ASYMMETRIC SCATTERING EFFECT OF SOLITARY WAVE IN A TWO-SECTION COMPOSITE GRANULAR CHAIN

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Introduction

Granular materials, consisting of discrete grains, are commonly encountered in agricultural and industrial activities. These materials exhibit various unique phenomena such as dilute-dense granular flow and segregation of binary mixtures. These phenomena have been extensively observed in grain storage, pharmaceutical mixing, mineral exploitation, and mechanical processing. The ordered granular chain (GC) is a fundamental component used to control and manipulate the propagation of mechanical waves in numerous applications. This includes energy management, signal processing, shock mitigation, and even the design of mechanical logic elements. Understanding the dynamics of nonlinear wave propagation, especially nonlinear scattering at mismatched interfaces, within an ordered granular chain is crucial for theoretical investigations and engineering applications in the field of acoustic metamaterials.

1. Simulation Model

Fig. 1 shows the sketch of a one-dimensional composite GC with two sections. A line of spherical grains N=N_L+N_R with identical diameter d=5 mm is arranged between two fixed walls, i.e., W_L and W_R. In the simulation, the precompression is achieved by adjusting the distance between the left and the right walls. The interaction force without dissipation between two adjacent grains is only considered in the normal direction. Hertz model is used to describe the nonlinear interaction,

$$F_i^{\mathrm{H}} = k_{\mathrm{n}}[\delta_0 + (u_{i-1} - u_i)]_+^{3/2}, \tag{1}$$

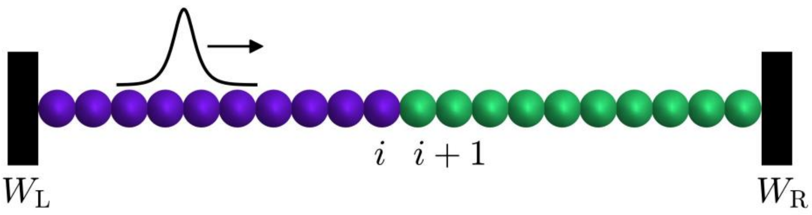


Fig. 1 Sketch of a two-section composite granular chain

In the simulation, the discrete element method is adopted to describe the motion of each grain. The position and velocity of each grain are updated by integrating Newton's second law of motion. The Verlet-velocity algorithm is used in each time step. According to Eq.(1), grain i moves under the following equation,

$$m\frac{d^2u_i(t)}{dt^2} = k_n[\delta_0 + (u_{i-1} - u_i)]_+^{3/2} - k_n[\delta_0 + (u_i - u_{i+1})]_+^{3/2}. \qquad (2)$$

In Table 1, the parameters of granular materials in the left-section GC used in the simulations are listed in Table 1.

Table 1 The parameters of the granular materials

Quantity	Symbol	Value
Diameter (mm)	d	5
Density for grain (g/cm^3)	ρ	7.9
Young's modulus (GPa)	E	193
Poison's ratio	ν	0.25
Simulation time-step (s)	dt	10^{-9}

2. Theoretical Arguments

For a strongly precompressed GC,(u_{i}-u_{i+1})/δ_0 <1. Nesterenko employs a continuum approximation with L>d, where L is the characteristic spatial size of the perturbation to make Eq.2 into a classical KdV equation, and the corresponding SW velocity is analytically derived:

$$V_S = c_0\left(1 + \frac{d\xi_{\max}}{12\delta_0}\right), \qquad (3)$$

The other analysis is conducted for the weakly precompressed GC with (u_i-u_i+1)/(δ_0)>1. It begins at δ_0 ~u_{i}-u_{i+1} as an initial condition. The SW velocity is written as,

$$V_{S0} = \frac{2}{\sqrt{5}}c_{\text{mat}}\xi_{\max}^{1/4}, \qquad (4)$$

When the ISW passes through a mismatched interface, $x=0$, the grains around the interface must be involved simultaneously because the SW has a certain width. In our simulation, the deformation induced by ISW is rather smaller contrasted with the initial precompression, δ_0 for a small amplitude of ISW. Thus, the ISW can be considered traveling across a precompressed two-section composite GC with different material parameters. Mass density and elastic coefficient are regarded to remain constant. In particular, Eqs.(3)(4) indicate that mass density and elastic coefficient have the same effects on the characteristic wave velocities c_0 and c_mat. Thus, we can make the following arguments on the asymmetric scattering effect taken from the mismatched mass density and elastic coefficient by using classical acoustic theory in a linear medium.

The characteristic wave velocity ratio of the $c_0^{\rm R}$ of the right section to $c_0^{\rm L}$ of the left section is introduced,

$$\gamma_{c_0} = c_0^{\rm R}/c_0^{\rm L} = \alpha_\rho^{-1/2}\alpha_{k_{\rm n}}^{1/3}, \tag{4}$$

in which the initial force equilibrium condition is used, $k_{\rm n}^{\rm L}(\delta_0^{\rm L})^{3/2} = k_{\rm n}^{\rm R}(\delta_0^{\rm R})^{3/2}$. .

The acoustic impedance ratio can be defined as:

$$\gamma_{R_{\rm G}} = R_{\rm G}^{\rm R}/R_{\rm G}^{\rm L} = \alpha_\rho^{1/2}\alpha_{k_{\rm n}}^{1/3}. \tag{7}$$

Introducing of the characteristic wave velocity ratio and the acoustic impedance ratio is useful for discussing the asymmetric scattering effect in the two$-$section composite GC with a mismatched interface. First, the mismatch of wave velocity can lead to the ``\emph{fracture}" and ``\emph{squeeze}" effects of the ISW at the interface for $\gamma_{c_0}>1$ and $\gamma_{c_0}<1$, respectively. At the former condition, the single$-$peak ISW decomposes into at least two parts because the leading front ISW first passing across the interface has a larger propagation velocity in the right$-$section GC than that the following part lagging in the left$-$section GC. Under the latter condition, the propagation velocity of the leading front ISW in the right$-$section GC is smaller than that of the following part in the left$-$section GC. The two parts would be squeezed and thus compressive TSWs and RSWs are generated on both sides of the interface of the two$-$section GC. Second, the mismatch of the acoustic impedance can also result in ISW breakdown. Compared with the ``\emph{soft}" right$-$section GC, the left$-$section GC can be considered as a ``\emph{hard}" one when $\gamma_{R_{\rm G}}<1$. In this case, the acoustic pressure of the transmitted wave $P_{\rm Tr}^{\rm R}$ is higher than the incident wave $P_{\rm In}^{\rm L}$. This indicates the occurrence of the overshooting phenomenon when the grains in the RSWs keep the same phase as that in the ISW. The RSWs should be an expansive wave. Under the condition of $\gamma_{R_{\rm G}}>1$, $P_{\rm Tr}^{\rm R}/P_{\rm In}^{\rm L}<1$, $P_{\rm Re}^{\rm L}$ is lower that $P_{\rm Re}^{\rm R}$. The left$-$section GC is ``\emph{soft}", making the RSWs and the TSWs being compressive\cite{Acousticbook01,Acousticbook02,Xu2017Phlow372.20130186

,Daraio2006PRL96.058002}. When $\gamma_{R_{\rm G}}=1$, the reflection is completely suppressed and total transmission occurs.

3. Simulation results

The precompression is set to δ_0 =0. In Fig.2(a), a heavy$-$light GC is used. The ratio of mass density is $\alpha_{\rho}=\rho^{\rm R}/{\rho^{\rm L}}=0.2$, i.e., $\gamma_{c_0}>1$ and $\gamma_{R_{\rm G}}<1$. The mismatched interface is placed between grains 400 and 401. The mass-mismatch interface leads to occurrence of a wave fracture in the right$-$section GC, in which a series of multipulse-structure TSWs arises. In Fig.2(b), the light-heavy chain extends the nonlinear scattering effect to both sides of the interface. In the right$-$section GC, a single FTSW appears, which occupies most of the energy of TSWs. In the left$-$section GC, one single front wave of RSWs(FRSW), following a series of secondary waves.

The precompression is set to δ_0 =10-3. In Fig.3(a), the fracture of the ISW occurs and the TSWs are a multipulse structure as expected. The mismatch of acoustic impedance i.e., $\gamma_{R_{\rm G}}<1$, results in the occurrence of the overshooting effect. A portion of initial potential energy in the left$-$section GC is released and enters into the right$-$section GC. A train of expansive oscillatory RSWs arises. Similar results are repeated for hard-soft chain,

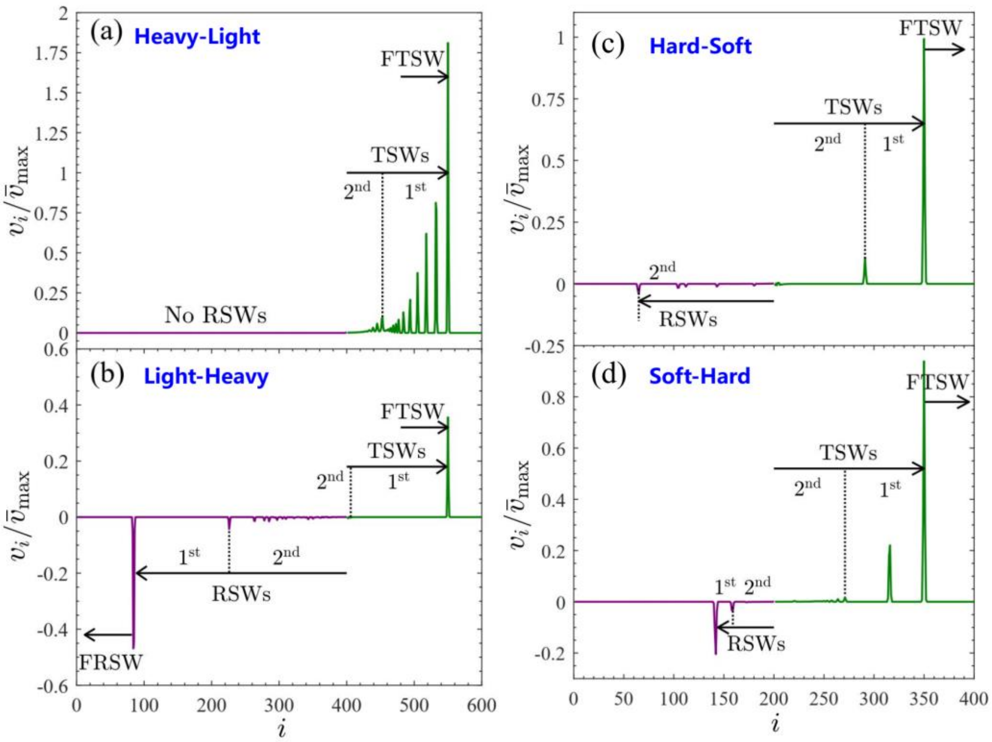
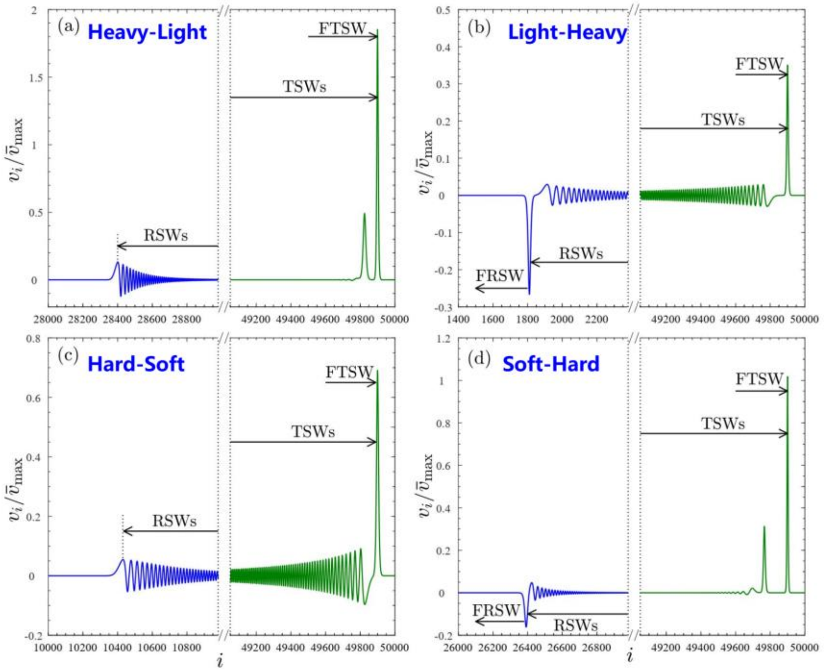


Fig. 2 Grain velocity as a function of grain number in mass$-$mismatch granular chain with precompression $\delta_0/d =0$. (a) Heavy-light chain

$\alpha_{\rho}=0.2$, (b) light-heavy chain $\alpha_{\rho}=5$. (c) Hard-soft chain $\alpha_{k_{\rm n}}=0.2$ (d) Soft-hard chain $\alpha_{k_{\rm n}}=5$.



(a) Heavy-light chain $\alpha_{\rho}=0.2$, (b) light-heavy chain $\alpha_{\rho}=5$. (c) Hard-soft chain $\alpha_{k_{\rm n}}=0.2$ (d) Soft-hard chain $\alpha_{k_{\rm n}}=5$.

Fig. 3 Grain velocity as a function of grain number in elastic$-$coefficient$-$mismatch granular chain with precompression $\delta_0/d=10^{-3}$.

$\alpha_{k_{\rm n}}=0.2$, as shown in Fig. 3(c). For light-heavy chain $\alpha_{\rho}=5$ and Soft-hard chain $\alpha_{k_{\rm n}}=5$, the ISW is decomposed into an FTSW and a multipulse structure SWs, which can exist stably. For the RSWs, the soft characteristic of the left$-$section GC produces a compressive RSWs, composed of a stable FRSW and a train of oscillatory waves dying away.

Conclusions

In this study, the asymmetric scattering effect of SW in a two$-$section composite GC is studied via numerical simulations. Building upon Nesterenko's work on solitary waves in monodisperse granular chains and using continuous acoustic wave theory in linear medium, we argue the intrinsic materials properties, i.e., mass density and elastic coefficient, dominate the asymmetric scattering effect of the mismatched interface. The numerical simulations are then conducted on granular chain.

The Nesterernko work on the characteristics of SW in monodisperse GC is briefly revisit at the unprecompressed and compressed conditions. According to continuous acoustic wave theory in a linear medium, the mismatched acoustic velocity and the impedance at the interface are used to explain the asymmetric scattering effect of the two$-$section GC. When the SW is incident from the section with a small acoustic velocity, the fracture effect occurs, and the TSWs are generated in a multipulse structure. On the contrary, the squeeze effect occurs at the mismatched interface with large$-$small acoustic velocity, in which the transmitted and reflected waves being compressive can be generated. However, the mismatched interface with high$-$low acoustic impedance leads to the occurrence of the overshooting effect, in which the RSWs appear as an expansive wave.

The simulation results confirm that the mismatched acoustic velocity and the impedance at the interface, which are intrinsically determined by the mass density and elastic coefficient, dominate the nonlinear asymmetric scattering effect. When $\gamma_{c_0}>1$, i.e., $\alpha_{\rho}<1$ and $\alpha_{k_{\rm n}}>1$, the fracture effect occurs for the TSWs in which an SW train is observed. The squeeze effect occurs at the mismatched interface with $\gamma_{c_0}<1$, i.e., $\alpha_{\rho}>1$ and $\alpha_{k_{\rm n}}<1$. The overshooting effect is validated for $\gamma_{R_{\rm G}}<1$. The mismatched interface, which is a hard$-$soft one, leads to the generation of the expansive wave in the RSWs. For the unprecompressed GC, the separation gaps arise, and the amplitude of the RSWS are very tiny. The expansive waves, which cannot live long, when the precompression is loaded. For the soft$-$hard interface, i.e., $\gamma_{c_0}>1$, the RSWs propagate in the compressive form.