. Analyticity. Examples of functions.*
Topic 1.3 *Integration on complex plane. Cauchy's theorem and formula.*

Topic 1.4 *Power series. Singularities. Residue theorem.*

Number series. Taylor series. Cauchy-Hadamard formulas. Taylor series for elementary functions. Isolated singularities and their classification. Meromorphic functions. Residue theorem.

Topic 1.5 *Harmonic functions. Poisson equation.*

Harmonic functions. Laplace operator. Relations between harmonic and analytic functions. Uniqueness theorem. Green's formula. Dirichlet problem. Poisson formula.

Section 2 *Integral transforms*

Topic 2.1 *Fourier transform.*
Topic 2.2 *Laplace transform.*
Topic 2.3 *Mellin transform.*

Section 3 *Basic special functions*

Topic 3.1 Special functions

TEACHING AND METHODOLOGICAL MAP OF THE DISCIPLINE

Full-time form of higher education

Title of section, topic	Title of section, topic	In-class hours					Independent work	Form of control
		Lectures	Practical classes	Seminar classes	Laboratory classes	Other		
1	2	3	4	5	6	7	8	9
1	Complex variable functions							
1.1	Complex numbers. Planar sets. Riemann sphere.	2	2					interview at the lecture, check of the solutions to exercises
1.2	Analytic functions of the complex variable.	2	2					interview at the lecture, check of the solutions to exercises
1.3	Integration on the complex plane.	2	2					interview at the lecture, check of the solutions to exercises
1.4	Power series.	2	2					interview at the lecture, check of the solutions to exercises
1.5	Harmonic functions.	2	2					interview at the lecture, check of the solutions to exercises

2	Integral transforms.							
2.1	Fourier integral transform.	2	2					interview at the lecture, check of the solutions to exercises
2.2	Laplace integral transform.	2	2					interview at the lecture, check of the solutions to exercises
2.3	Mellin integral transform.	2	2					interview at the lecture, check of the solutions to exercises
3	Special functions.	2	2					interview at the lecture, check of the solutions to exercises
	Total	18	18					

INFORMATION AND METHODOLOGICAL PART

List of basic literature

1. Asmar, Nakhle H. Complex Analysis with Applications / Nakhle H. Asmar, Loukas Grafakos. - Cham : Springer, 2018. - viii, 494 c.

2. Garling, D. J. H. A Course in Mathematical Analysis : [in 3 vol.] / D. J. H. Garling. - Cambridge : Cambridge University Press, 2014. - Vol. 3 : Complex Analysis, Measure and Integration. - 2014. - x c., C. 625–945.

List of additional literature

1. Debnath, L. Integral Transforms and their Applications / L. Debnath, D. Bhatta. Boca Raton, Fl: CRC Press, 3rd ed. – 2015. – 778 p.

2. Лунц, Г. Л. Функции комплексного переменного с элементами операционного исчисления: [учебник для вузов] / Г. Л. Лунц, Л. Э. Эльгольц. - 2-е изд. - Санкт-Петербург: Лань, 2002. - 299 с.

List of recommended diagnostic tools and methodology for final mark formation

The form of interim certification in the discipline "Integral transforms and complex variable functions" in accordance with the curriculum is exam.

A rating system of the student knowledge is used for the final mark formation, which makes it possible to trace and evaluate the dynamics within the process of achieving learning objectives. The rating system stipulates the use of weighting coefficients for current and interim certification of students in the academic discipline.

The following means of current certification can be used to diagnose competences: interview at the lecture, check of the solutions to exercises.

The final mark formation in the course of control measures for current certification (approximate weighting coefficients determining the contribution of current certification to the mark for passing interim certification) includes:

- answers at seminar classes – 50 %
- exercises – 50 %

The final mark for the discipline is calculated on the basis of the mark of current certification (rating system of knowledge) — 50 % and exam mark — 50 %.

Approximate topics of practical classes

1. Complex numbers. Sets.
2. Complex variable functions. Differentiability.
3. Integration. Cauchy integral theorem. Cauchy integral formula.
4. Power series. Isolated singularities. Residue theorem and its application.
5. Harmonic functions.
6. Fourier transform. Calculation and application to the solution of problems.
7. Laplace transform. Calculation and application to the solution of problems.
8. Mellin transform. Calculation and application to the solution of problems.
9. Special functions. Check of the properties.

Description of innovative approaches and methods for teaching the discipline

When conducting classes on the discipline "Integral Transformations and Theory of Functions of a Complex Variable", a practice-oriented approach is used, which involves:

- mastering the content of education through solving practical problems;
- acquiring skills for effectively performing various types of professional activity;
- focusing on generating ideas, implementing group student projects, developing an entrepreneurial culture;
- using procedures and assessment methods that record the formation of professional competencies.

Methodological recommendations for the organization of independent work

Modern information technologies are actively used to organize independent work of students; exam questions, exam assignment samples, and methodological recommendations are available to master's students in electronic form. The effectiveness of independent work of master's students is checked during current and final knowledge assessment. A rating system is used for the overall assessment of the quality of master's students' assimilation of educational material.

Approximate list of questions for the exam

Theoretical questions.

1. Complex numbers and operations with complex numbers.
2. Domains and curves on the complex plane and an extended complex plane. Riemann sphere.
3. Complex variable functions. Continuity.
4. Analytic functions of complex variable. Cauchy-Riemann equations.
5. Integration on the complex plane. Properties of the complex integrals.
6. Cauchy integral theorem.
7. Cauchy integral formula. Application to the calculation of integrals.
8. Power series.
9. Laurent series. Isolated singularities.
10. Residue Theorem. Calculation of integrals.
11. Harmonic functions. Connection with analytic functions.
12. Dirichlet problem for the Laplace equation.
13. Green's function for the Laplace equation. Poisson formula.
14. Fourier transform. Existence and other properties.
15. Calculation of the Fourier transform. Application to the solution of differential and integral equations.
16. Laplace transform. Existence and other properties.
17. Calculation of the Laplace transform. Application to the solution of differential and integral equations.
18. Mellin transform. Existence and other properties.
19. Calculation of the Mellin transform. Application to the solution of differential and integral equations.
20. Basic special functions and their differential equations.

Practical tasks.

1. Calculate:

   a) $(1+2i)\{3(2+i)-2(6+2i)\}$; b) $\frac{3+5i}{7+i}+\frac{1+i}{4+3i}$; c) $\left(\frac{i^9+2}{i^{11}+1}\right)^2$; d) $\frac{(1+i^5)^3}{(1+\sqrt{3}\,i)^8}$.
2. Calculate all values of the roots of complex numbers

   a) $\sqrt[3]{-1}$; b) $\sqrt[3]{-4+3i}$; c) $\left(-\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{1/4}$.
3. Determine what lines are described by the following relations

   a) $|z-2|+|z+2|=5$; b) $|z-z_1|=|z-z_2|$; c) $Re\,\frac{1}{z}=\frac{1}{a}$ $(a>0)$;

   d) $Im\,\frac{z-1}{z+1}=0$.
4. Determine what sets on the plane satisfied the following inequalities. Draw the pictures.

   a) $\frac{1}{|z|}\geq 1$; b) $|z-2|-|z+2|<2$; c) $1<|z+i|<2$; d) $0<Re\,(iz)<1$;

e) $\frac{\pi}{4} < arg\,(i+z) < \frac{\pi}{2}$.

5. Find the roots of the equations

   $a)\ z^3+8=0;\ b)\ e^z=-1;\ c)\ \cos z=\frac{3i}{4};\ d)\ \operatorname{sh} z=\frac{i}{2}$.

6. Calculate the values of the functions at the point z_0

   $a)\ f(z)=z+\sqrt[4]{z},\quad z_0=-1;$

7. Calculate the limits:

   $a)\ \lim\limits_{z\to i}\frac{z^2-4iz-3}{z-i}\ ;\quad b)\ \lim\limits_{z\to\frac{\pi i}{4}}\frac{\sin iz}{\operatorname{ch} z+i\operatorname{sh} z}\ ;\quad c)\ \lim\limits_{z\to\frac{\pi}{2}}\frac{e^{2iz}+1}{e^{iz}+i}\ .$

8. Prove that the following functions satisfy Cauchy-Riemann equation:

   $$\sin z,\ \operatorname{ch} z,\ e^z, z^n, \log z,$$

   and prove the identities

   $$(\sin z)'=\cos z\ ,(\operatorname{ch} z)'=\operatorname{sh} z,(e^z)'=e^z,(z^n)'=nz^{n-1},(\log z)'=\frac{1}{z}.$$

9. Calculate integrals $I_1=\int_\gamma\ x\,dz,\quad I_1=\int_\gamma\ y\,dz$

   a) along radius-vector of the point $z=2+i$;
   б) along semi-circle $|z|=1,\ 0\le\arg z\ \le\pi$ (initial point is $z=1$);
   в) along the circle $|z-a|=R$.

10. Calculate integrals using Cauchy integral theorem (all curves are positively oriented):

   (a) $\int_{|z|=2}\frac{dz}{z^2+1}$; (b) $\int_{|z|=4}\frac{\cos z\,dz}{z^2-\pi^2}$; (c) $\int_{|z+1|=3}\frac{dz}{(z+1)(1-z)^3}$.

11. Determine the radius of convergence of the following power series

   $$\sum_{n=1}^{\infty}\frac{z^n}{n};\qquad \sum_{n=1}^{\infty}\frac{z^n}{n^2+n};\qquad \sum_{n=1}^{\infty}\frac{(-1)^n z^n}{3^n+n^2};\qquad \sum_{n=1}^{\infty}\frac{nz^n}{3^n};$$

12. Investigate convergence of power series on the boundary of the disc of convergence

   $$\sum_{n=1}^{\infty}\frac{(-1)^n z^n}{1+n};\qquad \sum_{n=1}^{\infty}\frac{(z-1)^n}{(1+n+n^2)};$$

13. Represent following function in form of the Laurent series in the corresponding domains

   a) $\frac{1}{z(1-z)}$ in neighbourhood of points 1) $z=0$, 2) $z=1$, 3) $z=\infty$.

   b) $\frac{z^2-2z+1}{(z-2)(1+z^2)}$ 1) by powers of $(z-2)$; 2) in the ring $\{z: 1<|z|<2\}$.

14. Calculate integrals using Residue Theorem

   a) $\int_\gamma\ \frac{dz}{z^4+1}$, where γ is the circle $x^2+y^2=2x$.

   b) $\int_\gamma\ \frac{zdz}{(z-1)(z-2)^2}$, where γ is the circle $|z-2|=\frac{1}{2}$.

c) $\int_\gamma \frac{dz}{(z-3)(z^5-1)}$, where γ is the circle $|z| = 2$.

d) $\frac{1}{2\pi i}\int_\gamma \sin^2 \frac{1}{z} dz$, where γ is the circle $|z| = r,\ r > 0$.

15. Find harmonic function $u(z)$ in the disc $|z| < R$ for which

$$\lim_{r\to R} u(re^{i\theta}) = \begin{cases} 0, \text{ если } 0 < \theta < \pi, \\ 1, \text{ если } \pi < \theta < 2\pi. \end{cases}$$

16. Solve the Dirichlet problem for the Laplace equation

$\Delta u = 0, \quad -\infty < x < +\infty,\ y > 0,$

With boundary conditions $u(x,0) = 0,\ |x| > 1$, and $u(x,0) = 1,\ x \in (-1,1)$.

17. Calculate Fourier transform of the function e^{-ax^2}. Deduce from the obtained formula the relation $\int_{-\infty}^{+\infty} x^2 e^{-ax^2} dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$.

18. Prove the following relation $\mathcal{F}\{e^{-a|x|}\} = \frac{2}{\sqrt{\pi}} \frac{a}{a^2+k^2}, \quad a > 0$.

19. Solve the following integral equation (Electric current in a simplest chain). Find the current $I(t)$ in a simplest chain for given при заданном resistance R and induction L, satisfying equation

$$L\frac{dI}{dt} + RI = E(t).$$

Hint. $\hat{I}(k) = \frac{aE_0}{iL}\sqrt{\frac{2}{\pi}} \frac{1}{\left(k-\frac{Ri}{L}\right)(k^2+a^2)}$. $I(t) = \frac{aE_0}{i\pi L}\int_{-\infty}^{+\infty} \frac{\exp(ikt)dk}{\left(k-\frac{Ri}{L}\right)(k^2+a^2)} = \frac{E_0 e^{at}}{R+aL}$.

20. Determine solution of the integral equation

$$\int_{-\infty}^{+\infty} f(x-\xi)f(\xi)d\xi = \frac{1}{x^2+a^2}.$$

21. Calculate Laplace transform of the following functions

a) t, b) e^{at}, c) sin (at).

22. Find solution to the first order differential equation $\frac{dx}{dt} + px = q,\ t > 0$, satisfying initial condition $x(0) = a$, where $a,\ p,\ q$ – are constants.

23. Find solution to partial differential equation $xu_t + u_x = x,\ x > 0,\ t > 0$, satisfying initial and boundary conditions $u(x,0) = 0$ for $x > 0$, and $u(0,t) = 0$ for $t > 0$.

24. Calculate Mellin transform of the following functions

a) e^{-nx}, b) $\frac{1}{1+x}$, c) e^{-ikx}, d) $\cos kx$.

25. Find solution to the following boundary problem

$$x^2u_{xx} + xu_x + u_{yy} = 0, \quad 0 \le x < \infty, \quad 0 < y < 1,$$

$$u(x,\ 0) = 0,\ u(x,\ 1) = \left\{\begin{matrix} A, & 0 \le x \le 1, \\ 0, & 1 < x < \infty \end{matrix}\right\}$$

where A is a constant.

26. Legendre polynomials $P_n\ (x)$ are defined by the Rodriges

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n.$$

Define $P_3\ (x), P_5\ (x)$.

27. Represent the function $\frac{1-t^2}{1-2xt+t^2}$ in form of the Taylor series in t and determine the Chebyshev polynomial $T_3(x)$.

ПРОТОКОЛ СОГЛАСОВАНИЯ УЧЕБНОЙ ПРОГРАММЫ УО

Название учебной дисциплины, с которой требуется согласование	Название кафедры	Предложения об изменениях в содержании учебной программы учреждения высшего образования по учебной дисциплине	Решение, принятое кафедрой, разработавшей учебную программу (с указанием даты и номера протокола)
Дисциплина не требует согласования			

Заведующий кафедрой теоретической
и прикладной механики, доктор
физико-математических наук, профессор М.А. Журавков
28.05.2024

ДОПОЛНЕНИЯ И ИЗМЕНЕНИЯ К УЧЕБНОЙ ПРОГРАММЕ ПО ИЗУЧАЕМОЙ УЧЕБНОЙ ДИСЦИПЛИНЕ

на _____/_____ учебный год

№ п/п	Дополнения и изменения	Основание

Учебная программа пересмотрена и одобрена на заседании кафедры теоретической и прикладной механики, протокол № 12 от 28 мая 2024 г.

Заведующий кафедрой теоретической
и прикладной механики, доктор
физико-математических наук, профессор М.А. Журавков

УТВЕРЖДАЮ

Декан механико-математического факультета,
доктор физико-математических наук, профессор С.М. Босяков