UDC 519.2

OPERATOR METHOD IN BOUNDARY CROSSING PROBLEMS FOR RANDOM WALKS

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Problems related to crossing linear boundaries by the trajectory of random walk are considered.An analytic approach to solve those problems is developed. For various boundary functionals (first exit time, overshoot, the crossing number of a strip, sojourn time, some others) explicit expressions and asymptotic expansions are derived.

Keywords:random walk; trajectory; boundary crossing; first exit time; overshoot; crossing number of a strip; sojourn time; Laplace transform; Wiener-Hopf factorization.

ОПЕРАТОРНЫЙ МЕТОД В ЗАДАЧАХ ПЕРЕСЕЧЕНИЯ ГРАНИЦ ДЛЯ СЛУЧАЙНЫХ БЛУЖДАНИЙ

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Рассматриваются задачи, связанные с пересечением линейных границ траекторией случайного блуждания. Разработан аналитический подход для решения этих задач. Для различных функционалов, связанных с границами (время первого выхода, перескок, число пересечений полосы, время пребывания, некоторые другие) получены точные выражения и асимтотические разложения.

Ключевые слова: случайное блуждание; траектория; пересечение границы; время первого выхода; перескок; число пересечений полосы; время пребывания; преобразование Лапласа; факторизация Винера-Хопфа.

Since publication of the fundamental book of A. Wald [1] an intensive development of the sequential statistical analysis started, faced with of probability theory problems related to boundary crossing random events.

The results presented here generalize recent results of the author [2] – [7] related to the problem.

We demonstrate an analytical approach to a number of problems related to crossing linear boundaries by the trajectory of a random walk. Main results consist in finding explicit expressions and asymptotic expansions for distributions of various boundary functionals such as first exit time and overshoot, the crossing number of a strip, sojourn time, etc.

The method includes several steps. We start with the identities containing Laplace transforms of joint distributions under study. The use of the Wiener-Hopf factorization is the main instrument to solve these identities. Thus we obtain explicit expressions for the Laplace transforms in terms of factorization components. It turns out that in many cases Laplace transforms are expressed through the special factorization operators which are of particular interest. We further discuss possibilities of exact expressions for these operators, analyze their analytic structure, continuity theorem and obtain asymptotic representations for them under the assumption that the boundaries tend to infinity. After that we invert Laplace transforms asymptotically to get limit theorems and asymptotic expansions for the distributions under study.

Some asymptotic results for the distributions of boundary functionals will be presented.

References

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