ANTI-PLANE SHEAR WAVES IN ULTRATHIN ELASTIC LAYER ATTACHED TO AN ELASTIC HALF-SPACE TAKING INTO ACCOUNT INTERFACE SLIP AND SURFACE EFFECTS

G.I. Mikhasev[1], V.A. Eremeyev[2], M.G. Botogova[3]

[1]Harbin Institute of Technology, Harbin, China

[2]University of Cagliari, Cagliari, Italy

[3]Belarusian State University, Minsk, Belarus

The phenomenon of anti-plain shear surface waves is very useful for nondestructive damage evaluation of thin coatings, for estimation of surface properties, as well as for modelling of acoustic signal propagation in nanowires and nanofilms. It should be noted that anti-plane shear waves running along a free surface of an elastic body can be detected only under accounting for the surface microstructure. From the physical point of view, the surface elasticity models considering this microstructure describe coupled deformations of an elastic solid body with an elastic ultrathin membrane attached to the body surface. As for the mathematical point of view, an appropriate model predicting such waves includes the classic equation of motion and non-classic boundary conditions taking into account shear stresses and inertia on a free surface.

Nowadays, there are the two different models, Steigmann-Ogden and Gurtin-Murdoch [1] ones, which allow describing the wave motion in an elastic media and bodies considering the surface effects. The justification for the existence of such waves for an elastic half-space, based on a comparative analysis of dispersion relations within the framework of the two approaches "the Gurtin-Murdoch surface elasticity vis-a-vis the lattice dynamics" was given in paper [2]. In this paper, as well as in other contributions, studying anti-plane shear waves, solutions for the wave equations were found in the form of functions exponentially decaying from a free surface. Such modes of anti-plane waves were called the transversally exponential (TE) modes or the TE regime [3]. In [3], it was also revealed for the first time that for an elastic layer with although one free surface, in addition to the TE modes, the so-called TH modes of anti-plane waves are possible, which are characterized by a harmonic variation of amplitudes in the thickness direction of the layer. Later, these two modes were also observed in an elastic ultrathin plate perfectly attached to an elastic half-space [4] as well as in a two-layer elastic plate with rigid interface attachment [5].

The objective of this paper is to study the effect of interlayer slip on anti-plane shear waves in an elastic isotropic ultrathin layer (plate) of thickness h , not perfectly attached to an elastic half-space. Let $Ox_1x_2\,x_3$ be the orthogonal Cartesian coordinate system with an origin at the interface and the Ox_1 - axis coinciding with the direction of wave propagation, the Ox_2 – axis determining the only non-zero components u_1 and u_2 in the plate and half-space, respectively. The equations of shear motion for the two parts of the continuum take the form of wave equations

$$\mu_j\left(u_{j,11} + u_{j,22}\right) = \rho_j \ddot{u}_j, \;\; j = 1,2\;, \tag{1}$$

where μ_j are the shear moduli and ρ_j are the mass densities for the plate in the bulk and half-space, the subscripts $j = 1$ and $j = 2$ stand for the plate and half-space, respectively, the double dot means the second derivative with respect to time, and the subscript k following the comma denotes the differentiation with respect to the coordinate x_k.

The boundary condition on the free surface is given by the relation [1]:

$$\mu_1 u_{1,2} = \mu_s u_{1,22} - \rho_s \ddot{u}_1 \quad \text{at} \quad x_2 = h\;, \tag{2}$$

where μ_s and ρ_s are the surface shear modulus and mass density for the nano-film coating the plate surface, respectively.

At the interface $x_2 = 0$, we assume a slip in the x_3 - direction with the bonding stiffness k_s . Then the boundary conditions at the interface can be expressed by the following two equations:

$$\mu_1 u_{1,2} = \mu_2 u_{2,2}\;, \quad \mu_1 u_{1,2} = k_s(u_1 - u_2) \quad \text{at} \quad x_2 = 0\;. \tag{3}$$

Finally, for the half-space, we set the wave attenuation condition at infinity:

$$u_2 \to 0 \quad \text{as} \quad x_2 \to -\infty\;. \tag{4}$$

We arrived at the boundary-value problem (1)-(4). As follows from paper [4], it admits only two different solutions describing the propagation of anti-plane shear waves, which correspond to the TE-TE and TH-TE regimes of the amplitude variation in the x_2 – direction. For the TE-TE regime, this solution reads

$$u_1 = e^{i(kx_1 - \omega\, t)}\left[a_1 e^{\alpha_1(x_2 - h)} + a_2 e^{-\alpha_1 x_2}\right], \quad u_2 = b e^{i(kx_1 - \omega\, t)} e^{\alpha_2 x_2}\;, \tag{5}$$

while for TH-TE modes, we assume

$$u_1 = e^{i(kx_1 - \omega\, t)}[a_1 \sin\lambda x_2 + a_2 \cos\lambda x_2] \tag{6}$$

with u_2 found in the form of $(5)_2$.

The substitution of the above ansatz into the governing equation (1), with the boundary conditions (2)-(4) taken into account, results in the required dispersion equations related to TE-TE and TH-TE regimes. For instance, for the TH-TE regime, one obtains

$$\frac{|k|l_d(v^2-v_s^2)}{\sqrt{v^2-1}}=\frac{k_s\alpha\mu_2\cos\lambda h-\lambda(\alpha\ \mu_1\mu_2+k_s\mu_1)\sin\lambda h}{k_s\alpha\mu_2\cos\lambda h+\lambda(\alpha\ \mu_1\mu_2+k_s\mu_1)\sin\lambda h}\ , \tag{7}$$

where

$$\lambda=|k|\sqrt{\frac{c^2}{c_{T1}^2}-1},\quad \alpha=|k|\sqrt{1-\frac{c^2}{c_{T2}^2}},\quad v=\frac{c}{c_{T1}},\quad v_s=\frac{c_s}{c_{T1}},$$

$$c=\frac{\omega}{k},\ c_{Tj}=\sqrt{\frac{\mu_j}{\rho_j}},\ c_s=\sqrt{\frac{\mu_s}{\rho_s}},\ l_d=\frac{\rho_s}{\rho_1}.$$

Considering both of the possible regimes of anti-plane shear waves, we plotted the dispersion curves, i. e., the dimensionless velocity v versus the dimensionless wave number $|k|l_d$, for various relative characteristics of the layered continuum in the bulk to the surface counterparts under different values of the bonding stiffness k_s . The main outcomes of our study is the detail analysis of the influence of the interface slip on the dispersion curves for both regimes of anti-plane waves.

The provided analysis may be useful for nondestructive evaluation of materials at small scales and, particularly, for the detection of weak adhesion and delamination of layered materials.

References

1. Gurtin, M. E. , Murdoch, A. I. A continuum theory of elastic material surfaces // Archive for Rational Mechanics and Analysis.- 1975. - V. 57. – P. 291–323 .

2. Eremeyev VA, Sharma BL. Anti-plane surface waves in media with surface structure: Discrete vs. continuum model // International Journal of Engineering Science. - 2019. –V. 143. P. 33–38.

3. Mikhasev, G. I., Botogova, M. G., Eremeyev, V. A. Anti-plane waves in an elastic thin strip with surface energy. Philosophical Transactions of the Royal Society, Series A. – 2022.-V. 380. – P. 20210373–15.

4. Mikhasev, G.I., Erbas, B., Eremeyev, V. A. Anti-plane shear waves in an elastic strip rigidly attached to an elastic half-space // International Journal of Engineering Science. - 2023. – V.184. –P. 103809.

5. Mikhasev, G., Erbaş, B., Jia, F. Anti-plane Waves in an Elastic Two-Layer Plate with Surface Effects // Proceedings of 2023 the 6th International Conference on Mechanical Engineering and Applied Composite Materials. MEACM 2023. Mechanisms and Machine Science (eds.: Yue, X., Yuan, K.). - Singapore: Springer. - 2024. – V. 156.- pp. 33-40.