CLASSICAL SOLUTION OF AN INITIAL-BOUNDARY VALUE PROBLEM WITH A MIXED BOUNDARY CONDITION AND CONJUGATION CONDITIONS FOR A MILDLY QUASILINEAR WAVE EQUATION

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We consider an initial-boundary value problem for a mildly quasilinear wave equation in the first quadrant in which we pose the Cauchy conditions on the spatial half-line, a mixed boundary condition on the time half-line, and conjugation conditions on some characteristics. We prove the uniqueness of the solution and establish the conditions under which a classical solution exists.

In this report, we consider the following mixed problem

$$\begin{cases} \Box_a u(t,x) = f(t,x,u(t,x),\partial_t u(t,x),\partial_x u(t,x)), \quad (t,x)\in Q,\\ u(0,x)=\varphi(x), \quad \partial_t u(0,x)=\psi(x), \quad x\in[0,\infty),\\ u(t,0)=\mu_1(t), \quad t\in[0,t_*),\\ \partial_x u(t,0)=\mu_2(t), \quad t\in[t_*,\infty), \end{cases} \tag{1}$$

where $Q=(0,\infty)\times(0,\infty)$, $\Box_a=\partial_t^2-a^2\partial_x^2$ is the d'Alembert operator ($a>0$ for definiteness), t_* is a positive real number, f is a function given on the set $\overline{Q}\times\mathbb{R}^3$, φ and ψ are functions given on the half-line $[0,\infty)$, μ_1 is a function given on the segment $[0,t_*)$, and μ_2 is a function given on the half-line $[t_*,\infty)$.

Previously, we have announced the existence and uniqueness theorem of a classical solution of the problem (1) under some smoothness and compatibility conditions [1, 2]. But in the present work we won't assume that these compatibility conditions hold.

We present the main results of this report in the following theorem.

Theorem. *Let the conditions* $f\in C^1(\overline{Q}\times\mathbb{R})$, $\varphi\in C^2([0,\infty))$, $\psi\in C^1([0,\infty))$, $\mu_1\in C^2([0,t_*])$, $\mu_2\in C^1([t_*,\infty))$ *be fulfilled, and let the function* f *satisfy the Lipschitz condition*

$$|f(t,x,u,u_t,u_x)-f(t,x,z,z_t,z_x)|\leqslant L(t,x)(|u-z|+|u_t-z_t|+|u_x-z_x|)$$

with a continuous function $L:\overline{Q}\mapsto[0,\infty)$. *The mixed problem* (1) *with conjugation conditions*

$$[(u)^+-(u)^-](t,x=at)=\gamma(t;\varphi(0)-\mu_1(0)), \quad [(u)^+-(u)^-](t,x=at-at_*)=0,$$

where $\gamma:[0,\infty)\ni t\mapsto\gamma(t;\varphi(0)-\mu_1(0))\in\mathbb{R}$ *is a function with one parameter* $\varphi(0)-\mu_1(0)$ *that satisfies the natural comparability condition*

$$\gamma(t;0)=0, \quad t\in[0,\infty), \quad \gamma(0;s)=s, \quad s\in\mathbb{R},$$

has a unique solution u *in the class* $C^2(\widetilde{Q})\bigcap C(\widetilde{Q}_0)$, *where* $\widetilde{Q}=\overline{Q}\setminus\{(t,x): x-at = = 0\vee x-at=-at_*\}$ *and* $\widetilde{Q}_0=\overline{Q}\setminus\{(t,x): x-at=0\}$. *This solution depends continuously on the functions* φ,ψ,μ_1,μ_2, *and* γ.

The talk is based on a recent preprint [1]. The obtained results expand the previously known ones [3, 4].

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ANALYTICAL SOLUTIONS OF SOME SECOND-ORDER QUASI-LINEAR EQUATIONS WITH TWO INDEPENDENT VARIABLES IN HOMOGENEOUS ISOTROPIC MEDIA WITHOUT PERTURBATIONS

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Consider second-order quasi-linear equations with two independent variables in homogeneous isotropic media without perturbations [1]:

$$A_{11}u_{tt}+2A_{12}u_{tx}+A_{22}u_{xx}+A_1u_t+A_2u_x+\lambda f(u)=0, \tag{1}$$

where $A_i, A_{ij}, \lambda\in\mathbb{R}$; $\lambda\neq 0$, $f(u)$ is a function by u.

This quasi-linear equation has the following classification:

1) $A_{12}^2-A_{11}A_{22}>0$ – hyperbolic type;
2) $A_{12}^2-A_{11}A_{22}<0$ – elliptical type;
3) $A_{12}^2-A_{11}A_{22}=0$ – parabolic type.