БЕЛОРУССКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

И. В. Сташкевич
Р. И. Новицкая

Solid-State Laser Dynamics

Динамика твердотельных лазеров

*Рекомендовано
Учебно-методическим объединением
по естественно-научному образованию
в качестве пособия
для студентов учреждений высшего образования,
обучающихся по специальностям
«физика», «прикладная физика»,
«физико-математическое образование»,
«фундаментальная физика»*

МИНСК
БГУ
2024

УДК 621.373.826.038.825(076.5)
ББК 32.86-5я73
С78

Рецензенты:
кафедра лазерной техники и технологии
Белорусского национального технического университета
(заведующий кафедрой член-корреспондент НАН Беларуси,
доктор физико-математических наук, профессор *А. М. Маляревич*);
кандидат физико-математических наук, доцент *К. А. Саечников*;
старший преподаватеь английского языка гуманитарных
факультетов факультета социокультурных
коммуникаций БГУ *Г. А. Пусенкова*

Сташкевич, И. В.

С78 Динамика твердотельных лазеров = Solid-State Laser Dynamics : пособие / И. В. Сташкевич, Р. И. Новицкая. – Минск : БГУ, 2024. – 55 с.

ISBN 978-985-881-684-1.

Изложены свойства и параметры твердотельных лазеров, а также основные принципы формирования энергетических уровней редкоземельных ионов на примере неодима в различных кристаллических полях и лазерные переходы между ними. Приведены задачи, решение которых поможет студентам закрепить знания об особенностях генерации неодимовых и иттербиевых лазеров, а также о режимах модуляции добротности и синхронизации мод. В приложениях представлены листинги программ «Wolfram Mathematica», показаны примеры расчетов, связанных с Nd:YAG и Yb:YAG лазерами.

Предназначено для студентов учреждений высшего образования, обучающихся по специальностям «физика», «прикладная физика», «физико-математическое образование», «фундаментальная физика».

УДК 621.373.826.038.825(076.5)
ББК 32.86-5я73

ISBN 978-985-881-684-1

© Сташкевич И. В.,
Новицкая Р. И., 2024
© БГУ, 2024

CONTENTS

FOREWORD

In recent years, the field of laser technology has experienced unprecedented growth and development, becoming an integral part of various scientific and industrial applications. As we continue to unlock the potential of solid-state lasers, the need for a comprehensive understanding of their dynamics and parameters becomes increasingly critical. This book, "Solid State Laser Dynamics," has been crafted with this urgency in mind, serving as an essential resource for students and master's candidates eager to navigate the complexities of laser generation and performance analysis.

The structure of this book facilitates a progressive learning experience, guiding readers from fundamental principles to advanced concepts in laser dynamics. Each chapter is designed to build upon the previous one, ensuring a coherent and integrated approach to the subject matter. Topics encompass various laser modes of operation, including free-running mode, Q-switching – both active and passive – as well as mode locking, with detailed discussions on the mechanisms that govern each mode.

Moreover, the book delves into the methodologies for calculating laser parameters, providing students with the analytical tools necessary to evaluate and predict laser behavior. Emphasis on real-world applications and problem-solving exercises enriches the learning experience and equips students with practical skills that are highly sought after in both academic and industrial settings. We focus on the neodymium and ytterbium lasers, exploring their performance across different crystal host materials. By analyzing these configurations, we aim to highlight their unique properties and advantages, offering readers valuable insights into their operational dynamics and potential applications.

We hope this book inspires a new generation of laser scientists and engineers, fostering a deeper understanding of solid-state laser dynamics. May this book serve not only as a textbook but also as a source of inspiration in your academic journey and professional endeavors.

Happy reading and learning!

THEORETICAL MATERIAL

Some properties of rare-earth ions in a crystalline matrix

The splitting of energy levels of rare-earth ions into separate electronic states is determined by three types of interaction: Coulomb interaction – the interaction of nuclei with electrons and electrons with each other, spin-orbital interaction, and interaction with an intracrystalline field. The Coulomb interaction leads to the appearance of $(2S + 1)$-multiple degenerate levels with different values of the total orbital momentum L in the form of ^{2S+1}L $(L > S)$ with energy gaps between them of the order of $\sim 10^4$ cm^{-1}. The spin-orbit interaction leads to the splitting of each of the S-L multiplets into a number of levels with different values of the total angular momentum J. This splitting for rare-earth ions is an order of magnitude smaller than the Coulomb one. Interaction with the crystalline field leads to Stark splitting of energy levels with different J. This splitting is of the order of $\sim 10^2$ cm^{-1}, i. e., significantly less than the energy intervals between levels with different J. Thus, for all matrices, the structure of energy levels for the same rare-earth ion turns out to be similar, and differs only in the value of the Stark splitting. Next, we consider in more detail the formation of the structure of the energy levels of the ion in the crystal matrix using the example of neodymium.

Energy levels of neodymium ions in a crystal field

The electronic structure of the neodymium atom *Nd* consists of 60 electrons, which are located on 13 shells. Since the neodymium ion is found only in the oxidation state of +3, the final electronic configuration of the ion is [Xe] $4f^3$.

For this electronic configuration of the ion, there are several energy states (terms) (fig. 1). Taking into account the Coulomb repulsion between electrons and the exchange interaction, which is a consequence of the Pauli principle, the ground state with the lowest energy is characterized by the maximum value of total orbital momentum and the maximum value of the total spin. In this case, this is the term ^4I. The magnitude of the splitting is $\sim 10^4$ cm^{-1} [1].

The next perturbation that affects the splitting of energy levels is the spin-orbit interaction. For rare-earth ions, the states are best described by the Russell-Saunders states (the approximation of *LS-coupling*).

In such a model, the electron repulsion is much larger than the spin-orbit interaction, as a result of which the energy levels are split into $^{2S+1}L_J$ energy sublevels, and the total angular momentum J, according to the rule of addition of moments, is determined by the orbital L and spin S quantum numbers. Since the shell is filled with three electrons (less than half), according to the Hund rule, the underlying and basic state of the neodymium ion is the state with a minimum value of J. In this case, this is the term $^4I_{9/2}$. The amount of splitting is ~10^3 cm^{-1}.

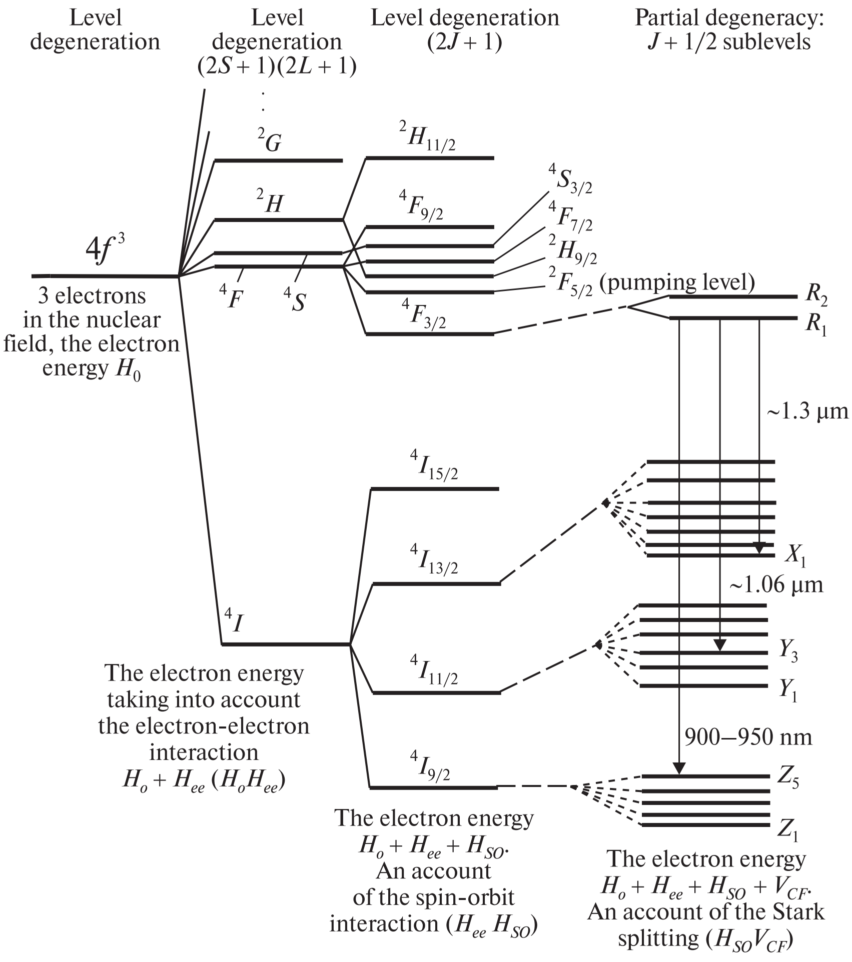


Fig. 1. Formation of energy levels of the neodymium ion in a crystalline matrix

When an ion is placed in a crystalline matrix, the spherical symmetry of the state functions, that are the eigenfunctions of the Hamiltonian of a free ion, is broken due to the surrounding ligands, which cause electrostatic interaction between the 4*f*-electrons of the neodymium ion and its static electric field. Individual terms are split into sublevels due to the Stark effect, and the magnitude of the splitting depends on the total quantum number J, the magnitude of the crystal field strength and its symmetry. However, since the radius of the 4*f* orbitals of the neodymium ion in space is smaller than that of the filled 5*s* and 5*p* orbitals, the latter shields 4*f* electrons from the external field, therefore, the splitting is very small and amounts to ~10^2 cm^{-1}.

Electronic transitions in the neodymium ion

The tuning range of the radiation of neodymium solid-state lasers is quite wide, starting from the generation at the high-energy transition $^2L_{15/2} \rightarrow {}^4I_{9/2}$, which is equivalent to a wavelength of electromagnetic radiation of approximately 340 nm, ending with the long-wavelength laser transition $^4F_{3/2} \rightarrow {}^4I_{15/2}$ with a wavelength of the order of 1.9 μm. However, the main transitions of the neodymium ion are $^4F_{3/2} \rightarrow {}^4I_{13/2}$, $^4F_{3/2} \rightarrow {}^4I_{11/2}$ and $^4F_{3/2} \rightarrow {}^4I_{9/2}$ (fig. 1).

It should be noted that the transitions of the neodymium ion are not limited to the ones indicated in fig. 1; in fact, other Stark sublevels can also participate. However, the greatest relative intensity is observed precisely for the indicated levels, in which case it is easier to achieve generation.

It can be seen from Fig. 1 that the main transitions of the neodymium ion ($^4F_{3/2} \rightarrow {}^4I_{13/2}$, $^4F_{3/2} \rightarrow {}^4I_{11/2}$, $^4F_{3/2} \rightarrow {}^4I_{9/2}$) are internal $4f^3 \leftrightarrow 4f^3$ transitions (*ff* transitions) occurring within the partially filled $4f^3$ configuration of the neodymium ion. Moreover, all the main terms of this configuration are odd. Therefore, these transitions are forbidden by the Laporte selection rule, according to which the parities of the initial and final states during the transitions must be opposite. This prohibition, which is strictly observed in the spectra of free ions, is partially lifted due to non centrosymmetric interactions of neodymium ions with a crystalline matrix, causing mixing of states of opposite parity. For such interactions in the matrix, both static (even terms in the expansion of the crystal field potential in spherical harmonics) and dynamic (lattice vibrations, which cause inverse symmetry breaking) perturbations of the crystal field potential are responsible, and a transition is called a stimulated dipole transition. The final selection rules for this type of transitions compared to the selection rule for Laporte-enabled transitions are shown in table 1.

Table 1

Rules for selecting *ff* transitions

Transition type	Selection rules	Oscillator strength
Electric dipole (ED) $4f^3$-$4f^25d$ junction	$\Delta S = 0$; $\lvert\Delta L\rvert \le 1$; $\lvert\Delta J\rvert \le 1$; Transitions $J = 0 \leftrightarrow J' = 0$ and $L = 0 \leftrightarrow L' = 0$ are forbidden	~ 0.01–1
Forced electric dipole $4f^3$-$4f^3$ junction	$\Delta S = 0$; $\lvert\Delta L\rvert \le 6$; $\lvert\Delta J\rvert \le 6$; $\lvert\Delta L\rvert = 2, 4, 6$ if $L = 0$ or $L' = 0$; $\lvert\Delta J\rvert = 2, 4, 6$ if $J = 0$ or $J' = 0$ Transitions $J = 0 \leftrightarrow J' = 0$ and $L = 0 \leftrightarrow L' = 0$ are forbidden	~ 10^{-4} of ED
Magnetic dipole $4f^3$-$4f^3$ junction	$\Delta S = 0$; $\Delta L = 0$; $\lvert\Delta J\rvert \le 1$; Transition $J = 0 \leftrightarrow J' = 0$ forbidden	~ 10^{-6} of ED
Electric quadrupole $4f^3$-$4f^3$ junction	$\Delta S = 0$; $\lvert\Delta L\rvert \le 2$; $\lvert\Delta J\rvert \le 2$; The transitions $J = 0 \leftrightarrow J' = 0,1$ and $L = 0 \leftrightarrow L' = 0,1$ are forbidden	~ 10^{-10} of ED

Since in the case of a stimulated dipole transition the oscillator strengths are small compared to the electric dipole transition (ED), this transition will have a lower intensity. A magnetic dipole transition is also possible, but its intensity is even lower than that of a stimulated dipole. It can be seen that the laser transitions classical for the neodymium ion $^4F_{3/2} \rightarrow {}^4I_{13/2}$, $^4F_{3/2} \rightarrow {}^4I_{11/2}$ and $^4F_{3/2} \rightarrow {}^4I_{9/2}$ are thus stimulated electrical dipole transitions.

Non-radiative relaxation

At low levels of doping with neodymium ions, the latter practically do not interact with each other, i. e. in each ion all processes occur independently of processes in the other ions. There are three such processes in total: light absorption, light emission and nonradiative transitions in which the ion excitation energy is transformed into the phonon spectrum excitation energy of the crystalline matrix.

After the transition of an ion to an excited state, a further change in the absorbed energy depends on the relationship between the probabilities of radiative and nonradiative transitions. In the case of rare-earth ions, after absorption of the pump radiation, the electron usually falls on the energy level, for which the non-radiative transition to a lower-lying level is most likely. Such nonradiative transitions occur until the neodymium ion is in the metastable state, for which the probability of the radiative transition is greater than the probability of the nonradiative transition. Since the neodymium ion transitions considered mainly are radiative transitions in the optical range, and the energy gap is several times greater than the phonon energy, the simultaneous participation of several phonons is necessary for the nonradiative process to occur.

Theoretical calculations and experimental studies give the following expression for the probability of nonradiative transitions:

$$W(\Delta E) = \frac{1}{\tau_{NR}} = Be^{(-\alpha\Delta E)}, \tag{1}$$

where B and α are parameters that depend only on the matrix; τ_{NR} is the nonradiative lifetime; ΔE is the energy gap between the considered level and the underlying level.

It can be seen that the probability of nonradiative transition W between energetically close levels strongly depends on the energy gap between them ΔE. The reason is that the probability W depends on the number of phonons of the lattice that participate in the nonradiative process. In a solid, the phonon energy is limited by the limiting value $h\Omega_{\text{max}}$. If the energy gap ΔE *is* greater than $h\Omega_{\text{max}}$, then the nonradiative transition requires the simultaneous participation of several phonons in the crystal matrix. The probability of such a process is much less than the probability of a process in which one phonon is involved. Therefore, the larger the number of phonons necessary for the implementation of the nonradiative process, the less likely such a process is going on.

As an example, the time of nonradiative relaxation in the case of Nd:YAG can be calculated. The parameter B is approximately 10^8 s^{-1}, α is $3.1 \cdot 10^{-3}$ cm, therefore, the nonradiative lifetime for the level $^4F_{3/2}$ in Nd:YAG is of the order of 50 ms with an energy gap of 5000 cm^{-1} between $^4F_{3/2}$ and next level $^4I_{15/2}$. For *YAG* the maximum phonon energy is ~700 cm^{-1}.

However, this law is not applicable for values ΔE two times smaller than the maximum phonon energy. In such cases, the nonradiative lifetime decreases faster (with ΔE decreasing), than the law (1). It was calculated that the relaxation time for levels within the term is in the picosecond range. This time is much shorter than any time characteristic of solid-state lasers. This fact is of great importance in modeling the operation of lasers, since it allows us to consider problems in the framework of thermodynamic equilibrium. The latter establishes that the relative populations at all levels of neodymium ions obey the Boltzmann distribution at any given time and are thus constants. Then the relative population of the sublevel Z_i of some multiplet l ($^4I_{9/2}$ or $^4F_{3/2}$, see fig. 1), considered later in the work, is equal to:

$$N_{li} = \frac{N_l}{Z_l}\exp\left(-\frac{E_{li}}{kT}\right) = f_{li}N_l, \tag{2}$$

where N_{li} is the population of the sublevel Z_i; f_{li} is its relative population; E_{li} is its energy; N_l is the total population of the multiplet; $k = 1.3806485 \cdot 10^{-23}$ J/K is the Boltzmann constant; T is the temperature; Z_l is the statistical sum (summation is performed over all sublevels of multiplet):

$$Z_l = \sum_{i=1}^{m}\exp\left(\frac{-E_{li}}{kT}\right).$$

Similarly, other sublevels of the multiplet can be analyzed.

It should be noted that for the upper laser level, this distribution is applicable in the case of stationary pumping, when the ion relaxation rate over the levels of the multiplet is considered to be higher than the pump speed.

Transition schemes of activator ions in crystal hosts

A quantitative difference between the type of a given transition and three- and four-level transitions is usually based on the analysis of the gain in the absence of saturation:

$$g_0 = \sigma[\gamma N_2 - (\gamma - 1)N_s], \tag{3}$$

where σ is the stimulated emission cross section; N_2 is the population of the upper laser level; N_s is the volume concentration of ion doping. The parameter $\gamma = 1 + f_1 / f_2$, where f_1 and f_2 are the relative populations of the lower and upper laser levels, respectively. If the parameter γ is 1, then the transition scheme is four-level. If γ is equal to 2, then it is three-level. If the value of γ lies in the range from 1 to 2, then this is a quasi-three-level scheme.

Four-level generation scheme

In the case of this scheme, the generation can occur between the lower sublevel $^4F_{3/2}$ (population N_2) and one of the sublevels of the multiplets $^4I_{11/2}$, $^4I_{13/2}$, $^4I_{15/2}$. We consider the case of the transition $^4F_{3/2} \rightarrow {}^4I_{11/2}$ (fig. 2).

According to formula (1), the time of nonradiative relaxation of neodymium ions from $^4I_{11/2}$ to $^4I_{9/2}$ is approximately several tens of nanoseconds, while the lifetime of a metastable state is measured by several tens of microseconds (see, for example, table A1). In a first approximation, the sublevel of the multiplet $^4I_{11/2}$ (to which the neodymium ion passes radiating a photon), can be considered uninhabited. However, in order to increase the accuracy of calculations, we will further take into account the population of the lower laser sublevel.

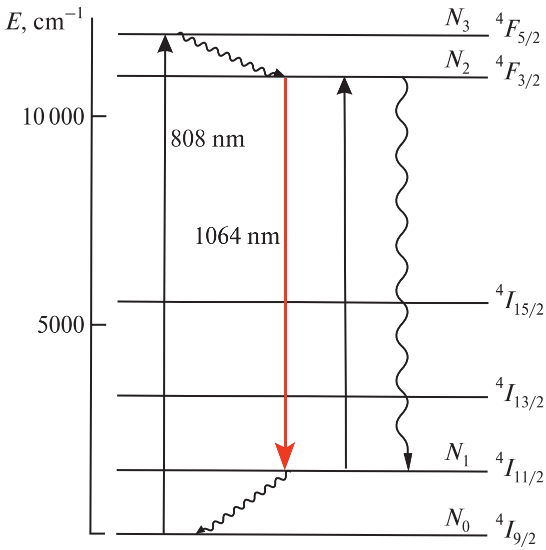


Fig. 2. Neodymium ion transitions in four-level lasing scheme. Wavy lines indicate spontaneous transitions

The lifetime of the pumped level $^4F_{5/2}$ is of the order of 10 ns; transitions to the metastable upper laser level $^4F_{3/2}$ are nonradiative.

To study the generation of solid-state lasers in a first approximation, we can use the point model of the laser based on the rate equations [2; 3]. The laser in this case is single-mode.

The approximation of the point model of the active medium is applicable in the case when the photon lifetime in the cavity is sufficiently large and the intensity in the cavity slowly changes with time (the relative change per cavity passage is much less than one). Assuming that the intensity in the cavity does not depend on spatial coordinates, we can write down the change in intensity during the round-trip of the cavity as

$$\Delta I\left(\Delta t = \frac{2L_{\text{opt}}}{c}\right) = I(t)\left[\rho_1\rho_2 e^{(k_{\text{gain}}2l - k_{\text{loss}}^{\text{inactive}}2L_{\text{opt}})} - 1\right] =$$

$$= I(t)\left[e^{(k_{\text{gain}}2l - k_{\text{loss}}^{\text{total}}2L_{\text{opt}})} - 1\right] \approx I(t)(k_{\text{gain}}2l - k_{\text{loss}}^{\text{total}}2L_{\text{opt}}),$$

where L_{opt} is the cavity optical length, l is the length of the active element; ρ_1, ρ_2 – the reflection coefficients of the mirrors. The total loss coefficient is equal to the sum of the active (output radiation through the cavity mirrors) and passive (absorption / scattering of radiation by optical elements in the cavity) losses:

$$k_{\text{loss}}^{\text{total}} = k_{\text{loss}}^{\text{inactive}} + \frac{1}{2L_{\text{opt}}} \ln\left(\frac{1}{\rho_1\rho_2}\right).$$

Averaging the change in intensity during the cavity round-trip, we obtain

$$\frac{dI}{dt} \approx \frac{\Delta I}{\Delta t} = \frac{I(t)(k_{\text{gain}} 2l - k_{\text{loss}}^{\text{total}} 2L_{\text{opt}})}{2L_{\text{opt}}/c} = I(t)(k_{\text{gain}} l - k_{\text{loss}}^{\text{total}} L_{\text{opt}}) \frac{c}{L_{\text{opt}}}.$$

The gain coefficient is equal to the number of emitted photons per unit length of the active medium:

$$k_{\text{gain}} = \sigma_{\text{em}}(\lambda, T) N_2 - \sigma_{\text{abs}}(\lambda, T) N_1,$$

where N_2 and N_1 are the population densities at the upper and lower laser levels, respectively (without taking into account degeneracy); $\sigma_{\text{em}}(\lambda, T)$ and $\sigma_{\text{abs}}(\lambda, T)$ are the effective emission and absorption cross sections at a certain wavelength. If the levels are nondegenerate, then the absorption and emission cross sections are equal. In the case when the levels are degenerate, the effective cross sections of stimulated emission and absorption at a certain wavelength and temperature can be written as

$$\sigma_{\text{abs}}(\lambda, T) = \sum_{i,j} f_{1i} \sigma_{1i,2j}(\lambda), \quad (4a)$$

$$\sigma_{\text{em}}(\lambda, T) = \sum_{i,j} f_{2j} \sigma_{2j,1i}(\lambda), \quad (4b)$$

$$\sigma_{1i,2j}(\lambda) = \sigma_{2j,1i}(\lambda),$$

where f_{1i} and f_{2j} are the relative populations of i and j sublevels of multiplets N_1 and N_2 at temperature T, $\sigma_{1i,2j}(\lambda) = \sigma_{2j,1i}(\lambda)$ are the cross sections of the transition between sublevels i and j of multiplets N_1 and N_2 at a certain wavelength.

Thus, the effective cross sections relate to the whole multiplet and are the sum of the transition cross sections between all pairs of sublevels, taking into account their relative populations. These cross sections can be quite simply determined from the experimental data (absorption and emission spectra), but it must be taken into account that they depend on temperature, because the energies of the sublevels and their relative populations vary with temperature. The cross sections for the transition between specific sublevels $\sigma_{1i,2j}(\lambda)$ cannot always be determined from the absorption and emission spectra, because the lines corresponding to these transitions are broadened and most often overlap. It is also worth noting that relations (4*a*, 4*b*) are valid in the case when the relative populations at all levels obey the Boltzmann distribution at any moment of time.

Thus, we can write the system of rate equations for the four-level lasing scheme. Instead of intensity we will use the photon flux density $S = I / h\nu$, because it does not depend on the wavelength, in contrast to the intensity, and we add another term to the first equation that will give the seed radiation of the generation, i. e. the fraction of spontaneous photons that go into the resonator mode. In the approximation of equal emission and absorption cross sections in the generation channel, the system of equations takes the following form:

$$\begin{cases} \dfrac{dS}{dt} = \mu \nu_c \sigma_{21}^{\mathrm{em}} S(N_2 - N_1) + K_S l \dfrac{N_2}{\tau_2} - S r_{tc}, \\ \dfrac{dN_3}{dt} = R_{\mathrm{pump}} \sigma_{03}^{\mathrm{abs}} (N_s - N_1 - N_2 - N_3) - \dfrac{N_3}{\tau_3}, \\ \dfrac{dN_2}{dt} = \sigma_{21}^{\mathrm{em}} S(N_2 - N_1) - \dfrac{N_2}{\tau_2} + \dfrac{N_3}{\tau_3}, \\ \dfrac{dN_1}{dt} = \sigma_{21}^{\mathrm{em}} S(N_2 - N_1) - \dfrac{N_1}{\tau_1} + \dfrac{N_2}{\tau_2}, \end{cases} \tag{5}$$

where S is the flux density of the generated radiation inside the resonator (photons / cm^2/μs); N_3 – the population of $^4F_{5/2}$ multiplet (cm^{-3}); N_2 is the population of the upper laser level $^4F_{3/2}$; N_1 is the population of the lower laser level $^4I_{11/2}$; τ_i – the lifetime of the i-th level (μs); R_{pump} – the rate of the upper laser level coherent pumping (photons/(cm^2· μs)); σ_{03} – the effective absorption cross section (cm^2); σ_{21} – the effective emission cross section (cm^2); N_s is the bulk density of neodymium ions in the crystal (cm^{-3}); v_c is the speed of light in the medium (cm/μs), K_s is the fraction of spontaneous emission in the generation channel, l is the length of the active element, μ is the parameter characterizing the value of the cavity filling with the active medium, r_{tc} is the reciprocal of the photon lifetime in the resonator (μs^{-1}), in our case, this value is proportional to the loss coefficient.

In this case, the parameter μ and the value of r_{tc} are defined as follows:

$$\mu = \frac{n_r l}{L_{\mathrm{opt}}}, \tag{6}$$

$$L_{\mathrm{opt}} = L_c + (n_r - 1)l, \tag{7}$$

where n_r is the refractive index of laser crystal; L_{opt} is the cavity optical length; l is the length of the active element; L_c is the cavity length (fig. 3);

$$r_{tc} = c\frac{\left(-\frac{\ln(\rho)}{2} + \gamma l\right)}{L_{\text{opt}}}, \tag{8}$$

where c is the speed of light; ρ is the reflection coefficient of the output mirro; γ is the coefficient of inactive radiation loss in the active medium (cm^{-1}).

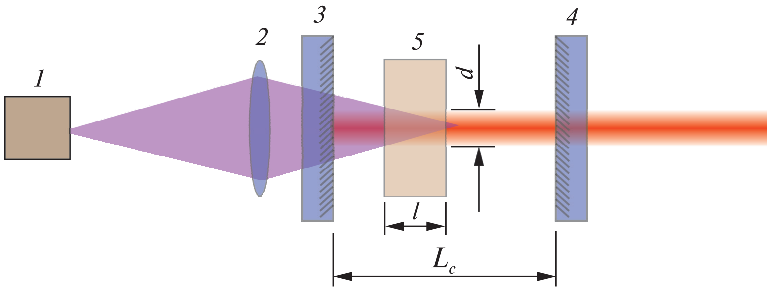


Fig. 3. The scheme of longitudinally pumped laser:
1 – diode pump laser; *2* – lens; *3*, *4* – cavity mirrors; *5* – laser crystal

The relationship between the pump rate and the incident pump power is determined as follows:

$$R_{\text{pump}} = \frac{P_{\text{inc}}\left(1 - \exp[-l\sigma_{13}N_s]\right)}{h\nu_p S_{\text{cr}}}, \tag{9}$$

where P_{inc} is the incident pump power on the crystal surface (longitudinal pump); $h\nu_p$ – pump photon energy; $S_{\text{cr}} = \pi d^2/4$ – pumped cross-section of the crystal with diameter d. For all matrices under consideration, the parenthesized expression is close to one, therefore, it can be omitted in the calculations.

Quasi-three-level generation scheme

In the case of a quasi-three-level scheme (fig. 4), the generation occurs between the lower sublevel of $^4F_{3/2}$ (total population of the multiplet N_2) and the upper sublevel of the split ground state $^4I_{9/2}$ (total population N_0).

The main difference from the four-level scheme is that the population of the lower laser level is not equal to zero due to the temperature distribution of neodymium ions over Stark sublevels. The fact is that the magnitude of the Stark splitting is several hundred cm^{-1}, which is comparable to the value of kT at room temperature. Therefore, according to the formula (2), all Stark sublevels are populated, but significantly to a different extent. At room temperature, the population of the upper sublevel is of the order of 1 %, which makes it easy to obtain an inverse population and, accordingly, lasing.

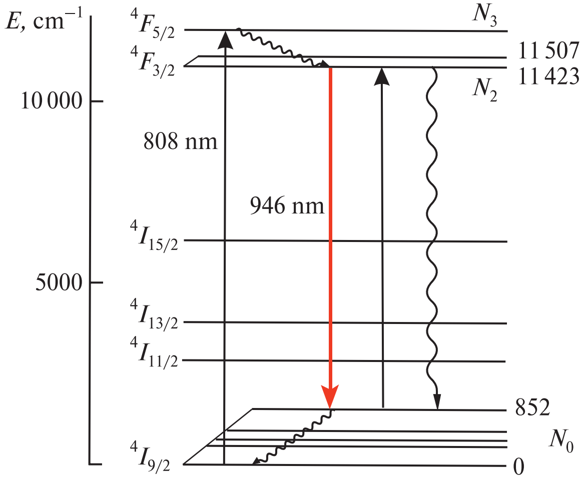


Fig. 4. Transitions of a neodymium ion in a quasi-three-level generation scheme

The system of the rate equations for the quasi-three scheme takes the following form:

$$\begin{cases} \dfrac{dS}{dt} = \mu v_c \sigma_{20} S(f_{21}N_2 - f_{05}N_0) + K_s l \dfrac{N_2}{\tau_2} - Sr_{tc}, \\ \dfrac{dN_3}{dt} = R_{\text{pump}} \sigma_{03}^{\text{abs}} N_0 - \dfrac{N_3}{\tau_3}, \\ \dfrac{dN_2}{dt} = -\sigma_{20} S(f_{21}N_2 - f_{05}N_0) - \dfrac{N_2}{\tau_2} + \dfrac{N_3}{\tau_3}, \\ N_s = N_0 + N_2 + N_3, \end{cases} \tag{10}$$

where S is generated flux density inside the resonator (photons / cm^2/μs); N_0 – population of the lower level $^4I_{9/2}$ (cm^{-3}); τ_i is the lifetime of the i-th level (μs); R_{pump} is the coherent pumping rate of the upper laser level (photon / (cm$^2 \cdot$ μs)); σ_{03}^{abs} is the effective absorption cross section (cm^2); σ_{20} is the cross section of the stimulated transition between the 1st sublevel of the multiplet N_2 and the

5th sublevel of the multiplet N_0 (cm^2); N_s is the bulk density of neodymium ions in the crystal (cm^{-3}); f_{05} is the relative population of the upper sublevel of the lower laser level $^4I_{9/2}$; f_{21} is the relative population of the lower sublevel of the upper laser level $^4F_{3/2}$; v_c is the speed of light in the medium (cm/μs); μ is the parameter characterizing the value of the cavity filling with the active medium, r_{tc} is the reciprocal of the photon lifetime in the cavity (μs^{-1}).

A specific example of modeling of a quasi-three-level solid-state laser operation is demonstrated for the Nd:YAG laser with specific parameters given in the Mathematica software package. The choice of a method for solving the ODE system numerically is also explained in the program itself.

Ytterbium laser

The dopant element in laser crystals and glasses with ytterbium is the trivalent ion Yb^{3+}. There is only one excited level $^2F_{5/2}$, which splits into three sublevels under the action of the electric field of the crystal lattice. The ground state $^2F_{7/2}$ also splits into four sublevels (fig. 5). Compared to the neodymium laser, the ytterbium laser has a much wider gain band, which makes it possible to generate ultrashort pulses. The lifetime at the upper laser level exceeds 1 ms, which makes it attractive for working in the Q-switching mode. Its energy levels scheme is actually two-level, therefore, concentration quenching is absent here and the concentration of ytterbium ions can be very high.

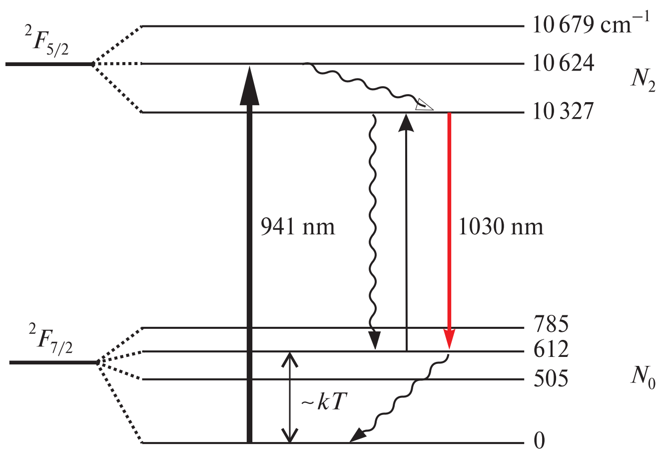


Fig. 5. Transitions of ytterbium ion during generation in a Yb:YAG crystal

From the aspect of calculating the lasing characteristics, the ytterbium laser is described by a quasi-three-level lasing scheme. In this case, the sublevel to which the transition occurs upon excitation and the sublevel from which the generation transition occurs belong to the same level $^2F_{5/2}$, therefore only two levels are considered in the system of equations (level 0 — $^2F_{7/2}$, level 2 — $^2F_{5/2}$):

$$\begin{cases} \dfrac{dS}{dt} = \mu v_c \sigma_{20} S(f_{21}N_2 - f_{03}N_0) + K_s l \dfrac{N_2}{\tau_2} - Sr_{tc}, \\ \dfrac{dN_2}{dt} = R_{\text{pump}} \sigma_{02} f_{01} N_0 - \dfrac{N_2}{\tau_2} - \sigma_{20} S(f_{21}N_2 - f_{03}N_0), \\ N_s = N_0 + N_2. \end{cases} \quad (11)$$

Q-Switching

Q-switching is a technique for generating high power pulses. The principle of generating such pulses is to control the quality factor (Q) of the resonator by introducing additional losses that depend on time or intensity inside the resonator. Initially, under the action of pumping, the gain (population inversion) in the active medium increases, but lasing does not start because of the large loss factor. Significant energy is accumulated in the active media in the form of excited particles. When the gain becomes close to its maximum value (for a given pump power), the loss factor decreases rapidly. Then generation begins and all the energy accumulated in the resonator is ejected from the laser in one short powerful pulse.

Active Q-switching

To obtain a powerful pulse in the active Q-switching mode, an active shutter is placed in the laser cavity. Usually this is an electro-optical crystal with a polarizer (fig. 6). To create a high level of losses in the cavity, a quarter-wave voltage is applied to the crystal. Linearly polarized radiation transmitted through the polarizer acquires circular polarization after passing the electro-optical crystal. Reflected from the mirror, circular polarization changes direction (e.g. left becomes right). Then, after passing through the electro-optical crystal a second time, the beam again becomes linearly polarized, however, the direction of polarization changes by 90 degrees. As a result, the polarizer does not pass such radiation and the shutter is in the closed state.

To open the shutter, the voltage is removed from the crystal and it ceases to affect the polarization of the transmitted light beam. As a result, the level of losses in the system is significantly reduced and a short and powerful laser pulse is generated.

To calculate the parameters of this pulse, the specific shutter design is not important for us. We also assume that the shutter opens quickly enough and its switching time can be neglected.

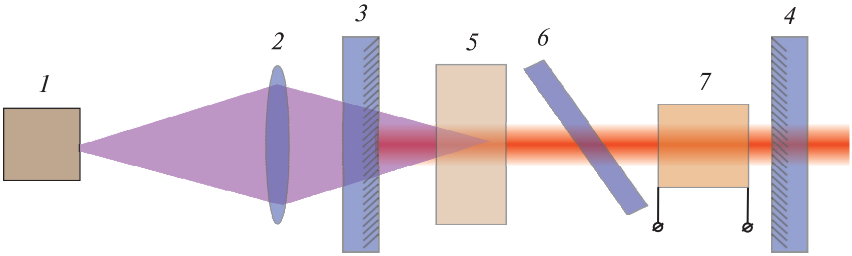


Fig. 6. The scheme of a laser with an electro-optical shutter:
1 – diode pump laser; *2* – a lens; *3*, *4* – resonator mirrors;
5 – laser crystal; *6* – polarizer; *7* – electro-optical crystal

To simulate the generation of a powerful pulse in a laser with a four-level scheme, one can use the system of rate equations (5), where the loss coefficient in the system depends on time. In the approximation of instantaneous shutter switching, the loss coefficient can be written in the form

$$r_{tc}(t) = c\frac{\left(-\dfrac{\ln(\rho)}{2} + \gamma l + r_{\mathrm{EOC}}(t)\right)}{L_{\mathrm{opt}}}, \qquad (12)$$

$$r_{\mathrm{EOC}}(t) = \begin{cases} r_0, & t < t_{\mathrm{EOC}}, \\ 0, & t \geq t_{\mathrm{EOC}}, \end{cases} \qquad (13)$$

where r_{EOC} – additional losses introduced by the electro-optical crystal (r_0 is a sufficiently large loss coefficient, ~50 % or more); t_{EOC} is the moment of time when the shutter is switched off [μs].

In this case, the processes in the system can be divided into two stages. At the first stage, the gain in the active medium increases under the constant pumping, but there is no generation, because losses in the cavity are very high. At the second stage, the losses are decreased and a laser pulse forms with parameters determined by the properties of the active medium, resonator, and pump power.

Passive Q-switching

In order to obtain a laser pulse in the passive Q-switching mode, a saturable absorber is placed in the laser cavity (fig. 7). It is called a passive shutter. The generation process proceeds as follows: under the influence of pumping, the gain of the active medium increases, but the generation does not start, because the loss coefficient is high due to the light absorption in the passive shutter. When the gain exceeds the losses, the generation begins. After the radiation intensity in the cavity becomes sufficient to bleach the shutter, the loss coefficient drops sharply and a shot powerful laser pulse forms.

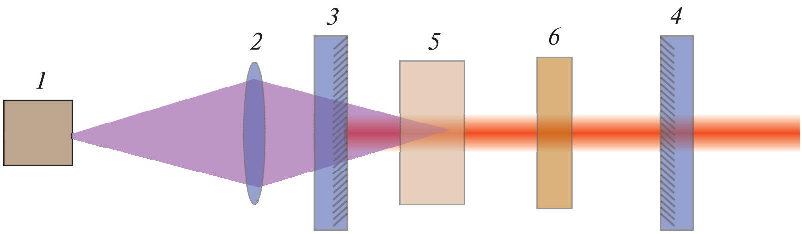


Fig. 7. The design of a laser with a passive shutter:
1 – diode pump laser; *2* – lens; *3*, *4* – cavity mirrors;
5 – laser crystal; *6* – passive shutter

Usually, the crystal Cr^{4+}:YAG is used as a passive shutter in neodymium lasers. Its initial transmission can vary widely. The parameters of this crystal used in the calculations are shown in table 2.

Table 2

Crystal Cr^{4+}:YAG parameters

The bulk density of chromium ions, cm^{-3}	$1.85 \cdot 10^{18}$
Effective absorption cross section from the ground state, cm^2	$8.7 \cdot 10^{-19}$
Upper level life time, μs	4
Refractive index at 1064 nm	1.83

Cr^{4+}:YAG has the energy level diagram presented on fig. 8. The transition from the ground energy level to the third corresponds to a wavelength of radiation from an yttrium aluminum garnet laser – 1064 nm. Relaxation from the third to the second level occurs in a much shorter time than the lifetime at the second level. As a result, we can assume that all excited ions immediately go to the second energy level. Also, the radiation wavelength of 1064 nm corresponds to the transition from the second to the fourth energy level. However, the lifetime at the fourth level is much shorter than the lifetime at the second level, so the transitions to the fourth level can be neglected in the simulations. Thus, the saturable absorber can be considered as a two-level medium.

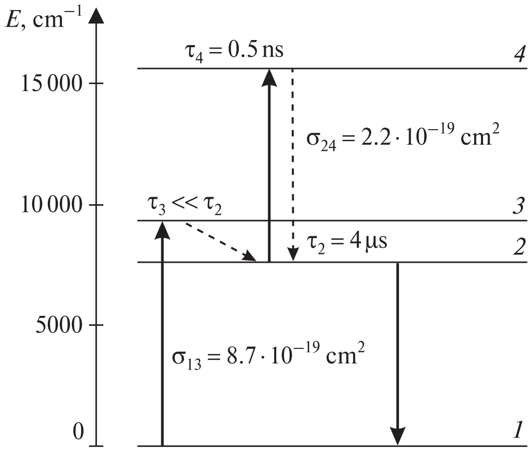


Fig. 8. The scheme of energy levels (*1–4*) of a passive shutter on a Cr^{4+} : YAG crystal

To calculate the parameters of a pulse, the system of rate equations for four-level scheme must be supplemented by equations for the saturable absorber:

$$\begin{cases} \dfrac{dS}{dt} = \mu \nu_c \sigma_{21}^{\text{em}} S(N_2 - N_1) + K_S l \dfrac{N_2}{\tau_2} - S r_{tc} - \mu_q \nu_{qc} \sigma_{13}^{\text{abs}} S(N_{qs} - N_{q2}), \\ \dfrac{dN_3}{dt} = R_{\text{pump}} \sigma_{03}^{\text{abs}} (N_s - N_1 - N_2 - N_3) - \dfrac{N_3}{\tau_3}, \\ \dfrac{dN_2}{dt} = \sigma_{21}^{\text{em}} S(N_2 - N_1) - \dfrac{N_2}{\tau_2} + \dfrac{N_3}{\tau_3}, \\ \dfrac{dN_1}{dt} = \sigma_{21}^{\text{em}} S(N_2 - N_1) - \dfrac{N_1}{\tau_1} + \dfrac{N_2}{\tau_2}, \\ \dfrac{dN_{q2}}{dt} = \sigma_{q13}^{\text{abs}} S(N_{qs} - N_{q2}) - \dfrac{N_{q2}}{\tau_{q2}}, \end{cases} \tag{14}$$

where N_{q2} is the population of the second level of the saturable absorber (cm^{-3}); τ_{q2} – its lifetime (μs); $\sigma_{q13}^{\text{abs}}$ – effective absorption cross section for the transition from the lower level of the saturable absorber (cm^2); N_{qs} is the bulk density of chromium ions in a crystal Cr^{4+}:YAG (cm^{-3}); v_{qc} is the speed of light in a Cr^{4+}:YAG crystal (cm/μs).

The parameter μ_q for the saturable absorber is calculated by formula (6), the parameters μ and r_{tc} for the active medium are calculated by formulas (6) and (8). It should be noted that the cavity optical length (7) will be

$$L_{\text{opt}} = L_c + (n_r - 1)l + (n_{rq} - 1)l_q, \tag{15}$$

where n_{rq} and l_q are the refractive index and the length of the saturable absorber, respectively.

Laser output power

Total laser output power P_{out} is given by the following formula

$$P_{\text{out}} = Sh\nu s_{cr}(1 - \rho), \tag{16}$$

where S is the photon flux density in the resonator, which is found from the system of rate equations; $h\nu$ is the photon energy for the generation wavelength; s_{cr} is the pumped area at the laser crystal, in our case it is equal to the area of the focused pump spot (this should be the output beam cross section area), ρ is the reflection coefficient of the output mirror.

Mode-Locking

Mode-locking is a method to produce short and high-power laser pulses of the pico- and femtosecond duration.

The laser cavity is actually a Fabry-Perot interferometer, where only the resonance beams corresponding to the standing waves with the nodes on the mirrors do not decay. On the laser generation, such standing waves (stable configurations of the electromagnetic field, called the longitudinal modes) are formed in the cavity. Each mode has its own frequency, and a generation spectrum as a whole looks like a set of frequencies with different amplitudes (fig. 9).

The frequency difference between adjacent modes is the same for the entire generation spectrum:

$$\Delta\omega = \frac{2\pi c}{2L_{\text{opt}}}, \tag{17}$$

where c is the speed of light; L_{opt} is the cavity optical length.

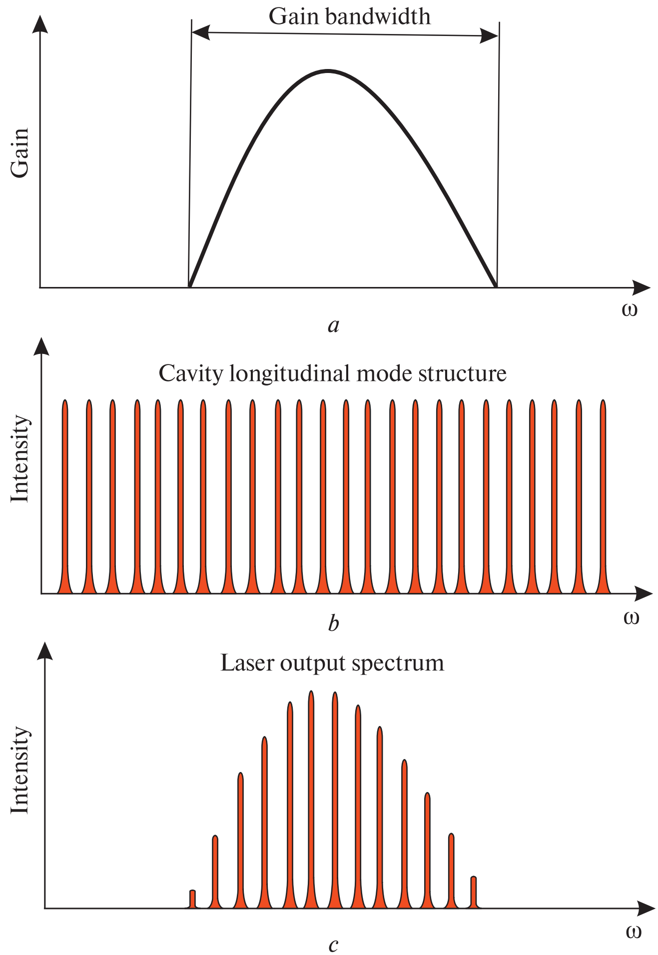


Fig. 9. Laser gain bandwidth (*a*), intracavity mode structure (*b*), and laser generation spectrum (*c*)

In the case of free-running lasing, the longitudinal modes have random initial phases, i.e., they are not coherent with each other. As a result, the output laser intensity is equal to the sum of the intensities of individual modes. However, the situation changes if the initial phases of all modes are equal (or different by $2\pi N$) – this state is called the mode-locking.

Let us find the total field in the cavity resulting from the interference of several longitudinal modes with equal initial phases and equidistant frequencies (17). Here we use the fact that a standing wave (longitudinal mode) is formed when two identical plane waves propagate in opposite directions (for example, along the z-axis).

The field of the *m-th* plane wave propagating along the z-axis can be expressed as

$$E_m^+(z,t) = A_m^0 \exp\left\{i(\omega_m t - k_m^+ z)\right\} =$$

$$= A_m^0 \exp\left\{i(\omega_0 t + \Delta\omega_m t - k_0 z - \Delta k_m z)\right\} = A_m^+(z,t)\exp\left\{i(\omega_0 t - k_0 z)\right\}, \quad (18)$$

$$A_m^+(z,t) = A_m^0 \exp\left\{i(\Delta\omega_m t - \Delta k_m z)\right\}, \quad (19)$$

$$k_0 = \frac{2\pi}{\lambda_0} = \frac{\omega_0}{c}, \quad (20)$$

where ω_0 is the center frequency of the lasing line; k_0 is the corresponding wave number; A_m^0 is the wave amplitude. The expression for the field of a wave propagating in the opposite direction (counter to the z-axis) is similar, considering that $k_m^- = -k_m^+$. Accordingly, the field of the *m-th* longitudinal mode is defined as a sum of plane waves with k_m^- and k_m^+:

$$E_m(z,t) = A_m^0 \exp\left\{i\left((\Delta\omega_m + \omega_0)t - (\Delta k_m + k_0)z\right)\right\} +$$

$$+ A_m^0 \exp\left\{i\left((\Delta\omega_m + \omega_0)t + (\Delta k_m + k_0)z\right)\right\}. \quad (21)$$

In the first approximation, we assume that the amplitudes of all modes are equal: $A_m^0 = A$. Then the light field in a laser cavity can be represented as a sum of plane waves with the frequencies corresponding to the longitudinal modes:

$$E(z,t) = \sum_{m=-M/2}^{M/2} E_m^+(z,t) + E_m^-(z,t) =$$

$$= A \sum_{m=-M/2}^{M/2} \mathrm{Exp}\left[i\left((\omega_0 + m)t - (k_0 + mk)z\right)\right] +$$

$$+ A \sum_{m=-M/2}^{M/2} \mathrm{Exp}\left[i\left((\omega_0 + m)t + (k_0 + mk)z\right)\right], \quad (22)$$

where $\Delta k_m = m\Delta k$; $\Delta\omega_m = m\Delta\omega$; M is the number of longitudinal modes, which is determined by the cavity length and the width of a generation spectrum $\Delta\nu_g$ (see table 6):

$$M = \frac{\Delta\nu_g c}{2L_{\mathrm{opt}}}, \quad \Delta\omega = 2\pi\Delta\nu = \frac{\pi c}{L_{\mathrm{opt}}}, \quad \Delta k = \frac{\pi}{L_{\mathrm{opt}}}. \quad (23)$$

The mode amplitudes A (note that in the first approximation they are considered identical) can be determined from the average value of the photon flux density within the cavity S in the free-running mode:

$$A = \sqrt{\frac{Sh\nu}{M}}. \tag{24}$$

The total amplitude of the intracavity light field can be written in a frame of reference associated with the principal mode (in this case $\omega_0 t - k_0 z = 0$ for a wave propagating along the z-axis and $\omega_0 t + k_0 z = 0$ for the one propagating in the counter direction):

$$A(z,t) = \sum_{m=-M/2}^{M/2} A_m^+(z,t) + A_m^-(z,t) =$$

$$= A \sum_{m=-M/2}^{M/2} \mathrm{Exp}\left[i(mt - mkz)\right] + \mathrm{Exp}\left[i(mt + mkz)\right]. \tag{25}$$

Here $A(z, t)$ is the amplitude of the envelope of the pulse formed in the cavity during the mode-locking; its intensity profile is given by $I(z,t) = $ $= \left|A(z,t)\right|^2$.

Thus, the waves corresponding to the longitudinal modes coherently add up and, as a result of the interference, form a single short pulse that propagates in the cavity from one mirror to another with the speed of light. While propagating in the cavity, this pulse is amplified in the laser active medium and is partly output from the cavity through the semitransparent mirror. The laser outputs periodic pulses with the repetition rate

$$f_{\mathrm{out}} \sim \frac{c}{2L_{\mathrm{opt}}}. \tag{26}$$

The peak amplitude of the pulse is proportional to the number of the locked modes M whereas the pulse length is inversely proportional to the width of a generation spectrum $\Delta\nu_g$:

$$\tau_g \sim \frac{1}{\Delta\nu_g}. \tag{27}$$

To implement passive mode-locking, a passive shutter must be added to the laser operating in the free-running mode (fig. 10). Semiconductor saturable absorber mirrors (SESAM, SAM) are usually used as passive shutters for mode-locking in solid-state lasers [14]. Such mirrors are placed instead of a highly reflective cavity mirror. The relaxation time of such an absorber is about 1 ps, and its absorption coefficient nonlinearly depends on the intensity of the incident radiation $|A(t)|^2$:

$$q(t) = \frac{q_0}{1 + |A(t)|^2 / I_{sat}}, \tag{28}$$

where I_{sat} is the saturation intensity of the absorber.

For the spikes with high intensity, the reflection coefficient is several percent higher than that for the low-intensity radiation (fig. 11). This insignificant difference is sufficient to separate and amplify the most intensive pulse from the fluctuation spikes in the cavity. Subsequently, the duration of this pulse is reduced to a minimum as its leading and trailing edges exhibit higher attenuation than the central part while passing through the absorber. Hence, the passive mode-locking is performed.

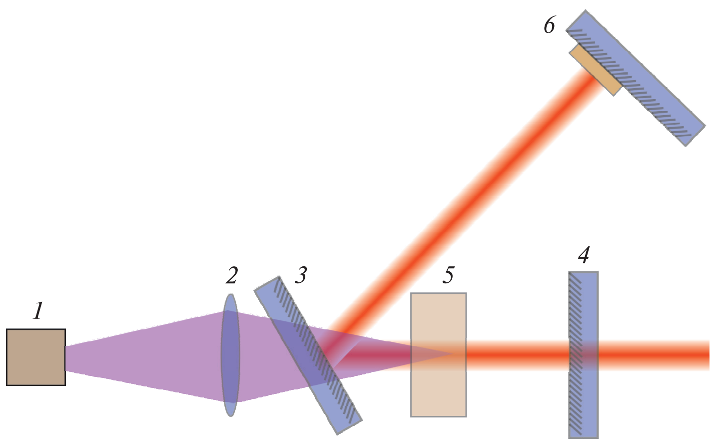


Fig. 10. The Scheme of the passively mode-locked laser:
1 – semiconductor pump laser; *2* – lens; *3* – high reflective mirror (transparent for pump light); *4* – cavity output mirror; *5* – laser crystal; *6* – semiconductor saturable absorber mirror (SESAM)

To simulate the mode-locking and calculate the parameters of the generated pulse, it is necessary to account for the change in the pulse shape during propagation in the cavity and the width of the laser gain line. However, to find the average intensity (photon flux density) and the population densities at the laser levels in the cavity, one can use the rate equations written in the approximation of the point model of the active medium (e. g., (4) for the neodymium laser and (6) for the ytterbium laser). Here it is considered that the relative population inversion and the average pulse intensity change insignificantly during the cavity round-trip time.

The influence of the saturable absorber could be included in the formula for the inverse photon lifetime in the cavity. The reflection coefficient ρ_2 is introduced for the mirror with the saturable absorber:

$$r_{tc} = c\frac{\left(-\frac{\ln(\rho_1\rho_2)}{2}+\gamma l\right)}{L_{\text{opt}}}. \quad (29)$$

Here, $\rho_2 = 0.98$ describes the saturated losses introduced by the absorber (see fig. 11).

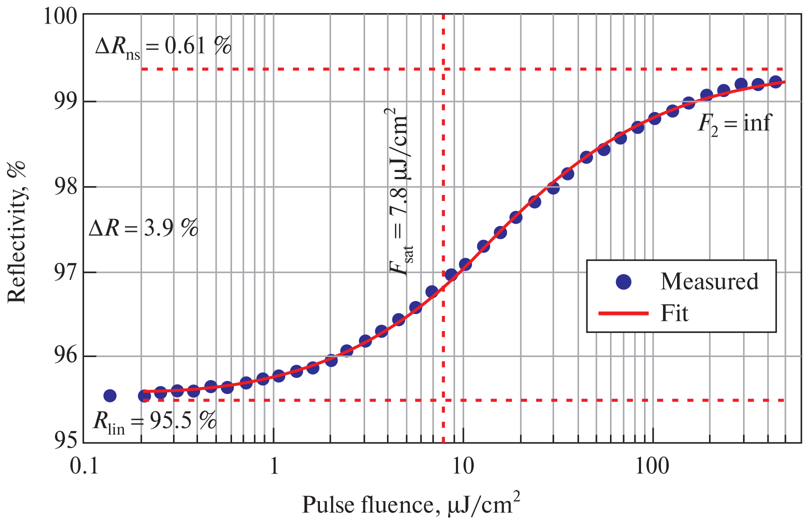


Fig. 11. Reflectivity of a semiconductor saturable absorber mirror (SESAM) as a function of the pulse fluence. Source: [15]

The amplitudes of the longitudinal modes can be estimated by such a model if their number and the average photon flux density in the cavity (24) are known.

The effect of the finite width of the laser gain line on the mode-locking pulse parameters can be estimated considering the distribution of amplitudes to be the same as the gain profile.

For example, a line with homogenous broadening has a Lorentz contour that can be approximated by a parabola near the center frequency:

$$g_{\text{Lorentz}}(\omega) = \frac{g_0}{1 + \left(\dfrac{\omega - \omega_0}{\Delta\omega_g / 2}\right)^2}, \tag{30}$$

$$g_{\text{parabolic}}(\omega) = g_0 \left(1 + \left(\frac{\omega - \omega_0}{\Delta\omega_g / 2}\right)^2\right), \tag{31}$$

where g_0 is the peak value of the gain (at the center frequency ω_0); $\Delta\omega_g$ is the width of the gain line.

The time dependence of the laser output power during the mode-locking is determined by the total intensity of the intracavity light field and by the reflection coefficient of the output mirror:

$$P_{\text{out}} = I(z, t) s_{cr} (1 - \rho_1). \tag{32}$$

PROBLEMS

Free lasing mode, neodymium laser, four-level scheme

1. Calculate the time needed to reach the steady-state lasing mode in the case of a neodymium laser with the given matrix for the given pump level.

2. Calculate the duration of the first spike in the dynamic generation mode in the case of a neodymium laser with the given matrix.

3. Plot the dependence of the output power of the laser on the pump power $P_{out}(P_{in})$ and calculate the generation efficiency (dP_{out}/dP_{in}) in the stationary mode. Find the threshold pump power for the given laser. The cross-section area of the output beam is assumed to be equal to the pumping area.

4. Calculate the generation efficiency (dP_{out}/dP_{in}) of the laser in steady-state mode in the case of transition ${}^4F_{3/2} \rightarrow {}^4I_{13/2}$ for a matrix No. 5 (lasing wavelength 1341.6 nm, $\sigma_{em} = 2.2 \cdot 10^{-19}$ cm^2).

5. Calculate the optimal reflection coefficient of the output mirror for a given pump power (corresponding to the maximal output power).

6. Plot the dependence of the output power on the loss coefficient.

For the task, take the following parameter values: cavity length $L = 20$ cm, crystal length $l = 0.3$ cm, inactive loss $\gamma = 0.002$ cm^{-1}, reflection coefficient $\rho = 0.97$. Spectroscopic parameters are shown in table A1. Temperature $T = 300$ K.

Free lasing mode, neodymium laser, quasi-three-level scheme

1. Calculate the time needed to reach the steady-state lasing mode in the case of a neodymium laser with the given matrix.

2. Calculate the duration of the first spike in the dynamic generation mode in the case of a neodymium laser with the given matrix.

3. Calculate the generation efficiency (dP_{out}/dP_{in}) of the laser in the stationary mode in the case of the transition to the fourth Stark sublevel of the ground multiplet ${}^4I_{9/2}$ for the given matrix.

4. Calculate the generation efficiency (dP_{out}/dP_{in}) of the laser in the stationary generation mode in the case of a neodymium laser with the given matrix.

5. Calculate the threshold pump power in the stationary generation mode in the case of a neodymium laser with the given matrix.

6. Find out how the generation threshold changes when the temperature increases by 40 °C

The spectroscopic parameters are shown in table A2.

Free-running mode, ytterbium laser

Ytterbium lasers (Yb^{3+}) operate only according to a quasi-three-level scheme. This is due to the fact that the ytterbium ion has only 2 energy multiplets – $^2F_{5/2}$ and $^2F_{7/2}$ (generation wavelength ~1040 nm).

1. Calculate the time needed to reach the steady-state lasing mode with the given matrix.

2. Calculate the threshold pump power in the stationary generation mode with the given matrix. Find out how the threshold changes when the temperature increases by 40 °C.

3. Calculate the generation efficiency (dP_{out}/dP_{in}) of the laser in the steady-state lasing mode with the given matrix.

4. Calculate how the power will change in the stationary generation mode if the temperature of the laser crystal is lowered by 30 °C.

5. Compare the start time of generation at 300 K and 270 K.

The spectroscopic parameters are shown in table A3.

Active Q-switching, neodymium laser, four-level scheme

1. Calculate the pulse duration and maximum intensity for matrix No. 4 and No. 5. Explain the effect of the lifetime in the excited state on the parameters of the pulse.

2. Calculate the pulse duration and intensity at maximum for matrix No. 2 and No. 6. Explain the effect of the radiation cross section on the pulse parameters.

3. Calculate the optimal time of the shutter switching (for which the pulse peak intensity is maximal) and the pulse duration for the given matrix.

4. Calculate the pulse peak intensity as a function of pump power.

5. Find energy in a single pulse at the laser output.

6. Find the reflection coefficient of the output mirror for a given pump power at which the peak intensity of the output pulses is maximal.

7. Plot the dependence of the output pulse peak power on the loss coefficient.

Passive Q-switching mode, neodymium laser, four-level scheme

1. Calculate the pulse duration, maximum intensity, and generation start time for the given matrix.

2. Find out the pulse repetition rate for the constant pumping for the given matrix. How will the repetition rate increase when the pump is doubled?

3. Find out the energy in one pulse at the laser output for the given matrix. How will the energy in one pulse increase when the pump is doubled?

4. Find the reflection coefficient of the output mirror for a given pump power at which the peak intensity of the output pulses is maximal for the given matrix.

5. Plot the dependence of the output pulse peak power on the loss coefficient for the given matrix.

Mode-Locking

1. Calculate the pulse duration and maximum intensity for the given laser crystal.

2. Calculate the pulse duration and maximum intensity for Nd : KGW and Nd : YAG laser crystal. Explain the effect of gain bandwidth on the parameters of the pulse.

3. Calculate the pulse duration, maximum intensity and the energy in one pulse for Nd : YAG laser crystal for cavity length L = 20 cm and L = 100 cm. Explain the effect of cavity length on the parameters of the laser pulse.

4. Calculate the pulse duration, maximum intensity and the energy in one pulse for Yb : YAG laser crystal for cavity length L = 20 cm and L = 100 cm. Explain the effect of cavity length on the parameters of the laser pulse.

For the task, take the following parameter values: cavity length L = 20 cm, crystal length l = 0.3 cm, inactive loss $\gamma = 0.002$ cm^{-1}, output mirror reflection coefficient ρ_1 = 0.97, SESAM saturable reflection coefficient ρ_2 = 0.98. Spectroscopic parameters are shown in table 3. Temperature T = 300 K.

APPENDICES

1. Spectroscopic parameters of laser crystals

Table A1

Spectroscopic parameters of neodymium ions in various matrices for a four level transition

Matrix No.	Matrix	Matrix (chem. Formula)	Refractive index *n*	Ion concentration Nd^{3+}, cm^{-3}	Lasing wavelength λ, nm	σ_{em}, cm^2 (T = 300K)	τ, μs	λ_{abs}, nm	σ_{abs}, cm^2 (T = 300 K)	Energy of sublevels $^4F_{3/2}$, cm^{-1}
1	YAG	$Y_3Al_5O_{12}$	1.816	1 % Nd $1.38 \cdot 10^{20}$	1064.2	$2.8 \cdot 10^{-19}$	230	808	$7.7 \cdot 10^{-20}$	11 512 11 427
2	YVO (a-cut)	YVO_4	$n_e = 2.17$ $n_o = 1.96$	1 % Nd $1.25 \cdot 10^{20}$	1064.2	$15.6 \cdot 10^{-19}$(‖ s)	110	808.5	$27 \cdot 10^{-20}$ (π)	11 384 11 366
3	GVO (a-cut)	$GdVO_4$	$n_e = 2.20$ $n_o = 1.99$	1 % Nd $1.25 \cdot 10^{20}$	1062.9	$7.6 \cdot 10^{-19}$(‖ s)	94	808.4	$52 \cdot 10^{-20}$ (π)	11 372 11 369
4	GGG	$Gd_3Ga_5O_{12}$	1.94	1.6 % Nd $2.0 \cdot 10^{20}$	1062	$2.2 \cdot 10^{-19}$	250	808	$4.8 \cdot 10^{-20}$	11 485 11 442
5	YAP (a-cut)	$YAlO_3$	$n_a = 1.929$ $n_b = 1.943$ $n_c = 1.952$	1 % Nd $1.96 \cdot 10^{20}$	1064.6	$1.7 \cdot 10^{-19}$(‖ c)	170	803	$5.0 \cdot 10^{-20}$(‖ c)	11 545 11 421
6	KGW (N_g-cut)	KGd $(WO_4)_2$	$n_g = 2.033$ $n_m = 1.986$ $n_p = 1.937$	3 % Nd $1.88 \cdot 10^{20}$	1067.2	$4.3 \cdot 10^{-19}$	110	811	$23 \cdot 10^{-20}$ (‖ N_m)	11 417 11 315

Source: [1; 4–7].

Table A2

Spectroscopic parameters of neodymium ions in various matrices for a quasi-three-level transition

Matrix No.	Matrix	Matrix (chem. Formula)	Refractive index *n*	Ion concentration Nd^{3+}, cm^{-3}	Lasing wavelength λ, nm	σ_{em}, cm^2 (T = 300 K)	τ, μs	λ_{abs}, nm	σ_{abs}, cm^2 (T = 300 K)	Energy of sublevels $^4I_{9/2}$, cm^{-1}	Energy of sublevels $^4F_{3/2}$, cm^{-1}
1	YAG	$Y_3Al_5O_{12}$	1.82	1 % Nd $1.38 \cdot 10^{20}$	946	$6.4 \cdot 10^{-20}$	230	808	$7.7 \cdot 10^{-20}$	857, 312, 199, 133,0	11 512 11 427
2	YVO (a-cut)	YVO_4	$n_e = 2.16$ $n_o = 1.95$	1% Nd $1.25 \cdot 10^{20}$	914	$5.0 \cdot 10^{-20}$ (‖ s)	110	808.5	$27 \cdot 10^{-20}$ (π)	439, 226, 173, 108, 0	11 384 11 366
3	GVO (a-cut)	$GdVO_4$	$n_e = 2.19$ $n_o = 1.97$	1 % Nd $1.25 \cdot 10^{20}$	912	$7.0 \cdot 10^{-20}$ (‖ s)	94	808.4	$52 \cdot 10^{-20}$ (π)	408, 267, 173, 107, 0	11 372 11 369
4	GGG	$Gd_3Ga_5O_{12}$	1.94	1.6 % Nd $2.0 \cdot 10^{20}$	937	$2.95 \cdot 10^{-20}$	250	808	$4.8 \cdot 10^{-20}$	772, 253, 178, 93, 0	11 485 11 442
5	YAP (a-cut)	$YAlO_3$	$n_a = 1.929$ $n_b = 1.943$ $n_c = 1.952$	1 % Nd $1.96 \cdot 10^{20}$	930	$5.3 \cdot 10^{-20}$ (‖ c)	170	803	$5.0 \cdot 10^{-20}$ (‖ c)	670, 499, 208, 115, 0	11 545 11 421
6	KGW (N_g-cut)	KGd $(WO_4)_2$	$n_g = 2.061$ $n_m = 2.010$ $n_p = 1.982$	8 % Nd $5.0 \cdot 10^{20}$	911.2	$7.0 \cdot 10^{-20}$ (‖ N_m)	90	811	$23 \cdot 10^{-20}$ (‖ N_m)	340, 295, 150, 102, 0	11 417 11 315

Source: [1; 8–11].

Table A3

Spectroscopic parameters of ytterbium ions in various matrices (ions go from the lower sublevel of the upper multiplet to the third sublevel of the ground multiplet)

Matrix No.	Matrix	Matrix (chemical formula)	Refractive index *n*	Concentration of ions Yb^{3+}, cm^{-3}	Lasing wavelength λ, nm	σ_{em}, cm^2 ($T = 300$ K)	τ, ms	λ_{abs}, nm	σ_{abs}, cm^2 ($T = 300$ K)	Energy of sublevels $^2F_{7/2}$, cm^{-1}	Energy of sublevels $^2F_{5/2}$, cm^{-1}
1	YAG	$Y_3Al_5O_{12}$	1.82	10 % Yb $13.8 \cdot 10^{20}$	1030	$2.0 \cdot 10^{-20}$	1.2	942	$7.8 \cdot 10^{-21}$	786, 619, 581, 0	10 927 10 634 10 327
2	CaF_2	CaF_2	1.429	2,59 % Yb $6.35 \cdot 10^{20}$	1049	$0.2 \cdot 10^{-20}$	2.4	980	$0.54 \cdot 10^{-20}$	1110, 635, 560, 0	10 900 10 471 10 234
3	KGW (N_g-cut)	$KGd(WO_4)_2$	$n_g = 2.037$ $n_m = 2.033$ $n_p = 1.936$	5 % Yb $3.125 \cdot 10^{20}$	1036	$2.6 \cdot 10^{-20}$ (‖ N_m)	0.3	981	$12 \cdot 10^{-20}$ (‖ N_m)	535, 385, 163, 0	10 682 10 471 10 188

Source: [4; 12; 13].

Table A4

Gain bandwidth, emission and absorption cross-sections for laser crystals with neodymium and ytterbium ions

Laser crystal	Fluorescence Lifetime (μs)	Absorption cross-sections ($10^{-20} cm^2$)	Emission cross-sections ($10^{-20} cm^2$)	Gain bandwidth
Nd:YAG	230	7.7 (808 nm)	28 (1064 nm)	0.6 nm
Nd:YVO	90	27 (808.5 nm)	250 (1064 nm)	0.96 nm
Nd:KGW	110	23 (811 nm)	43 (1067 nm)	2.73 nm
Yb:YAG	1200	0.77 (941 nm)	2.03 (1031 nm)	9 nm
Yb:KGW	600	12 (981 nm)	2.8 (1023 nm)	20 nm
Yb:CaF	2400	0.8 (980 nm)	0.3 (1030 nm)	70 nm

Source: [16].

2. Listing of the "Wolfram Mathematica" program for Nd:YAG free lasing mode

Nd:YAG 4 - Level Laser: free lasing mode

Laser and active medium parameters:

```
In[4]:= λa := 808.6 × 10^-9 (*[m]*) (*Pump wavelength*)
λe := 1064 × 10^-9 (*[m]*) (*Lasing wavelength*)
h := 6.626 × 10^-28 (*[J*μs]*) (*Planck constant*)
σ21 := 28 × 10^-20 (*[cm^2]*) (*Stimulated emission cross section*) (*6.0×10^-19*)
σ13 := 7.7 × 10^-20 (*[cm^2]*) (*Absorption cross section*) (*7.0×10^-20*)
t3 := 0.03(*[μs]*) (*Life time at the top level*)
t2 := 230(*[μs]*) (*Life time at the metastable level*) (*2.2×10^-4*)
t1 := 0.01(*[μs]*) (*Lower laser level life time*)
Ns := 1.38(*[cm^-3]*) (*Concentration of neodymium ions*) (*1.25×10^20*)
Lar := 0.3 (*[cm]*) (*Laser Crystal Length*)
Lc := 20 (*[cm]*) (*Cavity length*)
γ := 0.002 (*[cm^-1]*) (*Inactive loss ratio*)
ρ := 0.97 (*Reflection coefficient of the output mirror*)
nref := 1.82 (*Laser crystal refractive index*)
c := 3 × 10^4 (*[cm/μs]*) (*The speed of light in vacuum*)
vc := c / nref (*[cm/μs]*) (*The speed of light in matter*) (*1.64×10^10*)
Lopt := Lc + (nref - 1) * Lar (*Optical cavity length*)
rtc := c / Lopt * (γ * Lar - Log[ρ] / 2) (*[μs^-1]*)
(*Inverse photon lifetime in the cavity*) (*3×10^7*)
μ := nref * Lar / Lopt (*Resonator fill factor with active medium*) (*0.007*)
a := 0.25 (*[cm]*) (*The radius of the pump beam on the laser crystal*)
Sar := π * a^2 (*[cm^2]*) (*Pump area*)
Ppump := 10(*[W]*) (*Pump power*) (*Threshold pump power - 3.8 W*)
Rpump := Ppump / Sar / h / c * λa * 100 / 10^6 (*[phot/cm^2/μs]*)
(*Pump photon flux density*) (*5.0×10^20*)
Ks := 1 × 10^-10 (*The share of spontaneous photons that pass into the cavity mode
into the cavity mode *)
s0 := 10^-10 (*[phot/cm^2/μs]*) (*Seed spontaneous emission*) (*10^-10 *)
hne := h * c / λe / 100 (*[J]*) (*Radiated photon energy*)
T := 1000 (*[μs]*) (*Calculation time*)
```

The system of equations:

```
In[31]:= (*The system of equations taking into
 account the nonzero population of the lower laser
level n1*)
solvedDyn1 = NDSolve[
  {S'[t] - vc μ σ21 S[t] * (n2[t] - n1[t]) × 10^20 - Ks Lar n2[t] × 10^20 / t2 + S[t] rtc == 0,
   n3'[t] + n3[t] / t3 - Rpump σ13 (Ns - n1[t] - n2[t] - n3[t]) == 0,
   n2'[t] - n3[t] / t3 + n2[t] / t2 + σ21 S[t] (n2[t] - n1[t]) == 0,
   n1'[t] + n1[t] / t1 - n2[t] / t2 - σ21 S[t] (n2[t] - n1[t]) == 0,
   S[0] == s0, n1[0] == 0, n2[0] == 0, n3[0] == 0},
                              {S[t], n3[t], n2[t], n1[t]},
                              {t, 0, T}]
```

Out[31]= {{S[t] → InterpolatingFunction[Domain: {{0., 1.00×10^3}} Output: scalar][t],

n₃[t] → InterpolatingFunction[Domain: {{0., 1.00×10^3}} Output: scalar][t],

n₂[t] → InterpolatingFunction[Domain: {{0., 1.00×10^3}} Output: scalar][t],

n₁[t] → InterpolatingFunction[Domain: {{0., 1.00×10^3}} Output: scalar][t]}}

Output laser intensity and the relative population inversion :

In[32]:=

```
Intensity1[t_] := Evaluate[S[t] /. solvedDyn1[[1]]] * hne * (1 - ρ)
(*[MW/cm²]*) (*Laser output intensity*)
PopulationInversion1[t_] := Evaluate[(n₂[t] - n₁[t]) /. solvedDyn1[[1]]] / Ns
(*Relative population inversion*)
```

Laser radiation intensity at time T :

In[34]:=

```
SetPrecision[Intensity1[t] /. t → T, 10]
```

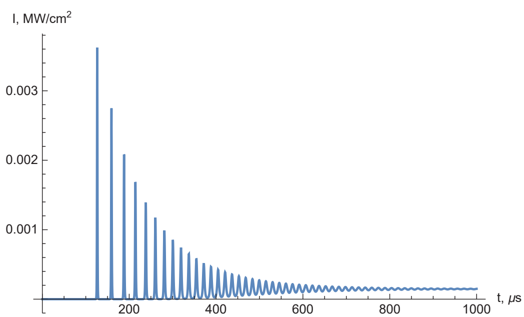
Out[34]= 0.0001494313101

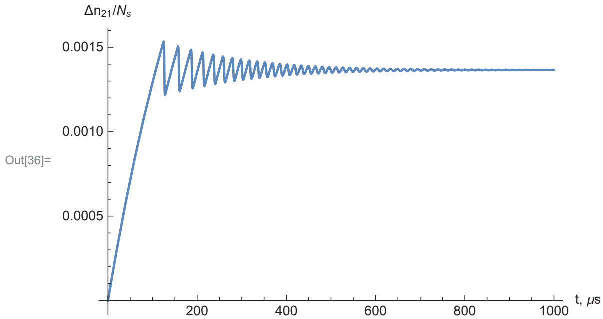
Graphs of the dependence of the intensity and density of populations at levels on time:

In[35]:=

```
Plot[{Intensity1[t]}, {t, 0, T}, AxesLabel → {"t, μs", "I, MW/cm²"},
 PlotRange → Full, PlotLegends → Automatic]
Plot[{PopulationInversion1[t]}, {t, 0, T}, PlotRange → Full,
 AxesLabel → {"t, μs", "Δn21/Ns"}, PlotLegends → Automatic]
```

Out[35]=





3. Listing of the “Wolfram Mathematica” program for Nd:YAG active Q-switch mode

Nd:YAG 4 - Level Laser: active Q-switching

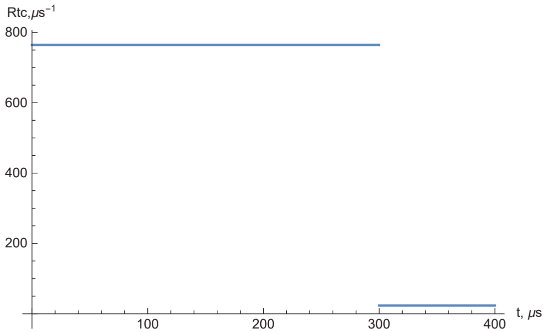
Laser and active medium parameters:

```
In[80]:= λa := 808.6 × 10^-9 (*[m]*) (*Pump wavelength*)
λe := 1064 × 10^-9 (*[m]*) (*Lasing wavelength*)
h := 6.626 × 10^-28 (*[J*mcs]*) (*Planck constant*)
σ21 := 28 × 10^-20 (*[cm^2]*) (*Stimulated emission cross section*) (*6.0×10^-19*)
σ13 := 7.7 × 10^-20 (*[cm^2]*) (*Absorption cross section*) (*7.0×10^-20*)
t3 := 0.03 (*[μs]*) (*Life time at the top level*)
t2 := 230 (*[μs]*) (*Life time at the metastable level*) (*2.2×10^-4*)
t1 := 0.01 (*[μs]*) (*Lower laser level life time*)
Ns := 1.38 (*[cm^-3]*) (*Concentration of neodymium ions*) (*1.25×10^20*)
Lar := 0.3 (*[cm]*) (*Laser Crystal Length*)
Lc := 20 (*[cm]*) (*Cavity length*)
γ := 0.002 (*[cm^-1]*) (*Inactive loss ratio*)
ρ := 0.97 (*Reflection coefficient of the output mirror*)
nref := 1.82 (*Laser crystal refractive index*)
c := 3 × 10^4 (*[cm/μs]*) (*The speed of light in vacuum*)
vc := c / nref (*[cm/μs]*) (*The speed of light in matter*) (*1.64×10^10*)
Lopt := Lc + (nref - 1) * Lar (*Optical cavity length*)
μ := nref * Lar / Lopt (*Resonator fill factor with active medium*) (*0.007*)
a := 0.25 (*[cm]*) (*The radius of the pump beam on the laser crystal*)
Sar := π * a^2 (*[cm^2]*) (*Pump area*)
Ppump := 50 (*[W]*) (*Pump power*) (*Threshold pump power - 3.8 W*)
Rpump := Ppump / Sar / h / c * λa * 100 / 10^6 (*[phot/cm^2/μs]*)
(*Pump photon flux density*) (*5.0×10^20*)
Ks := 1 × 10^-10 (*The share of spontaneous photons that pass into the cavity mode*)
s0 := 10^-10 (*[phot/cm^2/μs]*) (*Seed spontaneous emission*) (*10^-10 *)
hne := h * c / λe / 100 (*[J]*) (*Radiated photon energy*)
T := 400 (*[μs]*) (*Calculation time*)
T1 := 300 (*[μs]*) (*At this time the electro-optical shutter switches*)
```

The loss coefficient in the system depends on the time:

In[107]:=
```
l0[t_] := 50 * Boole[t < T1] (*[%]*)
(*Additional losses in the system before switching electro-optical
shutter*)
Rtc[t_] := c / Lopt * (γ * Lar - Log[ρ] / 2 + l0[t] / 100) (*[
μs^-1]*) (*Inverse photon lifetime in the cavity*)
Plot[Rtc[t], {t, 0, T}, AxesLabel → {"t, μs", "Rtc,μs^-1"}]
```

Out[109]=



The system of equations:

In[110]:=
```
(*The system of equations taking into
 account the nonzero population of the lower laser
level n1*)
solvedDyn1 = NDSolve[
  {S'[t] - vc μ σ21 S[t] * (n2[t] - n1[t]) × 10^20 - Ks Lar n2[t] × 10^20 / t2 + S[t] × Rtc[t] == 0,
   n3'[t] + n3[t] / t3 - Rpump σ13 (Ns - n1[t] - n2[t] - n3[t]) == 0,
   n2'[t] - n3[t] / t3 + n2[t] / t2 + σ21 S[t] (n2[t] - n1[t]) == 0,
   n1'[t] + n1[t] / t1 - n2[t] / t2 - σ21 S[t] (n2[t] - n1[t]) == 0,
   S[0] == s0, n1[0] == 0, n2[0] == 0, n3[0] == 0},
  {S[t], n3[t], n2[t], n1[t]}, {t, 0, T}]
```

Out[110]=
```
{{S[t] → InterpolatingFunction[ Domain: {{0., 400.}} Output: scalar ][t],
  n3[t] → InterpolatingFunction[ Domain: {{0., 400.}} Output: scalar ][t],
  n2[t] → InterpolatingFunction[ Domain: {{0., 400.}} Output: scalar ][t],
  n1[t] → InterpolatingFunction[ Domain: {{0., 400.}} Output: scalar ][t]}}
```

Output laser intensity and the relative population inversion:

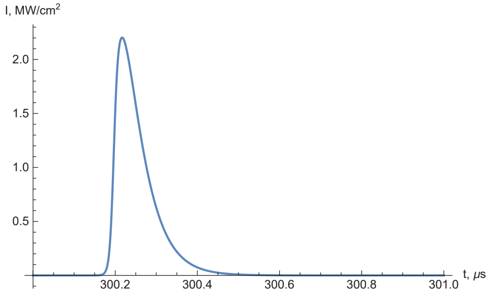
In[111]:=
```
Intensity1[t_] := Evaluate[S[t] /. solvedDyn1[[1]]] * hne * (1 - ρ)
(*[MW/cm^2]*) (*Laser output intensity*)

PopulationInversion1[t_] := Evaluate[(n2[t] - n1[t]) /. solvedDyn1[[1]]] / Ns
(*Relative population inversion*)
```

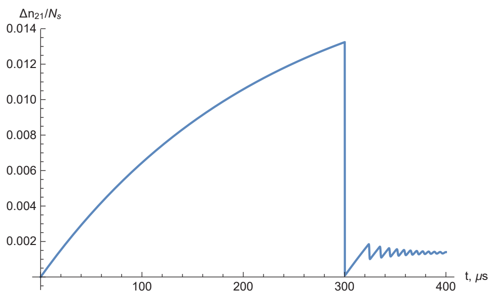
Graphs of the dependence of the intensity and density of populations at levels on time:

In[113]:=
```
Plot[{Intensity1[t]}, {t, T1, T1 + 1},
 AxesLabel → {"t, μs", "I, MW/cm^2"}, PlotRange → Full, PlotLegends → Automatic]
Plot[{PopulationInversion1[t]}, {t, 0, T}, PlotRange → Full,
 AxesLabel → {"t, μs", "Δn21/Ns"}, PlotLegends → Automatic]
```

Out[113]=



Out[114]=



4. Listing of the "Wolfram Mathematica" program for Nd:YAG passive Q-switch mode

Nd:YAG 4 - Level Laser: passive Q-switching

Laser and active medium parameters:

In[115]:=
```
λa := 808.6 × 10^-9 (*[m]*) (*Pump wavelength*)
λe := 1064 × 10^-9 (*[m]*) (*Lasing wavelength*)
h := 6.626 × 10^-28 (*[J*mcs]*) (*Planck constant*)
```

```
σ21 := 28 × 10^-20 (*[cm^2]*) (*Stimulated emission cross section*) (*6.0×10^-19*)
σ13 := 7.7 × 10^-20 (*[cm^2]*) (*Absorption cross section*) (*7.0×10^-20*)
t3 := 0.03 (*[μs]*) (*Life time at the top level*)
t2 := 230 (*[μs]*) (*Life time at the metastable level*) (*2.2×10^-4*)
t1 := 0.01 (*[μs]*) (*Lower laser level life time*)
Ns := 1.38 (*[cm^-3]*) (*Concentration of neodymium ions*) (*1.25×10^20*)
Lar := 0.3  (*[cm]*) (*Laser Crystal Length*)
Lc := 20 (*[cm]*) (*Cavity length*)
γ := 0.002  (*[cm^-1]*) (*Inactive loss ratio*)
ρ := 0.97 (*Reflection coefficient of the output mirror*)
nref := 1.82 (*Laser crystal refractive index*)
c := 3 × 10^4  (*[cm/μs]*) (*The speed of light in vacuum*)
vc := c / nref (*[cm/μs]*) (*The speed of light in matter*) (*1.64×10^10*)
Lopt := Lc + (nref - 1) * Lar (*Optical cavity length*)
rtc := c / Lopt * (γ * Lar - Log[ρ] / 2) (*[μs^-1]*)
(*Inverse photon lifetime in the cavity*) (*3×10^7*)
μ := nref * Lar / Lopt (*Resonator fill factor with active medium*) (*0.007*)
a := 0.25  (*[cm]*) (*The radius of the pump beam on the laser crystal*)
Sar := π * a^2  (*[cm^2]*) (*Pump area*)
Ppump := 100 (*[W]*) (*Pump power*) (*Threshold pump power - 3.8 W*)
Rpump := Ppump / Sar / h / c * λa * 100 / 10^6  (*[phot/cm^2/μs]*)
(*Pump photon flux density*) (*5.0×10^20*)
Ks := 1 × 10^-10  (*The share of spontaneous photons that pass
into the cavity mode *)
s0 := 10^-10  (*[phot/cm^2/μs]*) (*Seed spontaneous emission*) (*10^-10 *)
hne := h * c / λe / 100  (*[J]*) (*Radiated photon energy*)
T := 1000  (*[μs]*) (*Calculation time*)
```

Passive Shutter parameters (Cr^{4+}:YAG):

```
In[142]:= tq2 := 4  (*[μs]*) (*Life time at the top level*)
Nsq := 0.0185  (*[10^20 cm^-3]*) (*Concentration of chromium ions*)
σq12 := 87 × 10^-20  (*[cm^2]*) (*Absorption cross section*)
μq := 0.015 (*Resonator fill factor with saturable absorber*)
vcq := 1.64 × 10^4  (*[cm/μs]*) (*The speed of light in matter*)
```

The system of equations :

```
In[147]:= (*The system of equations taking into
 account the nonzero population of the lower laser
level n1*)
solvedDyn1 = NDSolve[
   {S'[t] - vc μ σ21 S[t] * (n2[t] - n1[t]) × 10^20 -
      Ks Lar n2[t] × 10^20 / t2 + S[t] rtc + vcq μq σq12 S[t] (Nsq - nq2[t]) × 10^20 == 0,
    n3'[t] + n3[t] / t3 - Rpump σ13 (Ns - n1[t] - n2[t] - n3[t]) == 0,
    n2'[t] - n3[t] / t3 + n2[t] / t2 + σ21 S[t] (n2[t] - n1[t]) == 0,
    n1'[t] + n1[t] / t1 - n2[t] / t2 - σ21 S[t] (n2[t] - n1[t]) == 0,
    nq2'[t] + nq2[t] / tq2 - σq12 S[t] (Nsq - nq2[t]) == 0,
    S[0] == s0, n1[0] == 0, n2[0] == 0, n3[0] == 0, nq2[0] == 0},
                                  {S[t], n3[t], n2[t], n1[t], nq2[t]},
                                  {t, 0, T}];
```

Output laser intensity and the relative population inversion :

In[148]:=

```
Intensity1[t_] := Evaluate[S[t] /. solvedDyn1[[1]]] * hne * (1 - ρ)
(*[MW/cm²]*) (*Laser output intensity*)

PopulationInversion1[t_] := Evaluate[(n₂[t] - n₁[t]) /. solvedDyn1[[1]]] / Ns
(*Relative population inversion*)

nq1[t_] := Evaluate[(nq2[t]) /. solvedDyn1[[1]]] / Nsq
(*The relative population of the upper level of saturable absorber*)
```

The radiation intensity in the cavity at time T :

In[151]:=

```
SetPrecision[Intensity1[t] /. t → T, 10]
```

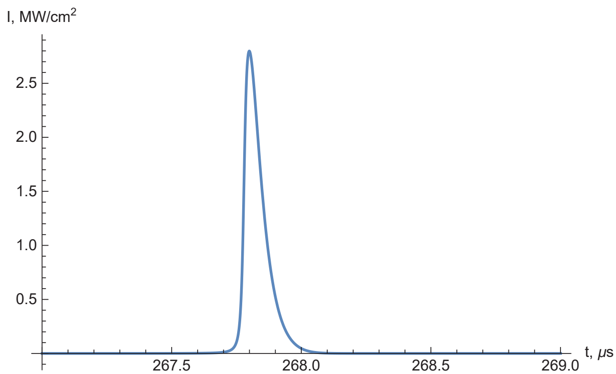
Out[151]= $3.458923563 \times 10^{-17}$

Graphs of the dependence of the intensity and density of populations at levels on time:

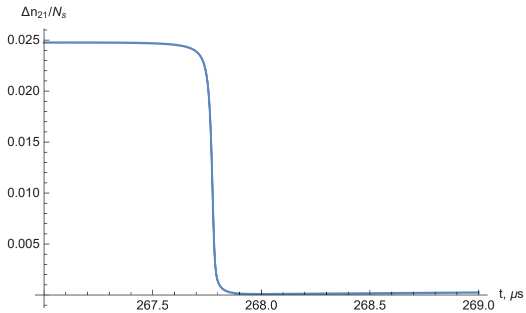
In[152]:=

```
t1 := 267
t2 := 269
Plot[{Intensity1[t]}, {t, t1, t2}, AxesLabel → {"t, μs", "I, MW/cm²"},
 PlotRange → Full, PlotLegends → Automatic]
Plot[{PopulationInversion1[t]}, {t, t1, t2}, PlotRange → Full,
 AxesLabel → {"t, μs", "Δn21/Ns"}, PlotLegends → Automatic]
Plot[{nq1[t]}, {t, t1, t2}, PlotRange → Full,
 AxesLabel → {"t, μs", "nq2/Nsq"}, PlotLegends → Automatic]
```

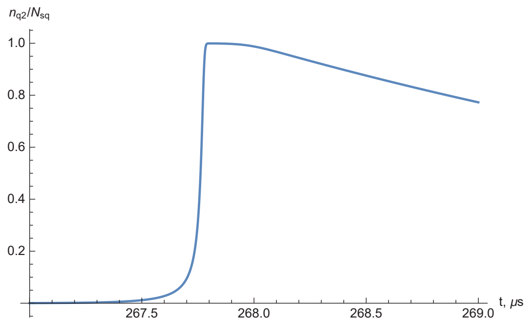
Out[154]=



Out[155]=



Out[156]=



5. Listing of the "Wolfram Mathematica" program for Nd:YAG Mode Locking

Mode Locking: Nd:YAG 4-level laser

Laser and active medium parameters:

```
In[157]:= λa := 808.6 × 10^-9 (*[m]*) (*Pump wavelength*)
λe := 1064 × 10^-9 (*[m]*) (*Lasing wavelength*)
h := 6.62607015 × 10^-28 (*[J*μs]*) (*Planck constant*)
σ21,em := 28 × 10^-20 (*[cm^2]*) (*Stimulated emission cross section*)
σ03,abs := 7.7 × 10^-20 (*[cm^2]*) (*Absorption cross section*)
τ3 := 0.03 (*[μs]*) (*Lifetime at the top level*)
τ2 := 230 (*[μs]*) (*Lifetime at the metastable level*)
τ1 := 0.01 (*[μs]*) (*Lifetime at the lower laser level*)
Ns := 1.38 (*[10^20 cm^-3]*) (*Concentration of neodymium ions*)
l := 0.3  (*[cm]*) (*Laser crystal length*)
Lc := 20  (*[cm]*)  (*Cavity length*)
γ := 0.002  (*[cm^-1]*) (*Coefficient of inactive losses*)
ρ1 := 0.97 (*Output mirror reflection coefficient*)
ρ2 := 0.95 (*SESAM saturable reflection coefficient*)
nr := 1.82 (*Laser crystal refractive index*)
```

```
c0 := 2.99792458 × 10^4 (*[cm/μs]*) (*Speed of light in the vacuum*)
vc := c0 / nr (*[cm/μs]*) (*Speed of light in the laser crystal*)
Lopt := Lc + (nr - 1) * l (*[cm]*) (*Optical cavity length*)
rtc := c0 / Lopt * (γ * l - Log[ρ1 ρ2] / 2) (*[μs^-1]*)
(*Inverse photon lifetime in the cavity*)
μ := nr * l / Lopt (*Resonator fill factor with active medium*)
d := 0.5 (*[cm]*) (*Diameter of the pump beam on the laser crystal*)
Scr := π * d^2 / 4 (*[cm^2]*) (*Pump area*)
Ppump := 50 (*[W]*) (*Pump power*) (*Threshold pump power - 3.8 W*)
Rpump := Ppump / Scr / h / c0 * λa * 100 / 10^6 (*[
phot/cm^2/μs]*) (*Pump photon flux density*)
Ks := 1 × 10^-10 (*The ratio of spontaneous photons that pass
into the cavity mode*)
S0 := 10^-10 (*[phot/cm^2/μs]*) (*Seed spontaneous emission*)
hνe := h * c0 / λe / 100 (*[J]*) (*Energy of the radiated photon*)
tcalc := 1000 (*[μs]*) (*Calculation time*)
```

The system of equations :

```
In[185]:= (*The system of equations taking into account nonzero population at the lower
laser level N1*)
solvedDyn = NDSolve[
  {S'[t] - vc μ σ21,em S[t] * (N2[t] - N1[t]) × 10^20 - Ks l N2[t] × 10^20 / τ2 + S[t] rtc == 0,
   N3'[t] + N3[t] / τ3 - Rpump σ03,abs (Ns - N1[t] - N2[t] - N3[t]) == 0,
   N2'[t] - N3[t] / τ3 + N2[t] / τ2 + σ21,em S[t] (N2[t] - N1[t]) == 0,
   N1'[t] + N1[t] / τ1 - N2[t] / τ2 - σ21,em S[t] (N2[t] - N1[t]) == 0,
   S[0] == S0, N1[0] == 0, N2[0] == 0, N3[0] == 0},
                          {S[t], N3[t], N2[t], N1[t]},
                          {t, 0, tcalc}]

Out[185]= {{S[t] → InterpolatingFunction[ Domain: {{0., 1.00 × 10^3}} Output: scalar ][t],
  N3[t] → InterpolatingFunction[ Domain: {{0., 1.00 × 10^3}} Output: scalar ][t],
  N2[t] → InterpolatingFunction[ Domain: {{0., 1.00 × 10^3}} Output: scalar ][t],
  N1[t] → InterpolatingFunction[ Domain: {{0., 1.00 × 10^3}} Output: scalar ][t]}}
```

Output laser intensity and relative population inversion :

```
In[186]:=
 AverageIntensity[t_] := Evaluate[S[t] /. solvedDyn[[1]]] * hνe
 (*[MW/cm^2]*) (*Average intensity in the cavity*)
 PopulationInversion[t_] := Evaluate[(N2[t] - N1[t]) /. solvedDyn[[1]]] / Ns
 (*Relative population inversion*)
```

Laser radiation intensity at time t_{calc} :

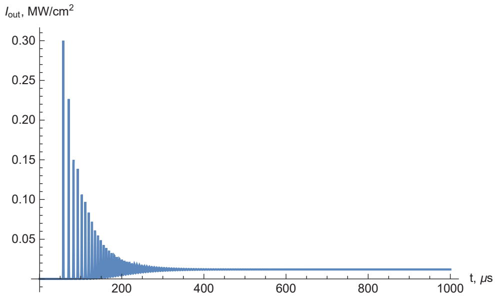
In[188]:=
```
SetPrecision[AverageIntensity[t] /. t → tcalc, 10]
(*[MW/cm²]*) (*Laser output intensity*)
```

Out[188]= 0.01192091664

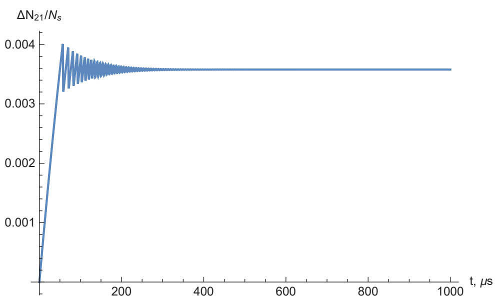
Graphs of the intensity (average value) and population inversion dependence on time:

In[189]:=
```
Plot[{AverageIntensity[t]}, {t, 0, tcalc},
 AxesLabel → {"t, μs", "Iout, MW/cm²"}, PlotRange → Full, PlotLegends → Automatic]
Plot[{PopulationInversion[t]}, {t, 0, tcalc}, PlotRange → Full,
 AxesLabel → {"t, μs", "ΔN21/Ns"}, PlotLegends → Automatic]
```

Out[189]=



Out[190]=



Mode–locking of longitudinal modes

Mode–locking parameters:

In[191]:=
```
ω0 := 2 * π * c0 / λe
Δω := π * c0 / Lopt  (*[μs⁻¹] = [MHz]*)
(*Frequency difference between two subsequent cavity modes*)
ωg := 200 × 10³  (*[MHz]*) (*Gain bandwidth*)
Mg := Floor[ωg / Δω]
(*The number of supported longitudinal modes for the given gain bandwidth*)
M = Mg
```

Out[195]= 42

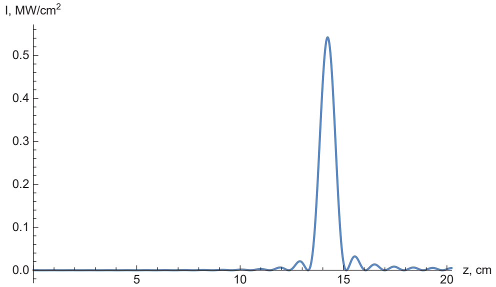
Amplitude of light field in the cavity is calculated as a sum of the longitudinal modes fields with equal amplitudes and equidistant frequencies:

In[196]:=
```
A0[t_] := Sqrt[AverageIntensity[t] / M] (*[MW/cm²]*) (*Intensity of the modes*)
A[z_, t_] := A0[t] * Sum[(Exp[ⅈ Δω * m * (t + z / c0)] + Exp[ⅈ Δω * m * (t - z / c0)]), {m, -M/2, M/2}]
(*Amplitudes of the modes are equal to the
 square root of the average intensity in the cavity*)
```

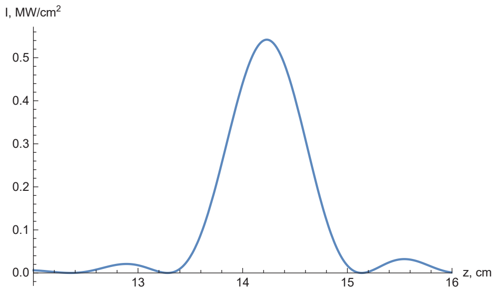
Intensity in the cavity at time t_{calc}:

In[198]:=
```
tcalc := 999.5 (*[μs]*)
Plot[{Evaluate[Abs[A[z, t]]^2 /. t → tcalc]}, {z, 0, Lopt},
 PlotRange → {{0, Lopt}, All}, AxesLabel → {"z, cm", "I, MW/cm²"}]
Plot[{Evaluate[Abs[A[z, t]]^2 /. t → tcalc]}, {z, 0, Lopt},
 PlotRange → {{12, 16}, All}, AxesLabel → {"z, cm", "I, MW/cm²"}]
```

Out[199]=

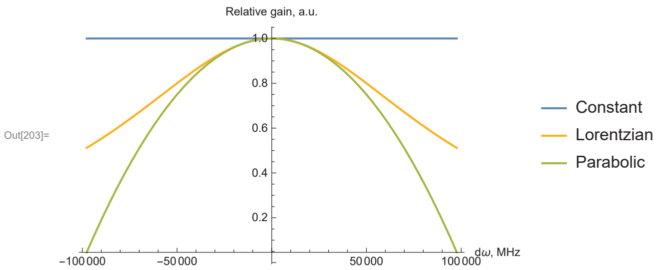


Out[200]=



Consider different mode amplitudes with distribution defined by the gain shape:

In[201]:=
```
gl[ω_] := 1/(1 + ω^2/(ωg/2)^2); (*Lorentzian gain shape*)
gp[ω_] := 1 - ω^2/(ωg/2)^2; (*Parabolic gain shape*)
Plot[{1, gl[ω], gp[ω]}, {ω, -Δω * M/2, Δω * M/2},
PlotRange → All, AxesLabel → {"dω, MHz", "Relative gain, a.u."},
PlotLegends → {"Constant", "Lorentzian", "Parabolic"}]
```



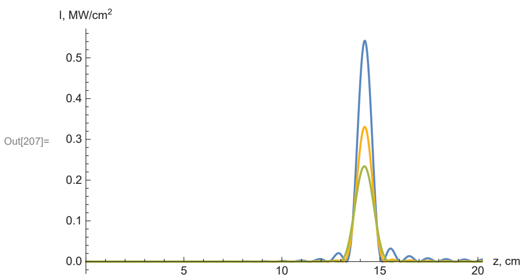
In[204]:=

```
Al[z_, t_] :=
 A0[t] * Sum[m=-M/2, M/2] gl[Δω * m] * (Exp[ⅈ Δω * m * (t + z / c0)] + Exp[ⅈ Δω * m * (t - z / c0)])
(*Lorentzian distribution of the amplitudes*)
Ap[z_, t_] :=
 A0[t] * Sum[m=-M/2, M/2] gp[Δω * m] * (Exp[ⅈ Δω * m * (t + z / c0)] + Exp[ⅈ Δω * m * (t - z / c0)])
(*Parabolic distribution of the amplitudes*)
```

Intensity in the cavity at time t_{calc} in the case of constant, parabolic and Lorentzian gain shape:

In[206]:=

```
tcalc := 999.5 (*[μs]*)
Plot[{Evaluate[Abs[A[z, t]]^2 /. t → tcalc],
  Evaluate[Abs[Al[z, t]]^2 /. t → tcalc], Evaluate[Abs[Ap[z, t]]^2 /. t → tcalc]},
 {z, 0, Lopt}, PlotRange → {{0, Lopt}, All}, AxesLabel → {"z, cm", "I, MW/cm^2"},
 PlotLegends → {"Constant", "Lorentzian", "Parabolic"}]
```



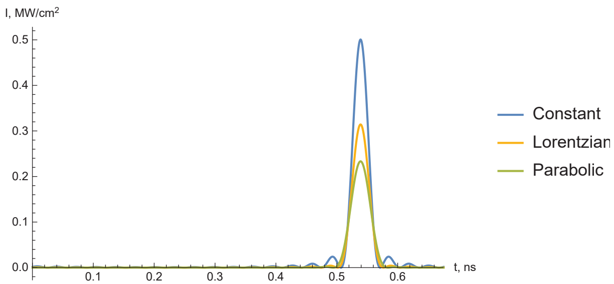
Time dependence of the intensity in the cavity for z=z_{calc} in the case of constant, parabolic and Lorentzian gain shape:

In[208]:=

```
zcalc := Lopt / 2 (*[cm]*)
dt := 2 Lopt / c0 (*[μs]*) (*Cavity round-trip time*)
dtmin = dt / M * 10^6 (*[ps]*) (*Minimal pulse length*)
Plot[{Evaluate[Abs[A[z, t]]^2 /. {z → zcalc, t → tcalc + t * 10^-3}],
  Evaluate[Abs[Al[z, t]]^2 /. {z → zcalc, t → tcalc + t * 10^-3}],
  Evaluate[Abs[Ap[z, t]]^2 /. {z → zcalc, t → tcalc + t * 10^-3}]}, {t, 0, dt / 2 * 10^3},
 PlotRange → {{0, dt / 2 * 10^3}, All}, AxesLabel → {"t, ns", "I, MW/cm^2"},
 PlotLegends → {"Constant", "Lorentzian", "Parabolic"}]
```

Out[210]= 32.1588

Out[211]=



6. Listing of the "Wolfram Mathematica" program for Nd:YAG quasi three level scheme

Nd:YAG Quasi-Three-Level Laser: free lasing mode

Laser and active medium parameters:

```
In[38]:= λa := 808 × 10^-9 (*[m]*) (*Pump wavelength*)
λe := 946 × 10^-9 (*[m]*) (*Lasing wavelength*)
h := 6.626 × 10^-28 (*[J*μs]*) (*Planck constant*)
σ20 := 6.4 × 10^-20 (*[cm^2]*) (*Stimulated emission cross section*) (*6.0×10^-19*)
σ03 := 7.7 × 10^-20 (*[cm^2]*) (*Absorption cross section*) (*7.0×10^-20*)
t3 := 0.03 (*[μs]*) (*Life time at the top level*)
t2 := 230 (*[μs]*) (*Life time at the metastable level*) (*2.2×10^-4*)
Ns := 1.38 (*[10^20 cm^-3]*) (*Concentration of neodymium ions*) (*1.25×10^20*)
Lar := 0.3 (*[cm]*) (*Laser Crystal Length*)
Lc := 20 (*[cm]*) (*Cavity length*)
γ := 0.002 (*[cm^-1]*) (*Inactive loss ratio*)
ρ := 0.97 (*Reflection coefficient of the output mirror*)
nref := 1.82 (*Laser crystal refractive index*)
c := 2.99792458 × 10^4 (*[cm/μs]*) (*The speed of light in vacuum*)
vc := c / nref (*[cm/μs]*) (*The speed of light in matter*) (*1.64×10^10*)
Lopt := Lc + (nref - 1) * Lar (*Optical cavity length*)
rtc := c / Lopt * (γ * Lar - Log[ρ] / 2) (*[μs^-1]*)
(*Inverse photon lifetime in the cavity*) (*3×10^7*)
μ := nref * Lar / Lopt (*Resonator fill factor with active medium*) (*0.007*)
a := 0.25 (*[cm]*) (*The radius of the pump beam on the laser crystal*)
Sar := π * a^2 (*[cm^2]*) (*Pump area*)
Ppump := 100 (*[W]*) (*Pump power*) (*Threshold pump power - 64 Вт*)
Rpump = Ppump / Sar / h / c * λa * 100 / 10^6 (*[phot/cm^2/μs]*)
(*Pump photon flux density*) (*5.0×10^20*)
Ks := 1 × 10^-10 (*The share of spontaneous photons that pass into the cavity mode*)
S0 := 10^-10 (*[phot/cm^2/μs]*) (*Seed spontaneous emission*) (*10^-10 *)
hne := h * c / λe / 100 (*[J]*) (*Radiated photon energy*)
t1 := 1000 (*[μs]*) (*Calculation time*)
Out[59]= 2.07162 × 10^15
```

The relative populations of sublevels depend on temperature:

```
In[64]:= k := 1.3806485 * 10^-23 (*J/K*) (*Boltzmann constant*)
T := 300 (*K*) (*Temperature*)
kT := k T / h / c (*cm^-1*)
E0 := {0, 133, 199, 312, 857} (*cm^-1*) (*Sublevels of 4I9/2 energy*)
f05 = Exp[-E0[[5]] / kT] / Sum[Exp[-E0[[i]] / kT], {i, 1, 5}]
(*Relative upper sublevel population 4I9/2*)
f01 = Exp[-E0[[1]] / kT] / Sum[Exp[-E0[[i]] / kT], {i, 1, 5}]
(*Relative population of lower sublevel 4I9/2*)
E2 := {11427, 11512} (*cm^-1*) (*Energy sublevels 4F3/2*)
f21 = Exp[-E2[[1]] / kT] / Sum[Exp[-E2[[i]] / kT], {i, 1, 2}]
  (*Relative population of lower sublevel 4F3/2*)
Out[68]= 0.00761743
Out[69]= 0.464287
Out[71]= 0.600524
```

The system of equations :

```
In[72]:= (*A system of equations describing generation according to a quasi-
 three-level scheme*)
solvedDyn = NDSolve[{
   {S'[t] - vc μ σ20 S[t] * (f21 n2[t] - f05 n0[t]) × 10^20 -
       Ks Lar n2[t] × 10^20 / t2 + S[t] rtc == 0,
     n3'[t] + n3[t] / t3 - Rpump σ03 n0[t] == 0,
     n2'[t] - n3[t] / t3 + n2[t] / t2 + σ20 S[t] (f21 n2[t] - f05 n0[t]) == 0} /.
    {n3[t] → Ns - n0[t] - n2[t], n3'[t] → -n0'[t] - n2'[t] },
   S[0] == S0, n0[0] == Ns, n2[0] == 0},
                                   {S[t], n2[t], n0[t]},
                                   {t, 0, t1}]

Out[72]= {{S[t] → InterpolatingFunction[ Domain: {{0., 1.00 × 10^3}} Output: scalar ][t],
   n2[t] → InterpolatingFunction[ Domain: {{0., 1.00 × 10^3}} Output: scalar ][t],
   n0[t] → InterpolatingFunction[ Domain: {{0., 1.00 × 10^3}} Output: scalar ][t]}}
```

Output laser intensity and the relative population inversion :

```
In[73]:=
Intensity[t_] := Evaluate[S[t] /. solvedDyn[[1]]] * hne * (1 - ρ)
(*[MW/cm^2]*) (*Laser output intensity*)
PopulationInversion[t_] := Evaluate[(f21 n2[t] - f05 n0[t]) /. solvedDyn[[1]]] /
  Ns (*Relative population inversion*)
```

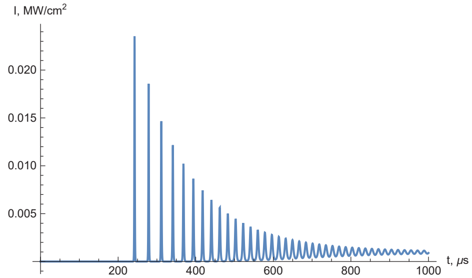
Laser radiation intensity at time t1

```
In[75]:= SetPrecision[Intensity[t] /. t → t1, 10]
Out[75]= 0.0008921400612
```

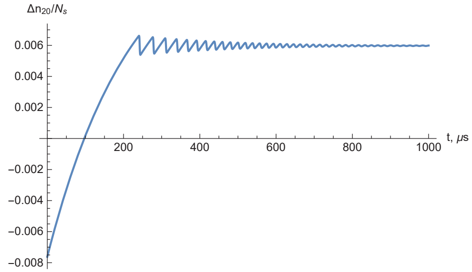
Graphs of the dependence of the intensity and density of populations at levels on time:

```
In[76]:= Plot[{Intensity[t]}, {t, 0, t1}, AxesLabel → {"t, μs", "I, MW/cm²"},
          PlotRange → Full, PlotLegends → Automatic]
         Plot[{PopulationInversion[t]}, {t, 0, t1}, PlotRange → Full,
          AxesLabel → {"t, μs", "Δn₂₀/Nₛ"}, PlotLegends → Automatic]
```

Out[76]=



Out[77]=



7. Listing of the "Wolfram Mathematica" program for Yb:YAG free lasing mode

Yb:YAG Quasi-Three-Level Laser: free lasing mode

Laser and active medium parameters:

```
In[275]:= λa := 940 × 10^-9 (*[m]*) (*Pump wavelength*)
          λe := 1030 × 10^-9 (*[m]*) (*Lasing wavelength*)
          h := 6.626 × 10^-28 (*[J*μs]*) (*Planck constant*)
          σ20 := 2.0 × 10^-20 (*[cm²]*) (*Stimulated emission cross section*)
          σ03 := 7.0 × 10^-20 (*[cm²]*) (*Absorption cross section*)
          t3 := 0.001 (*[μs]*) (*Life time at the top level*)
          t2 := 1200 (*[μs]*) (*Life time at the metastable level*)
          Ns := 1.38 (*[10^20 cm^-3]*) (*Concentration of ytterbium ions*)
          Lar := 0.3  (*[cm]*) (*Laser Crystal Length*)
          Lc := 20  (*[cm]*)  (*Cavity length*)
          γ := 0.002  (*[cm^-1]*) (*Inactive loss ratio*)
          ρ := 0.97 (*Reflection coefficient of the output mirror*)
          nref := 1.82 (*Laser crystal refractive index*)
```

```
c := 2.99792458 × 10^4 (*[cm/μs]*) (*The speed of light in vacuum*)
vc := c / nref (*[cm/μs]*) (*The speed of light in matter*) (*1.64x10^10*)
Lopt := Lc + (nref - 1) * Lar (*Optical cavity length*)
rtc := c / Lopt * (γ * Lar - Log[ρ] / 2) (*[μs^-1]*)
(*Inverse photon lifetime in the cavity*) (*3x10^7*)
μ := nref * Lar / Lopt (*Resonator fill factor with active medium*) (*0.007*)
a := 0.25 (*[cm]*) (*The radius of the pump beam on the laser crystal*)
Sar := π * a^2 (*[cm^2]*) (*Pump area*)
Ppump := 100(*[W]*) (*Pump power*) (*Threshold pump power - 64 Вт*)
Rpump = Ppump / Sar / h / c * λa * 100 / 10^6 (*[phot/cm^2/μs]*)
(*Pump photon flux density*) (*5.0x10^20*)
Ks := 1 × 10^-10 (*The share of spontaneous photons that pass
into the cavity mode*)
S0 := 10^-10 (*[phot/cm^2/μs]*) (*Seed spontaneous emission*) (*10^-10 *)
hne := h * c / λe / 100 (*[J]*) (*Radiated photon energy*)
t1 := 1500 (*[μs]*) (*Calculation time*)
```

Out[296]= 2.41005 × 10^15

The relative populations of sublevels depend on temperature:

```
In[301]:= k := 1.3806485 * 10^-23 (*J/K*) (*Boltzmann constant*)
T := 300 (*K*) (*Temperature*)
kT := k T / h / c (*cm^-1*)
E0 := {0, 581, 619, 786} (*cm^-1*) (*Sublevels of 2F7/2 energy*)
f03 = Exp[-E0[[3]] / kT] / Sum[Exp[-E0[[i]] / kT], {i, 1, 4}]
(*Relative third sublevel population 2F7/2*)
f01 = Exp[-E0[[1]] / kT] / Sum[Exp[-E0[[i]] / kT], {i, 1, 4}]
(*Relative population of lower sublevel 2F7/2*)
E2 := {10327, 10634, 10927} (*cм^-1*) (*Energy sublevels 2F5/2*)
f21 = Exp[-E2[[1]] / kT] / Sum[Exp[-E2[[i]] / kT], {i, 1, 3}]
  (*Relative population of lower sublevel 2F5/2*)
```

Out[305]= 0.0452195

Out[306]= 0.880222

Out[308]= 0.777808

The system of equations:

```
In[309]:= (*A system of equations describing generation according to a quasi-
 three-level scheme*)
solvedDyn = NDSolve[{
   {S'[t] - vc μ σ20 S[t] * (f21 n2[t] - f03 n0[t]) × 10^20 -
      Ks Lar n2[t] × 10^20 / t2 + S[t] rtc == 0,
Ns := 1.38(*[10^20 cm^-3]*) (*Concentration of ytterbium ions*)
Lar := 0.3 (*[cm]*) (*Laser Crystal Length*)
Lc := 20 (*[cm]*) (*Cavity length*)
γ := 0.002 (*[cm^-1]*) (*Inactive loss ratio*)
ρ := 0.97 (*Reflection coefficient of the output mirror*)
nref := 1.82 (*Laser crystal refractive index*)
```

Out[309]= {{S[t] → InterpolatingFunction[ Domain: {{0., 1.50 × 10^3}} Output: scalar][t],

```
n2[t] → InterpolatingFunction[ Domain: {{0., 1.50×10^3}} Output: scalar ][t],
n0[t] → InterpolatingFunction[ Domain: {{0., 1.50×10^3}} Output: scalar ][t]}}
```

Output laser intensity and the relative population inversion :

In[310]:=

```
Intensity[t_] := Evaluate[S[t] /. solvedDyn[[1]]] * hne * (1 - ρ)
(*[MW/cm²]*) (*Laser output intensity*)
PopulationInversion[t_] := Evaluate[(f21 n2[t] - f03 n0[t]) /. solvedDyn[[1]]] /
  Ns (*Relative population inversion*)
```

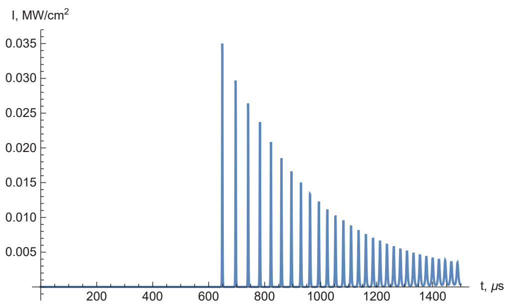
Laser radiation intensity at time t1

```
In[312]:= SetPrecision[Intensity[t] /. t → t1, 10]
Out[312]= 0.0003601702559
```

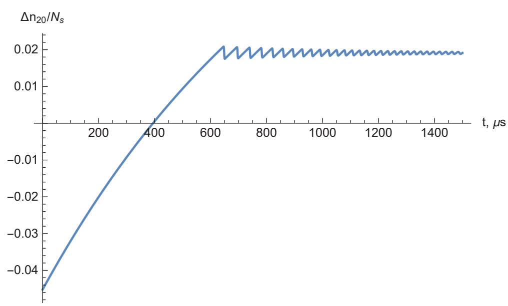
Graphs of the dependence of the intensity and density of populations at levels on time:

```
In[313]:= Plot[{Intensity[t]}, {t, 0, t1}, AxesLabel → {"t, μs", "I, MW/cm²"},
  PlotRange → Full, PlotLegends → Automatic]
Plot[{PopulationInversion[t]}, {t, 0, t1}, PlotRange → Full,
  AxesLabel → {"t, μs", "Δn20/Ns"}, PlotLegends → Automatic]
```

Out[313]=



Out[314]=



8. Listing of the “Wolfram Mathematica” program for Yb:YAG Mode Locking

Yb:YAG Quasi-Three-Level Laser Mode-locking

Average intensity in the cavity

Laser and active medium parameters:

```
In[890]:= λa := 940 × 10^-9 (*[m]*) (*Pump wavelength*)
λe := 1030 × 10^-9 (*[m]*) (*Lasing wavelength*)
h := 6.626 × 10^-28 (*[J*μs]*) (*Planck constant*)
σ20 := 2.0 × 10^-20 (*[cm^2]*) (*Stimulated emission cross section*)
σ03 := 7.0 × 10^-20 (*[cm^2]*) (*Absorption cross section*)
t3 := 0.001 (*[μs]*) (*Life time at the top level*)
t2 := 1200 (*[μs]*) (*Life time at the metastable level*)
Ns := 13.8 (*[10^20 cm^-3]*) (*Concentration of Itterbium ions 10%*)
Lar := 0.3  (*[cm]*) (*Laser Crystal Length*)
Lc := 20  (*[cm]*)  (*Cavity length*)
γ := 0.002  (*[cm^-1]*) (*Inactive loss ratio*)
ρ := 0.97  (*Reflection coefficient of the output mirror*)
nref := 1.82  (*Laser crystal refractive index*)
c := 2.99792458 × 10^4  (*[cm/μs]*) (*The speed of light in vacuum*)
vc := c / nref (*[cm/μs]*) (*The speed of light in matter*) (*1.64×10^10*)
Lopt := Lc + (nref - 1) * Lar  (*Optical cavity length*)
rtc := c / Lopt * (γ * Lar - Log[ρ] / 2) (*[μs^-1]*)
(*Inverse photon lifetime in the cavity*)
μ := nref * Lar / Lopt (*Resonator fill factor with active medium*) (*0.007*)
a := 0.25  (*[cm]*) (*The radius of the pump beam on the laser crystal*)
Sar := π * a^2  (*[cm^2]*) (*Pump area*)
Ppump := 40 (*[W]*) (*Pump power*)
Rpump = Ppump / Sar / h / c * λa * 100 / 10^6
(*[phot/cm^2/μs]*) (*Pump photon flux density*)
Ks := 1 × 10^-10  (*The share of spontaneous photons that pass into the cavity mode*)
S0 := 10^-10  (*[phot/cm^2/μs]*) (*Seed spontaneous emission*)
hne := h * c / λe / 100  (*[J]*) (*Radiated photon energy*)
t1 := 5000  (*[μs]*) (*Calculation time*)

Out[911]= 9.6402 × 10^14
```

The relative populations of sublevels depend on temperature:

```
In[916]:= k := 1.3806485 * 10^-23  (*J/K*) (*Boltzmann constant*)
T := 300 (*K*) (*Temperature*)
kT := k T / h / c  (*cm^-1*)
E0 := {0, 581, 619, 786}  (*cm^-1*) (*Sublevels of ^2F_7/2 energy*)
f03 = Exp[-E0[[3]] / kT] / Sum[Exp[-E0[[i]] / kT], {i, 1, 4}]
(*Relative third sublevel population ^2F_7/2*)
f01 = Exp[-E0[[1]] / kT] / Sum[Exp[-E0[[i]] / kT], {i, 1, 4}]
(*Relative population of lower sublevel ^2F_7/2*)
E2 := {10327, 10634, 10927}  (*cм^-1*) (*Energy sublevels ^2F_5/2*)
f21 = Exp[-E2[[1]] / kT] / Sum[Exp[-E2[[i]] / kT], {i, 1, 3}]
  (*Relative population of lower sublevel ^2F_5/2*)
```

Out[920]= 0.0452195

Out[921]= 0.880222

Out[923]= 0.777808

The system of equations :

```
In[924]:= (*A system of equations describing generation according to a quasi-
 three-level scheme*)
solvedDyn = NDSolve[{
   {S'[t] - vc μ σ20 S[t] * (f21 n2[t] - f03 n0[t]) × 10^20 - Ks μ n2[t] × 10^20 / t2 + S[t] rtc == 0,
     n2'[t] + n2[t] / t2 + σ20 S[t] (f21 n2[t] - f03 n0[t]) - Rpump σ03 f01 n0[t] == 0} /.
    {n2[t] → Ns - n0[t], n2'[t] → -n0'[t] },
   S[0] == S0, n0[0] == Ns , n2[0] == Ns - n0[0]},
                                    {S[t], n2[t], n0[t]},
                                    {t, 0, t1}]
```

```
Out[924]= {{S[t] → InterpolatingFunction[ Domain: {{0., 5.00 × 10^3}} Output: scalar ][t], n2[t] → n2[t],
   n0[t] → InterpolatingFunction[ Domain: {{0., 5.00 × 10^3}} Output: scalar ][t]}}
```

Output laser intensity
and the relative population inversion :

```
In[925]:=
Intensity[t_] := Evaluate[S[t] /. solvedDyn[[1]]] * hne * (1 - ρ)
(*[MW/cm^2]*) (*Laser output intensity*)
PopulationInversion[t_] :=
 Evaluate[(f21 (Ns - n0[t]) - f03 n0[t]) /. solvedDyn[[1]]] / Ns
(*Relative population inversion*)
```

Laser radiation intensity at time t1

```
In[927]:= SetPrecision[Intensity[t] /. t → t1, 10]
SetPrecision[PopulationInversion[t] /. t → t1, 10]
```

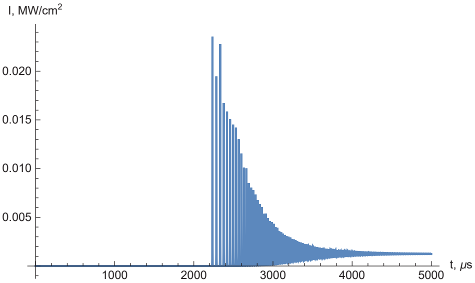
Out[927]= 0.001229412454

Out[928]= 0.001911903402

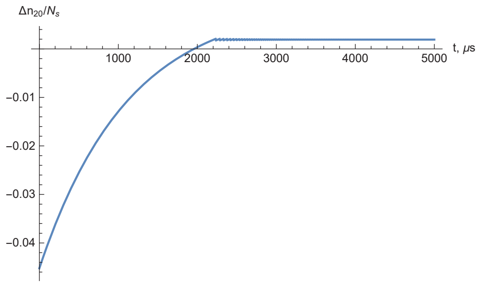
Graphs of the dependence
of the intensity and density-
of populations at levels on time:

```
In[929]:= Plot[{Intensity[t]}, {t, 0, t1}, AxesLabel → {"t, μs", "I, MW/cm^2"},
  PlotRange → Full, PlotLegends → Automatic]
Plot[{PopulationInversion[t]}, {t, 0, t1}, PlotRange → Full,
  AxesLabel → {"t, μs", "Δn20/Ns"}, PlotLegends → Automatic]
```

Out[929]=



Out[930]=



In[931]:=
In[932]:=

Mode–locking of longitudinal modes

Mode–locking parameters:

In[933]:=
```
ω0 := 2 * π * c0 / λe
Δω := π * c / Lopt (*[μs^-1] = [MHz]*)
(*Frequency difference between two subsequent cavity modes*)
ωg := 26 600 × 10^3 (*[MHz]*) (*Gain bandwidth*)
Mg := Floor[ωg / Δω]
(*The number of supported longitudinal modes for the given gain bandwidth*)
M = Mg
```
Out[937]= 5718

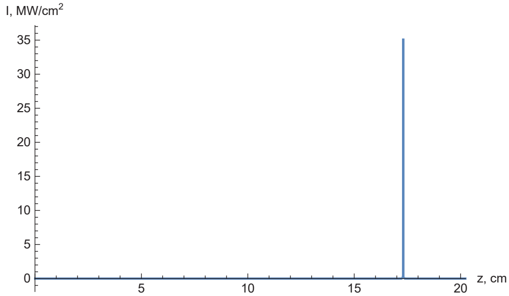
Amplitude of light field in the cavity is calculated as a sum of the longitudinal modes fields with equal amplitudes and equidistant frequencies:

In[938]:=
```
A0[t_] := Sqrt[Intensity[t] / M] (*[MW/cm^2]*) (*Intensity of the modes*)
A[z_, t_] := A0[t] * Sum[(Exp[ⅈ Δω * m * (t + z / c)] + Exp[ⅈ Δω * m * (t - z / c)]), {m, -M/2, M/2}]
(*Amplitudes of the modes are equal to the
 square root of the average intensity in the cavity*)
```

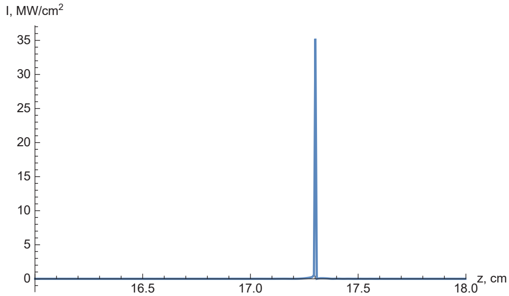
Intensity in the cavity at time t_{calc}:

In[940]:=
```
tcalc := 2999.5 (*[μs]*)
Plot[{Evaluate[Abs[A[z, t]]^2 /. t → tcalc]}, {z, 0, Lopt},
 PlotRange → {{0, Lopt}, All}, AxesLabel → {"z, cm", "I, MW/cm^2"}]
Plot[{Evaluate[Abs[A[z, t]]^2 /. t → tcalc]}, {z, 0, Lopt},
 PlotRange → {{16, 18}, All}, AxesLabel → {"z, cm", "I, MW/cm^2"}]
```

Out[941]=



Out[942]=



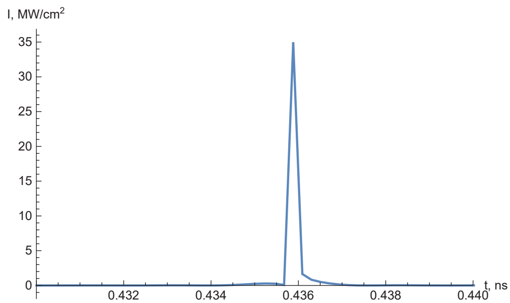
Time dependence of the intensity in the cavity for z=z_{calc} :

In[943]:=

```
zcalc := Lopt / 2 (*[cm]*)
dt := 2 Lopt / c (*[μs]*) (*Cavity round-trip time*)
dtmin = dt / M * 10^6 (*[ps]*) (*Minimal pulse length*)
Plot[{Evaluate[Abs[A[z, t]]^2 /. {z → zcalc, t → tcalc + t * 10^-3}]},
  {t, 0, dt / 2 * 10^3}, PlotRange → {{0.43, 0.44(*dt/2*10^3*)}, All},
  AxesLabel → {"t, ns", "I, MW/cm^2"}, PlotLegends → Automatic]
```

Out[945]= 0.236213

Out[946]=



In[947]:=

In[948]:=

REFERENCES

1. *Kaminskii A. A.* Laser Crystals: Their Physics and Properties. Berlin : Springer, 1990. 459 p.

2. *Svelto O.* Principles of Lasers. 5th ed. London : Springer, 2010. 620 p.

3. *Navitskaya R. I., Stashkevich I. V.* Cavity Dumping by the Second Harmonic Generation in the Mode-Locked Nd : YAG Laser // Nonlinear Phenomena in Complex Systems. 2019. Vol. 22, No. 2. P. 177–189.

4. *Kuleshov N. V., Yasyukevich A. S.* Active media of solid-state lasers: a training manual. Minsk : BNTU, 2010. 133 p.

5. Diode-pumped Lasers on $GdVO_4$ Crystal / V. A. Mikhailov [et al.] // Las Physics. 2003. Vol. 13, No. 3. P. 311–318.

6. Spectroscopic parameters for Nd : YAP [Electronic resource]. URL: www.laser-crylink.com/laser-products/laser-crystal/ndyap (date of access: 07.05.2023).

7. Diode-pumped Nd : GGG laser at 937 nm under direct pumping / Z. G. Zhang [et al.] // Laser Physics. 2011. Vol. 21, No. 10. P. 1745–1749.

8. *Fan T. Y., Byer R. L.* Modeling and CW Operation of a Quasi-Three-Level 946 nm Nd : YAG Laser // IEEE J. Quantum Electron. 1987. Vol. 23, No. 5. P. 605–612.

9. Spectroscopic characterization of Nd : Y_2O_3: application toward a differential absorption lidar system for remote sensing of ozone / B. Walsh [et al.] // J. Opt. Soc. Am. B. 2002. Vol. 19, No. 12. P. 2893–2903.

10. *Zavartsev Yu. D., Zagumenny A. I., Zerouk F.* Quasi-three-level Nd : $GdVO_4$ laser λ = 456 nm with diode pumping // J. Quantum Electronics. 2003. T. 33, No. 7. P. 651–654.

11. *Novitskaya R. I., Stashkevich I. V.* Lasing characteristics of a quasi-three-level Nd : KGW diode-excited laser // Bulletin of BSU. 2016. Ser. 1, No. 3. P. 71–75.

12. *Taira T., Tulloch W. M., Byer R. L.* Modeling of quasi-three-level lasers and operating of cw Yb : YAG lasers // IEEE J. of Quantum Electronics. 1997. Vol. 36, No. 9. P. 1867–1874.

13. Yb : CaF_2 – a new old laser crystal / M. Siebold [et al.] // Applied Physics B. 2009. Vol. 97, No. 2. P. 327–338.

14. *Brovelli L. L. R. , Keller U., Chiu T. H.* Design and operation of antiresonant Fabry – Pérot saturable semiconductor absorbers for mode-locked solid-state lasers // J. Opt. Soc. Am. B. 1995. Vol. 12, No. 2. P. 311–322.

15. Recent advances in ultrafast semiconductor disk lasers / W. Tilma Bauke [et al.] // Light: Science & Applications. 2015. No. 4. P. 310.

16. Laser Crystal – Gain Medium For Solid State Laser [Electronic resource]. URL: https://www.laser-crylink.com/laser-products/laser-crystal (date of access: 06.11.2023).

Учебное издание

Сташкевич Игорь Вячеславович
Новицкая Роза Игоревна

Solid-State Laser Dynamics

Динамика твердотельных лазеров

Пособие

Ответственный за выпуск *Т. М. Турчиняк*
Художник обложки *И. В. Сташкевич*
Дизайн обложки *А. А. Рабкевич*
Технический редактор *В. П. Явуз*
Компьютерная верстка *О. Ю. Шантарович*

Подписано в печать 30.10.2024. Формат 60×84/16. Бумага офсетная.
Печать цифровая. Усл. печ. л. 3,25. Уч.-изд. л. 2,76.
Тираж 40 экз. Заказ 560.

Белорусский государственный университет.
Свидетельство о государственной регистрации издателя, изготовителя,
распространителя печатных изданий № 1/270 от 03.04.2014.
Пр. Независимости, 4, 220030, Минск.

Республиканское унитарное предприятие
«Информационно-вычислительный центр
Министерства финансов Республики Беларусь».
Свидетельство о государственной регистрации издателя, изготовителя,
распространителя печатных изданий № 2/41 от 29.01.2014.
Ул. Кальварийская, 17, 220004, Минск.