**БЕЛОРУССКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ /**
**BELARUSIAN STATE UNIVERSITY**

УТВЕРЖДАЮ / APPROVED
Проректор по учебной работе и образовательным инновациям
/Vice-rector for academic affairs and education innovations
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# ГИДРОДИНАМИЧЕСКАЯ НЕУСТОЙЧИВОСТЬ И ТУРБУЛЕНТНОСТЬ/

# HYDRODYNAMIC INSTABILITY AND TURBULENCE

Учебная программа учреждения образования по учебной дисциплине для специальности:
The program of the educational institution of the discipline for the speciality:

**7-06-0533-06 Механика и математическое моделирование /**
**7-06-0533-06 Mechanics and mathematical modelling**

Профилизация: Теоретическая и прикладная механика /
Profilization: Theoretical and applied mechanics

2023

Учебная программа составлена на основе ОСВО 7-06-0533-06-2023; примерного учебного плана, регистрационный № 7-06-05-017/пр. от 18.01.2023 и учебного плана № М54а-5.4-114/уч. от 11.04.2023.

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Научно-методическим советом БГУ (протокол № 9 от 29.06.2023)

Заведующий кафедрой ____________________ М.А.Журавков

## ПОЯСНИТЕЛЬНАЯ ЗАПИСКА

### Цели и задачи учебной дисциплины

**Целью учебной дисциплины** является получение магистрантами новых знаний по специальным разделам механики жидкости и газа – теории гидродинамической устойчивости и турбулентности с приобретением современных навыков математического и механического моделирования разнообразных физических явлений, аналитического и численного решения задач.

**Задачи учебной дисциплины** «Hydrodynamic instability and turbulence»:

- ознакомиться с основными задачами теории гидродинамической устойчивости и теории турбулентности;
- научиться моделировать задачи устойчивости и турбулентных течений;
- обучиться методам аналитического и численного исследования поставленных задач.
-

**Место учебной дисциплины** в системе подготовки специалиста с углубленным высшим образованием (магистра).

Учебная дисциплина «Гидродинамическая неустойчивость и турбулентность» относится к модулю «Актуальные вопросы современной механики» государственного компонента.

**Связи** с другими учебными дисциплинами, включая учебные дисциплины компонента учреждения высшего образования, дисциплины специализации и др.

Данная дисциплина опирается и использует изученные ранее сведения из дисциплин «Partial differential equations» и «Continuum mechanics».

### Требования к компетенциям

Освоение учебной дисциплины «Гидродинамическая неустойчивость и турбулентность» должно обеспечить формирование следующей углубленной профессиональной компетенции:

УПК. Находить, формулировать и решать актуальные проблемы гидроаэродинамики.

В результате изучения дисциплины «Гидродинамическая неустойчивость и турбулентность» магистрант должен

**знать:**

- общую постановку задачи теории гидродинамической устойчивости и турбулентности;
- методы решения линейных и нелинейных задач теории гидродинамической устойчивости;
- методы решения задач теории гидродинамической устойчивости с неизвестной областью течения;
- модели и методы исследования турбулентных течений.

**уметь:**

- применять полученные знания для исследования устойчивости реальных гидродинамических течений;

- решать классические и новые задачи устойчивости и турбулентности.

**иметь навык:**

- в исследованиях устойчивости гидродинамических течений в линейной и нелинейной постановке;
- в методах решения задач турбулентных течений.

**Структура учебной дисциплины**

Дисциплина изучается во 2 семестре. Всего на изучение учебной дисциплины «Гидродинамическая неустойчивость и турбулентность» отведено:

– для очной формы получения углубленного высшего образования – 216 часов, в том числе 88 аудиторных часов, из них: лекции*– 36 часов, лабораторные занятия*– 52 часа.

Трудоемкость учебной дисциплины составляет 6 зачетных единиц.

Форма промежуточной аттестации по учебной дисциплине – экзамен.

*****Лекции и лабораторные занятия могут проходить с применением дистанционных образовательных технологий (ДОТ).

# EXPLANATORY NOTE

**Aim and tasks of the discipline**

**The purpose of the discipline** is to obtain new knowledge on special sections of fluid and gas mechanics - the theory of hydrodynamic stability and turbulence with the acquisition of modern skills of mathematical and mechanical modelling of various physical phenomena, analytical and numerical solution of problems.

**Objectives** of the academic discipline «Hydrodynamic instability and turbulence»:

- to familiarise with the basic tasks of the theory of hydrodynamic stability and turbulence theory;
- to learn how to model problems of stability and turbulent flows;
- to learn the methods of analytical and numerical investigation of the set problems.

**Place of the academic discipline** in the system of training a specialist with advanced higher education (Master).

The academic discipline belongs to the module «Actual issues of modern mechanics» of the state component.

**Connections** with other academic disciplines, including academic disciplines disciplines of the component of the higher education institution, disciplines of specialisation, etc.

This discipline is based on and uses the previously learnt information from the following disciplines «Partial differential equations» and «Continuum mechanics» disciplines.

**Requirements for competences**

Mastering the academic discipline «Hydrodynamic instability and turbulence» should ensure the formation of the following advanced professional competence:

DPS. Find, formulate und solve actual problems of hydroaerodynamics.

As a result of studying the discipline «Hydrodynamic instability and turbulence» the master student must

**know**:

- general statement of the problem of the theory of hydrodynamic stability and turbulence;
- methods of solution of linear and nonlinear problems of the theory of hydrodynamic stability;
- methods of solving problems of the theory of hydrodynamic stability with unknown flow area;
- models and methods of research of turbulent flows.

**be able to:**

- apply the acquired knowledge to study the stability of real hydrodynamic flows;
- solve classical and new problems of stability and turbulence.

**have skills in:**

- methods of investigation of stability of hydrodynamic flows in the linear and nonlinear formulation;
- models and methods for solving problems of turbulent flows.

**Structure of the academic discipline**

The discipline is studied in the 2nd semester. The total time for studying the academic discipline «Hydrodynamic instability and turbulence» is:

- for full-time higher education - 216 hours, including 88 classroom hours, including: lectures* - 36 hours, laboratory classes* - 52 hours.

The labour intensity of the discipline is 6 credit units.

The form of intermediate certification on the discipline is an exam.

*Lectures and laboratory classes can be conducted with the use of distance learning technologies (DLT).

# CONTENT OF THE STUDY MATERIAL

## Section 1. Problem statement of the hydrodynamic stability theory

Main current. Method of small oscillations. Orr-Sommerfeld equation. The problem on eigenvalues. Squire's theorem.

## Section 2. Methods of solving the linear stability problem

Direct method. Finite difference method. Asymptotic method.

## Section 3. Non-viscous instability

Instability of open currents. Rayleigh equation. Examples of investigation of instability in the mixing layer and jet flow based on the Rayleigh method.

## Section 4. Instability in flows with an interface

Boundary conditions at the interface of viscous and ideal fluids. Stability of two ideal unbounded fluids. Currents in a bounded region. Helmholtz and Taylor-Laplace instabilities.

## Section 5. Flow with velocity gradient

Investigations of steady flow under the action of a tangential force. Stability of flow under the action of tangential force.

## Section 6. A fluid layer on an inclined surface

Study of stability in the nonlinear approximation of the layer under the action of gravity and surface forces. Direct method. Kapitsa-Shkadov equations.

## Section 7. Stability of flow between rotating cylinders

Basic flow. Stability in linear formulation. Formation of Taylor vortices.

## Section 8. Instability and collapse of capillary jets

Basic flow. Investigation of stability in the case of potential flow in the jet taking into account external influence in the linear approximation. Stability of a free jet. Rayleigh solution. Connection of spatially and temporally growing perturbations. Study of stability in the linear approximation in the boundary layer type approximation. Nonlinear development of perturbations in the jet. Development of perturbations taking into account variable external pressure.

**Section 9. Stability and decay of a viscous fluid layer on the outer and inner surface of a rotating cylinder**

Experimental and theoretical studies of layer decay on inner and outer surfaces of a rotating cylinder. Equilibrium forms of axisymmetric and planar layers on the inner and outer surfaces. First integrals. Bifurcations of equilibrium layers on the outer and inner surfaces of a rotating cylinder. Investigation of the stability of plane and axisymmetric layers in the linear approximation. Motion of a layer on the inner and outer surfaces of a cylinder. Kapitsa-Shkadov method for hydrodynamic flows in the field of centrifugal forces. Evolutionary equations and their numerical solution. Nonisothermal problem.

**Section 10. Equilibrium forms, stability and decay of a viscous fluid layer on a rotating disk**

Equilibrium forms of axisymmetric and non-axisymmetric layers on the surface of a rotating disk. Investigation of the steady-state motion of the layer. Experimental and theoretical studies of the motion of a layer on the surface of a rotating disk. Motion of a layer and a viscous liquid drop on a rotating disk. Derivation of the surface evolution equation, its numerical and analytical solutions.

**Section 11. The concept of turbulent flows**

Molecular and turbulent transport. Laminar flow. Instability of laminar flow. Qualitative scenarios of transition to turbulent form of motion. Signs of turbulent flows. Navier-Stokes equations. Hydromechanical similarity laws, similarity criteria, Reynolds number. Critical Reynolds number.

**Section 12. Probabilistic description of turbulent transport**

Methods of averaging of random fields. Probability distribution density functions, averaged and pulsation characteristics of random hydrodynamic quantities. Reynolds equations. Turbulent stress tensor. Ergodicity theorem.

**Section 13 Theory of homogeneous and isotropic turbulence**

Homogeneous, isotropic turbulence. Determination of the spectrum of turbulent pulsations. Estimation of turbulence scales. Intervals of turbulence scales. Kolmogorov's hypotheses of similarity and their consequences. The Law of Five Thirds. Richardson's cascade process of energy transfer.

**Section 14. The turbulence closure problem**

The problem of closure of the Reynolds equations. Closure strategy of higher statistical moments of random hydrodynamic quantities. The turbulent boundary layer and its equations. Semiempirical theories of turbulence. Boussinesq approximation. Prandtl and Karman models. Flow near a wall. Logarithmic velocity profile.

## Section 15. Kinetic energy of turbulence. Turbulent scalar transfer

Definition of the kinetic energy of turbulence. Turbulent energy balance equation. Rate of dissipation of turbulent energy. Turbulent viscosity and thermal conductivity coefficients. Kolmogorov-Prandtl formula. Relationship between heat (mass) and momentum transfer. Model of turbulent scalar transport.

## Section 16. Methods of mathematical modeling of turbulent flows

Methods of modeling turbulent flows. Direct numerical modeling. Method of large vortices. Method of statistical moments. Method of probability distribution density function of random variables.

## Section 17. Differential closure models of the Reynolds equations

First order (algebraic) turbulence models. Second order models. Closure of the equations of kinetic energy of turbulent pulsations and its isotropic dissipation. $k$-ε, $k$-ω models, SST model of shear stresses.

## Section 18. Modeling of large eddies

Filtered Navier-Stokes equations and subgrid stresses. Smagorinsky model. Hermano dynamic model.

## Section 19. Numerical methods for calculating turbulent flows

Numerical methods for modeling turbulent mass, momentum and energy transfer: spectral method, finite volume method, large particle method. Finite volume method for solving hydrodynamic problems. SIMPLE algorithm. Finite volume method for solving problems of convective heat and mass transfer in turbulent flows. Approximation of boundary conditions.

**TEACHING AND METHODOLOGICAL MAP OF THE DISCIPLINE**

Full-time form of higher education with the use of distance learning technologies (DLT)

| Title of section, topic | Title of section, topic | In-class hours | | | | | Independent work | Form of control |
|---|---|---|---|---|---|---|---|---|
| | | Lectures | Practical classes | Seminar classes | Laboratory classes | Other | | |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | **Problem statement of the hydrodynamic stability theory** Main current. Method of small oscillations. Orr-Sommerfeld equation. The problem on eigenvalues. Squire's theorem. | 2 | | | | | | |
| 2 | **Methods of solving the linear stability problem** Direct method. Finite difference method. Asymptotic method. | | | | 2 | | | problem solving |
| 3 | **Non-viscous instability** Instability of open currents. Rayleigh equation. Examples of investigation of instability in the mixing layer and jet flow based on the Rayleigh method. | | | | 2 | | | problem solving |
| 4 | **Instability in flows with an interface** Boundary conditions at the interface of viscous and ideal | 2 | | | 2 | | | written report on laboratory work |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | fluids. Stability of two ideal unbounded fluids. Currents in a bounded region. Helmholtz and Taylor-Laplace instabilities. | | | | | | | |
| 5 | **Flow with velocity gradient** Investigations of steady flow under the action of a tangential force. Stability of flow under the action of tangential force. | 2 | | | 2 | | | problem solving |
| 6 | **A fluid layer on an inclined surface** Study of stability in the nonlinear approximation of the layer under the action of gravity and surface forces. Direct method. Kapitsa-Shkadov equations. | 2 | | | 2 | | | abstract |
| 7 | **Stability of flow between rotating cylinders** Basic flow. Stability in linear formulation. Formation of Taylor vortices. | | | | 2 | | | problem solving |
| 8 | **Instability and collapse of capillary jets** Basic flow. Investigation of stability in the case of potential flow in the jet taking into account external influence in the linear approximation. Stability of a free jet. Rayleigh solution. Connection of spatially and temporally growing | 4 | | | 6 | | | written report on laboratory work |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | perturbations. Study of stability in the linear approximation in the boundary layer type approximation. Nonlinear development of perturbations in the jet. Development of perturbations taking into account variable external pressure. | | | | | | | |
| 9 | **Stability and decay of a viscous fluid layer on the outer and inner surface of a rotating cylinder** Experimental and theoretical studies of layer decay on inner and outer surfaces of a rotating cylinder. Equilibrium forms of axisymmetric and planar layers on the inner and outer surfaces. First integrals. Bifurcations of equilibrium layers on the outer and inner surfaces of a rotating cylinder. Investigation of the stability of plane and axisymmetric layers in the linear approximation. Motion of a layer on the inner and outer surfaces of a cylinder. Kapitsa-Shkadov method for hydrodynamic flows in the field of centrifugal forces. Evolutionary equations and their | 4 | | | 6 | | | problem solving |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | numerical solution. Nonisothermal problem. | | | | | | | |
| 10 | **Equilibrium forms, stability and decay of a viscous fluid layer on a rotating disk** Equilibrium forms of axisymmetric and non-axisymmetric layers on the surface of a rotating disk. Investigation of the steady-state motion of the layer. Experimental and theoretical studies of the motion of a layer on the surface of a rotating disk. Motion of a layer and a viscous liquid drop on a rotating disk. Derivation of the surface evolution equation, its numerical and analytical solutions. | 2 | | | 4 | | | abstract |
| 11 | **The concept of turbulent flows** Molecular and turbulent transport. Laminar flow. Instability of laminar flow. Qualitative scenarios of transition to turbulent form of motion. Signs of turbulent flows. Navier-Stokes equations. Hydromechanical similarity laws, similarity criteria, Reynolds number. Critical Reynolds number. | 2 | | | 2 | | | problem solving |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 12 | **Probabilistic description of turbulent transport** Methods of averaging of random fields. Probability distribution density functions, averaged and pulsation characteristics of random hydrodynamic quantities. Reynolds equations. Turbulent stress tensor. Ergodicity theorem. | 2 | | | 2 | | | written report on laboratory work |
| 13 | **Theory of homogeneous and isotropic turbulence** Homogeneous, isotropic turbulence. Determination of the spectrum of turbulent pulsations. Estimation of turbulence scales. Intervals of turbulence scales. Kolmogorov's hypotheses of similarity and their consequences. The Law of Five Thirds. Richardson's cascade process of energy transfer. | 2 | | | 4 | | | problem solving |
| 14 | **The turbulence closure problem** The problem of closure of the Reynolds equations. Closure strategy of higher statistical moments of random hydrodynamic quantities. The turbulent boundary layer and its equations. Semiempirical | 2 | | | 4 | | | report |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | theories of turbulence. Boussinesq approximation. Prandtl and Karman models. Flow near a wall. Logarithmic velocity profile. | | | | | | | |
| 15 | **Kinetic energy of turbulence. Turbulent scalar transfer** Definition of the kinetic energy of turbulence. Turbulent energy balance equation. Rate of dissipation of turbulent energy. Turbulent viscosity and thermal conductivity coefficients. Kolmogorov-Prandtl formula. Relationship between heat (mass) and momentum transfer. Model of turbulent scalar transport. | 2 | | | 2 | | | problem solving |
| 16 | **Methods of mathematical modeling of turbulent flows** Methods of modeling turbulent flows. Direct numerical modeling. Method of large vortices. Method of statistical moments. Method of probability distribution density function of random variables. | 2 | | | | | | problem solving |
| 17 | **Differential closure models of the Reynolds equations** First order (algebraic) turbulence models. Second order models. | 2 | | | 2 | | | written report on laboratory work |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Closure of the equations of kinetic energy of turbulent pulsations and its isotropic dissipation. *k*-ε, *k*-ω models, SST model of shear stresses. | | | | | | | |
| 18 | **Modeling of large eddies** Filtered Navier-Stokes equations and subgrid stresses. Smagorinsky model. Hermano dynamic model. | 2 | | | 2 | | | problem solving |
| 19 | **Numerical methods for calculating turbulent flows** Numerical methods for modeling turbulent mass, momentum and energy transfer: spectral method, finite volume method, large particle method. Finite volume method for solving hydrodynamic problems. SIMPLE algorithm. Finite volume method for solving problems of convective heat and mass transfer in turbulent flows. Approximation of boundary conditions. | 2 | | | 6 | | | abstract |
| | **TOTAL HOURS** | **36** | | | **52** | | | |

## INFORMATION AND METHODOLOGICAL PART

### List of basic literature


1. Ландау, Л. Д. Теоретическая физика: учебное пособие для студентов физических специальностей университетов : в 10 т. / Л. Д. Ландау, Е. М. Лифшиц; под ред. Л. П. Питаевского. - Изд. 6-е, испр. - Москва : Физматлит, 2021.–Т. 6 : Гидродинамика. - 2021. - 727 с. - URL: https://znanium.com/catalog/document?id=369178.
2. Александров, Д. В. Прикладная гидродинамика: учебное пособие для вузов / Д. В. Александров, А. Ю. Зубарев, Л. Ю. Искакова. - Москва : Издательство Юрайт, 2024. - 109 с.
3. Швед, Г. М. Введение в динамику и энергетику атмосферы: учебное пособие / Г. М. Швед ; Санкт-Петербургский гос. ун-т. - Санкт-Петербург : Изд-во Санкт-Петербургского ун-та, 2020. - 394 с. - URL: https://znanium.com/catalog/document?id=373760.


### List of additional literature


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2. Лойцянский Л.Г. Механика жидкости и газа. Изд. 5. М.: Наука, 1978.
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5. Фриш У. Турбулентность. Наследие А.Н. Колмогорова. М.: Фазис. 1998.
6. Седов Л.И. Механика сплошных сред. Т. 1, т. 2, изд. 5. М.: Наука, 1994.
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9. Черный Г.Г. Газовая динамика. М.: Наука, 1988.
10. Себиси Т., Брэдшоу П. Конвективный теплообмен. М.: Мир. 1987.
11. Шкадов В.Я., Запрянов З.Д. Течения вязкой жидкости. Изд. МГУ, 1984.
12. Дразин Ф. Введение в теорию гидродинамической устойчивости. М.: Физматлит, 2005.
13. Гершуни Г.З., Жуховицкий Е.М. Конвективная устойчивость несжимаемой жидкости. М., Наука, 1972.
14. Бетчов Р., Криминале В. Вопросы гидродинамической устойчивости. М., Мир, 1971.
15. Ван-Дайк М. Альбом течений жидкости и газа. М.: Мир. 1986.
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17. Ши Д. Численные методы в задачах теплообмена. М.: Мир. 1988.
18. Линь Цзя-Цзяо. Теория гидродинамической устойчивости. М: ИЛ, 1958.
19. Гайфуллин А.М. Вихревые течения. М.: Наука, 2015.

## List of recommended diagnostic tools and methodology for final mark formation

The object of diagnostics of Master's students' competences is knowledge, skills acquired by them as a result of studying the academic discipline. Master's students' learning achievements are revealed by means of current control and interim certification.

The following means of current control can be used for diagnostics of competences: written report on laboratory work, abstract, problem solving.

The form of intermediate certification for the discipline «Hydrodynamic instability and turbulence» is an **exam**.

Control activities are carried out in accordance with educational-methodical map of the discipline.

For Master's students who missed the control events or received an unsatisfactory mark, the decision on the repeated carrying out of the control event is made in accordance with the Regulations on the rating system of evaluation of students' knowledge rating system of assessment of students' knowledge of the academic discipline in the Belarusian State University.

The final mark formation in the course of control measures for current certification (approximate weighting coefficients determining the contribution of current certification to the mark for passing interim certification) includes:

- problem solving - 30%;
- abstract - 30 %;
- written report on laboratory work - 40 %.

The final mark for the discipline is calculated on the basis of marking current attestation and examination mark, taking into account their weights coefficients. The weighted mark of the current assessment is 40%, the exam mark is 60%.

## Approximate topics of laboratory classes

**Laboratory work No. 1.** Investigation of jet profile stability by the Rayleigh method.

**Laboratory work No. 2.** Investigation of the stability of the profile of the mixing layer of two streams by the Rayleigh method.

**Laboratory work No. 3.** Instability of flow with the interface of two non-viscous unbounded liquids.

**Laboratory work No. 4.** Instability of flow with the interface of two non-viscous confined liquids.

**Laboratory work No. 5**. Instability of a circular jet of a non-viscous liquid at potential flow in linear approximation.

**Laboratory work No. 6.** Investigation of free surface forms of a layer stationary relative to a rotating cylinder for the plane and axisymmetric case by means of the first integral.

**Laboratory work No. 7.** Investigation of linear instability of a flat liquid layer on the outer surface of a rotating cylinder in a non-viscous formulation.

**Laboratory work No. 8.** Investigation of the universal velocity profile in a viscous sublayer, turbulent boundary layer and trace layer.

**Laboratory work No. 9.** Study of velocity profile in turbulent motion of medium in pipes.

**Laboratory work No. 10.** Study of velocity profile in turbulent motion of medium in jet streams.

## Description of innovative approaches and methods for teaching the discipline

When organising the educational process the following approach is used practice-oriented approach, which implies:

- mastering the content of education through solving practical problems;
- cquisition of skills for effective fulfilment of different types of professional activities;
- orientation to the generation of ideas, implementation of group student projects, development of entrepreneurial culture;
- use of procedures, methods of evaluation, fixing the formation of professional competences.

## Methodological recommendations for the organization of independent work

When studying the discipline the following forms of independent work are assumed:

- search of literature and electronic sources on the individually posed problem of the discipline;
- study of the material assigned for independent work;
- preparation for lectures and laboratory classes;
- work related to the preparation of laboratory reports with oral defence.

Thus, it implies a gradual transformation of learning into independent work, when a master's student should obtain knowledge mainly in the process of creative independent work, independently finding the necessary information and creatively working through it in order to make the necessary inferences and obtain knowledge.

In this case, while performing academic tasks, master's students independently acquire new knowledge, skills and abilities, in particular, the ability to analyse and make decisions in non-standard situations, which is very important for effective future independent professional activity.

## Topics of abstracts

1. Investigations of instability of flows in traces and jets by the Rayleigh method.
2. Investigations of instability in flows with interface.
3. A fluid layer on an inclined surface and its stability.
4. Instability and decay of capillary jets.
5. Investigation of equilibrium forms of a liquid layer on the outer surface of a rotating cylinder.
6. Bifurcation of quasi-stationary solutions of the motion of a liquid layer on the outer surface of a rotating cylinder.
7. Investigation of the stability of a flat layer on the outer surface of a swirling cylinder in the non-viscous approximation.
8. Experimental and theoretical investigations of the decay of a layer on the outer surface of a rotating cylinder.
9. Theory of homogeneous and isotropic turbulence. A.N. Kolmogorov's hypotheses of similarity and their consequences.
10. The problem of turbulence closure. Turbulent Boundary Layer and Its Equations.
11. Methods of mathematical modelling of turbulent flows.
12. Closure models of the Reynolds equations.
13. Modelling of large eddies.
14. Numerical methods for modelling turbulent mass, momentum and energy transport: spectral method, finite volume method, large particle method.

## Approximate list of questions for the exam

1. Basic equations and formulation of the problem on stability of stationary flows. The method of small oscillations.
2. The Orr-Sommerfeld equation. The problem on eigenvalues. Squire's theorem.
3. Investigations of stability of jet flow with profile $U=(ch\ y)^{-2}$
4. Instability of flows in traces and jets. Non-viscous instability. The Rayleigh equation.
5. Rayleigh's method of investigation of non-viscous instability. Examples.
6. Instability in flows with an interface. The Taylor-Laplace instability. Helmholtz instability.
7. Flow under the action of tangential stress, its stability.
8. Layer of fluid on an inclined surface and its stability.
9. Instability and collapse of capillary jets. Stationary solution.

Stability study in linear approximation under potential flow.

10. Instability and collapse of capillary jets. Stability study in the linear approximation at constant pressure in the cross-section.
11. Nonlinear Instability. Nonlinear development of perturbations in an ideal liquid jet.

12. Motion of a liquid layer on the outer surface of a rotating cylinder.Problem statement. Investigation of the forms of relative equilibrium. First integrals in axisymmetric and plane cases. A study of equilibrium forms in natural coordinates.
13. Fluid layer on the outer surface of a rotating cylinder. Branching of quasi-stationary solutions in the linear approximation.
14. Motion of a fluid layer on the outer surface of a rotating cylinder. Bifurcation of quasi-stationary solutions in a nonlinear problem.
15. Investigation of the stability of a flat layer on the outer surface of a twisted cylinder in the non-viscous approximation.
16. Experimental and theoretical studies of the decay of a layer on the outer surface of a rotating cylinder. Derivation of the surface evolution equation.
17. Theoretical investigations of the layer decay on the external surface of a rotating cylinder. Numerical solution of the evolution equations and results.
18. The concept of turbulent flows. Qualitative scenarios of transition to turbulent form of motion. Signs of turbulent flows.
19. Probabilistic description of turbulent transport. Reynolds equations. Turbulent stress tensor. Ergodicity theorem.
20. Theory of homogeneous and isotropic turbulence. Kolmogorov's hypotheses of similarity and their consequences.
21. The problem of turbulence closure. Turbulent boundary layer and its equations. Semi-empirical theories of turbulence. Boussinesq Approximation. Prandtl and Karman models.
22. Kinetic energy of turbulence. Turbulent scalar transport.
23. Methods of mathematical modeling of turbulent flows.
24. Differential models of closure of Reynolds equations.
25. Modeling of large eddies.
26. Numerical methods for modeling turbulent mass, momentum and energy transport: spectral method, finite volume method, large particle method.

## ПРОТОКОЛ СОГЛАСОВАНИЯ УЧЕБНОЙ ПРОГРАММЫ УО

| Название учебной дисциплины, с которой требуется согласование | Название кафедры | Предложения об изменениях в содержании учебной программы учреждения высшего образования по учебной дисциплине | Решение, принятое кафедрой, разработавшей учебную программу (с указанием даты и номера протокола)[1] |
|---|---|---|---|
| дисциплина не требует согласования | | | |

Заведующий кафедрой
теоретической и прикладной механики
д.ф.-м.н., профессор____ ________________ М.А.Журавков

19.05.2023

## ДОПОЛНЕНИЯ И ИЗМЕНЕНИЯ К УЧЕБНОЙ ПРОГРАММЕ ПО ИЗУЧАЕМОЙ УЧЕБНОЙ ДИСЦИПЛИНЕ

на _____/_____ учебный год

| № п/п | Дополнения и изменения | Основание |
|---|---|---|
| | | |

Учебная программа пересмотрена и одобрена на заседании кафедры (протокол № от 202 г.)

Заведующий кафедрой
Д.ф.-м.н., профессор____ ________________ _ М.А.Журавков _____
(степень, звание) (подпись) (И.О.Фамилия)

УТВЕРЖДАЮ
Декан факультета
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