ПОДХОДЫ К ДОКАЗАТЕЛЬСТВУ ГИПОТЕЗЫ КОЛЛАЦА

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На сегодняшний день гипотеза Коллатца является одной из математических проблем, которая до сих пор остается нерешенной. Она названа в честь немецкого математика Лотара Коллатца, который сформулировал похожую проблему. Несмотря на простоту формулировки, за 90 лет никто не смог доказать правильность гипотезы. В статье рассматриваются подходы к доказательству гипотезы Коллатца и алгоритм вычисления градовых чисел.

Ключевые слова: гипотеза Коллатца; длина пути; закон Бенфорда; градовые числа.

THE IDEAS OF PROVING THE COLLATZ CONJECTURE

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Today the Collatz conjecture is one of the problems that has remained unresolved. It is named after the German mathematician Lothar Collatz, who formulated a similar problem. Despite the simplicity of the formulation, no one has been able to prove the correctness of the hypothesis for 90 years. The article covers the ideas of the Collatz conjecture and algorithm for calculating hailstone numbers.

Keywords: Collatz conjecture; path length; Benford's law; hailstone numbers.

The Collatz conjecture is, but incredibly difficult to prove. So difficult in fact that many outstanding mathematicians have tried and failed to prove. There are more than 30 tasks that have remained partially or completely unsolved for today.

The Collatz conjecture is named after the German mathematician Lothar Collatz (1910 - 1990). As a professor at the Technical University of Hanover and then at the University of Hamburg, scientist worked in numerical analysis and published many important papers in this area. However, it is the conjecture for which he is best remembered.

In this article we will analyze the Collatz conjecture.

The Collatz hypothesis states that if we take any number n, and perform the following operations: divide the number by 2 if n is even; multiply the number by 3 and increase by 1 if the number is odd; then after a finite number of transformations we get 1.

Similar calculations were performed with a huge set of integers. An employee of the University of Tokyo, Nabuo Yoneda, studied all integers up to 2^{40}, or in other words up to 1.1×10^{12}. The result in all cases was the same: after a finite number of steps, the sequence converged to 1. Of the first 50 integers, the number 27 had the longest return path to 1. Integers that would generate an infinite sequence going up indefinitely were not found. And yet, we cannot claim that all integers behave the same way, since there is no theoretical justification for this yet.

After all that has been said, there seems to be no point in looking for a number that would behave in any other way. But there are some scientific works, which tried to prove the Collatz conjecture and came up with some result [1][2]. Also, the very fact of the existence of hailstone numbers raises many curious questions.

Perhaps the most intriguing properties of such numbers are the pronounced distribution of maximum values and path lengths. If such a small number as 27 is held "in limbo" for 111 steps and reaches a maximum of 9232, then one would expect that the path length and maximum values grow as fast as N. In reality, the path lengths grow extremely slowly; the growth of the maximum values is faster, but also very disordered.

Out of the first 100,000 numbers, the longest path is only 350 steps (N=77,031).

Thus, when N is increased by a factor of 1000, the path length only triples. There seems to be a logarithmic relationship here. As for the maximum value, the record value of 9232, reached by the number (N= 27, was beaten only by the number (N=255: its maximum is 13,120. Other recorded maxima were distributed in the most disorderly way. Hailstone 77,671 broke all records, reaching 1,570,824,736.

One of the approaches to shed light on the problem of hailstone number`s is to reverse its formulation. Suppose that all positive integers really converge to 1. Then they should form an unbroken chain connecting the entire infinite countable set of integers with a cycle at the base of this chain. Therefore, it is also possible to reverse the transformation over the hailstone numbers: starting from 1, we apply the transformation, as if backing away, to obtain large numbers. If any number cannot be obtained in this way, it cannot reach one.

This method would be perfect for obtaining a general solution to the problem of hailstone number`s, if only it could be completed. It turns out that the procedure is not as straightforward as it seems. The usual hailstone transformation function is unambiguous: any value of N has only one successor at any point in the calculations. If, for example, N=40, then the next (generated by it) number can be only 20. When we go through the path in the opposite direction, ambiguity arises. Regarding the number 20, it is clear that it could have turned out to be 40, which, therefore, should now follow it. But after the number 40, you can get either 80 or 13: the stream spreads into two arms, each of which needs to be explored. Branching occurs at all points of the form 6K+4, where K is any non-negative integer, including zero [3].

In conclusion, we studied the Collatz conjecture, the hailstone numbers, sequences of maximum values, path lengths and value of the Collatz conjecture in mathematics, physics and law. Even if we won't be able to confirm the Collatz conjecture, we still can use the function at proving Benford`s law (the probability of the appearance of a certain first significant digit). It proves that 1 appears as first digit (~30%), then 9 (~5%). If we take first 50 integers and their paths to 1, then integers starting with 1 will be 357, starting with 2- 213, with 3 – 102, with 4 – 153, 5 – 105, 6 – 31, 7 – 47, 8 – 77, 9 – 31. If we take more integers, we`ll get results closer to Benford`s law.

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