has a unique solution u *in the class* $C^2(\widetilde{Q})\bigcap C(\widetilde{Q}_0)$, *where* $\widetilde{Q} = \overline{Q} \setminus \{(t,x) : x - at = = 0 \vee x - at = -at_*\}$ *and* $\widetilde{Q}_0 = \overline{Q} \setminus \{(t,x) : x - at = 0\}$. *This solution depends continuously on the functions* $\varphi, \psi, \mu_1, \mu_2$, *and* γ.

The talk is based on a recent preprint [1]. The obtained results expand the previously known ones [3, 4].

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ANALYTICAL SOLUTIONS OF SOME SECOND-ORDER QUASI-LINEAR EQUATIONS WITH TWO INDEPENDENT VARIABLES IN HOMOGENEOUS ISOTROPIC MEDIA WITHOUT PERTURBATIONS

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Consider second-order quasi-linear equations with two independent variables in homogeneous isotropic media without perturbations [1]:

$$A_{11}u_{tt} + 2A_{12}u_{tx} + A_{22}u_{xx} + A_1 u_t + A_2 u_x + \lambda f(u) = 0, \tag{1}$$

where $A_i, A_{ij}, \lambda \in \mathbb{R}$; $\lambda \neq 0$, $f(u)$ is a function by u.

This quasi-linear equation has the following classification:

1) $A_{12}^2 - A_{11}A_{22} > 0$ – hyperbolic type;
2) $A_{12}^2 - A_{11}A_{22} < 0$ – elliptical type;
3) $A_{12}^2 - A_{11}A_{22} = 0$ – parabolic type.

By some transformations, equation (1) is reduced to the following four types of equations:

$$u_{tx} + A_1 u_t + A_2 u_x + \lambda f(u) = 0, \tag{2}$$

$$u_{tt} - u_{xx} + A_1 u_t + A_2 u_x + \lambda f(u) = 0, \tag{3}$$

$$u_{tt} + u_{xx} + A_1 u_t + A_2 u_x + \lambda f(u) = 0, \tag{4}$$

$$u_t + A_{22} u_{xx} + A_2 u_x + \lambda f(u) = 0. \tag{5}$$

Equation (2) is a hyperbolic nonlinear Klein–Gordon equation in the first form. Equation (3) is a hyperbolic nonlinear Klein–Gordon equation in the second form. Equation (4) is an elliptic nonlinear Klein–Gordon equation. The equation (5) is a nonlinear convection–diffusion equation.

The reduction for equation (1) will be carried out using symmetries and substitution of variables $u = y(z(t,x))$.

We will look for infinitesimal transformations for equation (1) in the form [2–5]

$$\widetilde{t} = t + \varepsilon\tau(t,x,u) + \ldots, \quad \widetilde{x} = x + \varepsilon\xi(t,x,u) + \ldots, \quad \widetilde{u} = u + \varepsilon\eta(t,x,u) + \ldots,$$

and $\widetilde{f}(\widetilde{u}) = f(u) + \varepsilon f^{(1)}(u)\eta + \ldots$

Then the group generator for equation (1) has the form

$$X = \tau(t,x,u)\frac{\partial}{\partial t} + \xi(t,x,u)\frac{\partial}{\partial x} + \eta(t,x,u)\frac{\partial}{\partial u}.$$

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ON FRACTIONAL DERIVATIVE OF THE MITTAG-LEFFLER TYPE FUNCTIONS

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The report is devoted to the calculation of the fractional derivatives of the Mittag-Leffler function with respect to parameters.

We calculate the fractional derivative of the Mittag-Leffler function. We use both main types of fractional derivatives, namely, the Riemann–Liouville fractional derivative

$$\left({}^{RL}D^{\mu} f(\tau)\right)(t) := \frac{1}{\Gamma(n-\mu)}\frac{d^n}{dt^n}\int\limits_0^t \frac{f(\tau)\,d\tau}{(t-\tau)^{\mu-n+1}}, \quad n = [\mu]+1,$$