NONPARAMETRIC STATISTICAL INFERENCE FOR COMPOUND MODELS

D.V. Belomestny[1], E.A. Morozova[2], V.A. Panov[3]

[1] *University of Duisburg-Essen, Thea-Leymann-Str. 9,*
45127 Essen, Germany
[2], [3] *HSE University, Pokrovsky boulevard 11,*
109028 Moscow, Russia

Compound models as sums of random variables with random number of summands are studied in terms of statistical inference. The probability distribution of the random number of summands is estimated. The nonparametric estimator for the probability distribution of N based on the superposition of the Mellin and Laplace transforms is proposed and analyzed. Simulation results illustrate the theory.

Keywords: decompounding, compound models, nonparametric inference, Mellin transform.

НЕПАРАМЕТРИЧЕСКИЕ СТАТИСТИЧЕСКИЕ ВЫВОДЫ ДЛЯ СОСТАВНЫХ МОДЕЛЕЙ

Д.В. Беломестый[1], Е.А. Морозова[2], В.А. Панов[3]

[1]*Университет Дуйсбург-Эссен, ул. Теа-Лейманн 9,*
45127 Эссен, Германия
[2], [3]*Национальный исследовательский университет «Высшая школа экономики»,*
Покровский бульвар 11, 109028 Москва, Россия

Изучаются составные модели в виде сумм случайных величин со случайным числом слагаемых в контексте статистических выводов. Оценивается распределение вероятностей случайного числа слагаемых. Предложена и исследована непараметрическая оценка распределения вероятностей N, основанная на суперпозиции преобразований Меллина и Лапласа. Результаты имитационного моделирования иллюстрируют теорию.

Ключевые слова: разбор составления, составные модели, непараметрические выводы, преобразование Меллина.

The current research is devoted to the statistical inference for the compound models of the form

$$X = \sum_{k=1}^{N} \xi_k, \tag{1}$$

where $\xi_1, \xi_2, \ldots$ is a sequence of i.i.d. random variables, and N is a non-negative integer-valued random variable independent of $\xi_1, \xi_2, \ldots$. The particular case of the model (1) with N being distributed according to the Poisson law is known as the compound Poisson model and finds numerous applications in such areas as the queuing theory and actuarial science; see, e.g., Embrechts et al. [7]. For that case, there is a wide range of estimation techniques available; to name a few, see [4], [5], [6], [8]. However, very little research has been devoted to the case when N does not necessarily have a Poisson distribution. At the same time, it appears to be of great interest since this would allow for much more flexibility. For example, as was pointed out in [1] and [9], the use of the Poisson law in insurance is not sufficiently justified, and some of the few papers trying to analyse the goodness-of-fit of that model found it inappropriate for the description of the data. Hence, it can be expected that the model (1) will provide a more accurate fit in many practical situations.

In the present paper, we aim at estimating the distribution of N based on a sample $X_1, X_2, \ldots, X_n$ from the model (1). The distribution of ξ_1 is assumed to be known and infinitely divisible. In addition, due to the identifiability issues, it is assumed that the distribution of N is strictly positive. We propose the nonparametric estimator for the distribution of N based on the superposition of the Mellin and Laplace transforms. Such an approach has been introduced by Belomestny and Schoenmakers [3] for the statistical inference for time-changed Lévy processes, and further successfully employed by Belomestny and Panov [2] for the semiparametric inference for normal variance-mean mixture model, but to the best of our knowledge, has never been applied to the considered problem. We establish the rates of convergence for the proposed estimator and demonstrate its performance on a simulated study.

References

1. *Asmussen S. and Albrecher H.* Ruin probabilities // World scientific, 2010. Vol. 14.
2. *Belomestny D. and Panov V.* Semiparametric estimation in the normal variance-mean mixture model // Statistics, 2018. Vol. 52, No. 3. P. 571–589.
3. *Belomestny D. and Schoenmakers J.* Statistical inference for time-changed Lévy processes via Mellin transform approach // Stochastic Processes and their Applications. 2016. Vol. 126, No. 7. P. 2092–2122.
4. *Buchmann B. and Grübel R.* Decompounding: an estimation problem for Poisson random sums // The Annals of Statistics. 2003. Vol. 31, No. 4. P. 1054–1074.
5. *Coca A. J.* Efficient nonparametric inference for discretely observed compound Poisson processes // Probability Theory and Related Fields. 2018. Vol. 170, No. 1. P. 475–523.
6. *Comte F., Duval C. and Genon-Catalot V.* Nonparametric density estimation in compound Poisson processes using convolution power estimators // Metrika. 2014. Vol. 77, No. 1. P. 163–183.
7. *Embrechts P., Klüppelberg C., and Mikosch T.* Modelling extremal events for insurance and finance. Springer, 1997.
8. *Gugushvili S., Mariucci E. and van der Meulen F.* Decompounding discrete distributions: A nonparametric Bayesian approach // Scandinavian journal of statistics. 2020. Vol. 47, No. 2. P. 464–492.
9. *Seal H. L.* The Poisson process: Its failure in risk theory // Insurance: Mathematics and Economics. 1983. Vol. 2, No. 4. P. 287–288.