

# COUPLED SPH-DEM-FEM MODEL FOR INTERACTION BETWEEN WATER, SEA ICE AND MARINE STRUCTURES

Dongbao Yang<sup>1</sup>, Lu Liu<sup>1</sup>, Shunying Ji<sup>1,2</sup>

1. State Key Laboratory of Structural Analysis, Optimization and CAE Software for Industrial Equipment, Dalian University of Technology, Dalian, China

2. DUT-BSU Joint Institute, Dalian University of Technology, Dalian, China

## 1. Introduction

In the interaction between offshore structures and sea ice in polar environment many different physical processes occur e.g. the failure of sea ice, structural vibration, the drift of sea ice, and the collision between sea ice, structure and fluids. These processes include two media: solid and fluid, and three materials: water, ice and structure. From the perspective of different media, this process belongs to a fluid-solid interaction (FSI) problem. However, conventional FSI in ocean engineering mainly focuses on structural elastic response under fluid load, e.g. violent free-surface flow, slamming issues and underwater explosion problems<sup>[1]</sup>. And to solve complex fluid and solid dynamics problems, a series of coupled CFD-FEM methods are developed. In polar environment, sea ice shows strong discrete characteristics, which makes sea ice exist as polygonal discrete particles in space. From the perspective of different materials, this process can be more finely divided into a fluid-particle-structure interaction (FPSI) problem. The FPSI govern a wide range of natural and engineering phenomena in recent years. For example, the impact of debris flow on barriers/structures<sup>[2-4]</sup>, modeling the mill behavior<sup>[5]</sup>, abrasive wear on the mill, and abrasive water jet cutting nozzles<sup>[6]</sup>.

This paper presents a introduces a novel coupled SPH-DEM-FEM model for FPSI is proposed to study water, ice and offshore structures interactions. The offshore structure is modelled using the FEM, the ice media using the DEM and the water using the SPH. Finally, the proposed model is employed to investigate a multi-habitat ship running from water to ice and ice-gonging ships navigated in the broken ice.

## 2. Coupled SPH-DEM-FEM model description

### 2.1 Governing equations for DEM

The DEM used in dynamics model of the granular system is based on the motion of a single particle, and the translation and rotation of the particle are solved according to Newton's second theorem. Equations of particle  $i$ :

$$m_i \frac{d^2 \mathbf{u}_i}{dt^2} = \mathbf{F}_i, \quad (1)$$

$$\mathbf{I}_i \frac{d^2 \boldsymbol{\theta}_i}{dt^2} = \mathbf{M}_i, \quad (2)$$

where,  $\mathbf{u}_i$  and  $\boldsymbol{\theta}_i$  are position and orientation vector for a given particle,  $m_i$  is the mass of particle,  $\mathbf{I}_i$  is the rotary inertia,  $\mathbf{F}_i$  and  $\mathbf{M}_i$  are the external forces and moments acting on the particle, such as body force, buoyancy forces, drags forces and collision force and its moments.

For the simulation of ice model, the 3D sphere and dilated polyhedral DEM with bond and fracture model are employed in the breaking process of ice in ocean engineering, as shown in Figure 1.

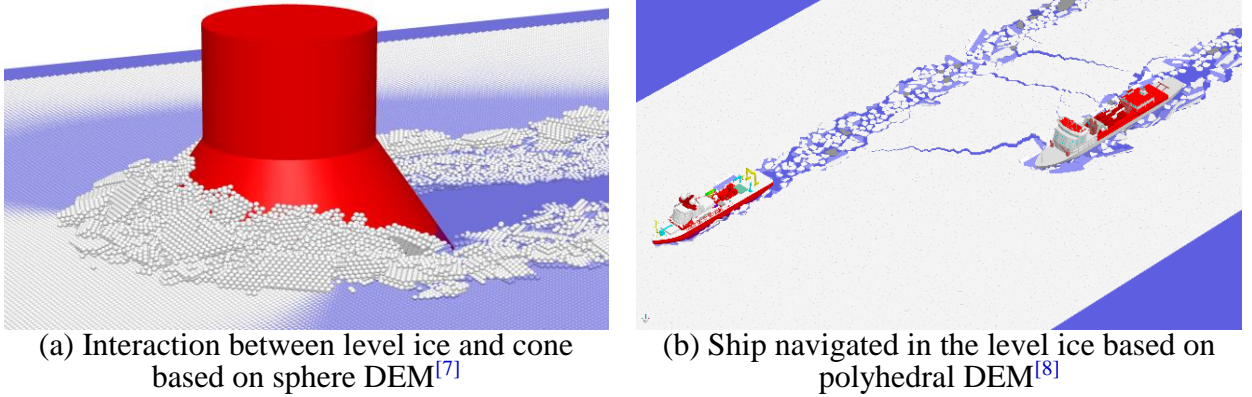


Figure 1. Different element of ice mode based on DEM are applied in ocean engineering

## 2.2 Governing equations for SPH

For incompressible Newtonian fluids, without considering energy loss, the fluid governing equation is as follows:

$$\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0, \quad (3)$$

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v} + \mathbf{g} \quad (4)$$

where,  $\rho$ ,  $p$  and  $\nu$  is the density, pressure and kinematic viscosity of fluids,  $\mathbf{v}$  and  $\mathbf{g}$  is the vector of the fluid velocity and gravity,  $t$  is time. The SPH method discretizes the range of fluid domain by the kernel function, and the fluid governing equation (1) and (2) are transformed from new form according to the particle approximation:

$$\frac{D\rho}{Dt} = \sum_{j=1}^N m_j (\mathbf{v}_i - \mathbf{v}_j) \cdot \nabla_i W_{ij} \quad (5)$$

$$\frac{D\mathbf{v}_i}{Dt} = -\sum_{j=1}^N m_j \left( \frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} \right) \nabla_i W_{ij} + \nu \nabla^2 \mathbf{v} + \mathbf{g} \quad (6)$$

where,  $W_{ij}$  is the kernel function,  $m_j$  is the mass of particle  $j$ ,  $p_i$  and  $p_j$  is the pressure of particle  $i$  and  $j$ . Then, the explicit integration method is used to quickly solve the field variables of the particles to simulate the fluid domain changes.

### 2.3 Governing equations for FEM

The offshore structures in the current work are modelled using the FEM. In this method, the continuous structures are discretized into smaller and simpler “finite elements”. In this way, the partial differential equations on the whole structure dynamics does not need to be solved on the complete spatial domain, but on each small element. In matrix form, the dynamic equation of the structure is expressed as follows:

$$M\ddot{u} + C\dot{u} + Ku = Q \quad (7)$$

where,  $M$ ,  $C$  and  $K$  is mass, damping and stiffness matrix,  $Q$  is the vector of the external force,  $\ddot{u}$ ,  $\dot{u}$ , and  $u$  are the acceleration, velocity and displacement vectors. The explicit central difference method or implicit Newmark method is used to solve the dynamic equation.

### 2.4 Fluid-solid coupling

The solid materials including sea ice DEM and offshore structures FEM at any time is regarded as a solid phase without deformation and motion, and both are considered as the solid boundary of the SPH particle. In the fluid-solid boundary model of SPH, fixed or ghost particles method, boundary integral method, and repulsive force mode have all been used to deal with the contact of SPH particles with irregularly shaped boundaries. For the first two methods, the construction of irregularly DEM elements requires a large number of fixed or ghost particles and integral computation, and the construction algorithm can be cumbersome. In this paper, repulsive force model<sup>[9]</sup> is used to calculate the force between SPH particles and discrete elements, finite element structures, which is expressed as follows.

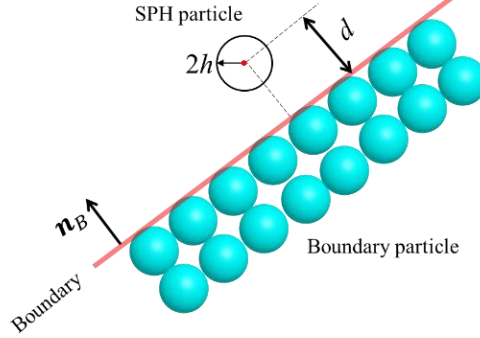
$$f_B = n_B \varepsilon(z) R(d) \quad (8)$$

$$\varepsilon(z) = \begin{cases} 0.02, & z \geq 0 \\ |z/h_0| + 0.02, & -h_0 \leq z < 0 \\ 1, & |z/h_0| > 1 \end{cases} \quad (9)$$

$$R(d) = A \frac{1}{\sqrt{q}} (1 - q) \quad (10)$$

where,  $f_B$  is the repulsive force,  $n_B$  is the normal of contact plane,  $\varepsilon(z)$  is the

balances the pressure at different depths,  $R(d)$  is the repulsive force function,  $h_0$  is the initial fluid depth,  $z$  is the water level and free surface level is defined as the zero, i.e.,  $z = 0$ ,  $A$  is related to the sound speed.



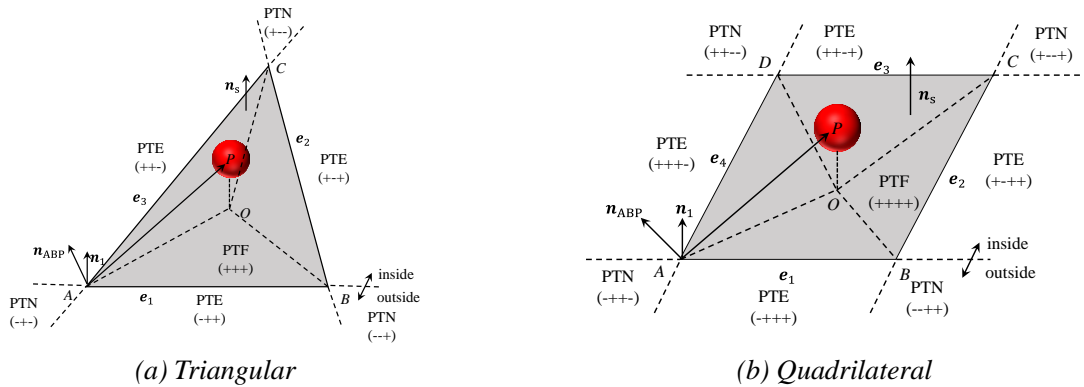
**Figure 2. Repulsive force model of SPH particle and arbitrary solid boundary**

### 2.5 Particles-FEM coupling

Both DEM and SPH are particle methods, and the particles-FEM coupling method mainly includes contact search and load equivalence. Contact search is mainly to calculate the distance and force between the particle and the finite element, as shown in Figure 3. A nonlinear contact model is used to calculate the contact forces between the DEM particles and the finite elements. The contact force is usually applied to any contact point on the element outside the model, and the contact force between the particles and the finite element is equivalent to the element node based on the shape function difference technique. And the function difference technique is defined as follows:

$$\mathbf{f}_i = \mathbf{N}_i \mathbf{F} \quad (i=1, 2, 3 \dots N) \quad (11)$$

where,  $\mathbf{f}_i$  is node force,  $\mathbf{N}_i$  is shape function,  $\mathbf{F}$  is the contact force between the particles and the finite element.

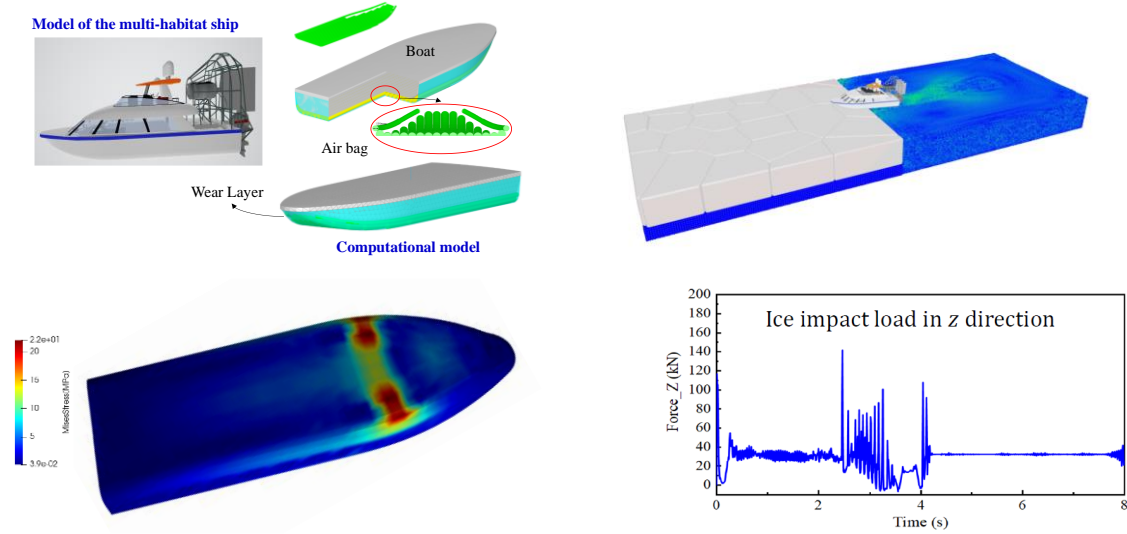


**Figure 3. Contact-search and load equivalence model of spherical element and different element**

### 3. Application for interaction between ice, water, and offshore Structure

#### 3.1 Case1: Simulation of a multi-habitat ship running from water to ice

The coupled SPH-DEM-FEM method is used to simulate the navigation of multi-habitat ship from water to ice. Among them, the multi-habitat ship is composed of four parts, the wear-layer made of PVC at the bottom, the buffer air bag in the middle, the upper hull and the propulsion system. In the computational model, the wear-layer and air bag are modelled using the FEM, the hull is a rigid body, and the propulsion system is simplified. The sea ice is composed of polyhedral discrete elements, and the water is calculated by SPH. The whole simulation process and results are shown in Figure 4.

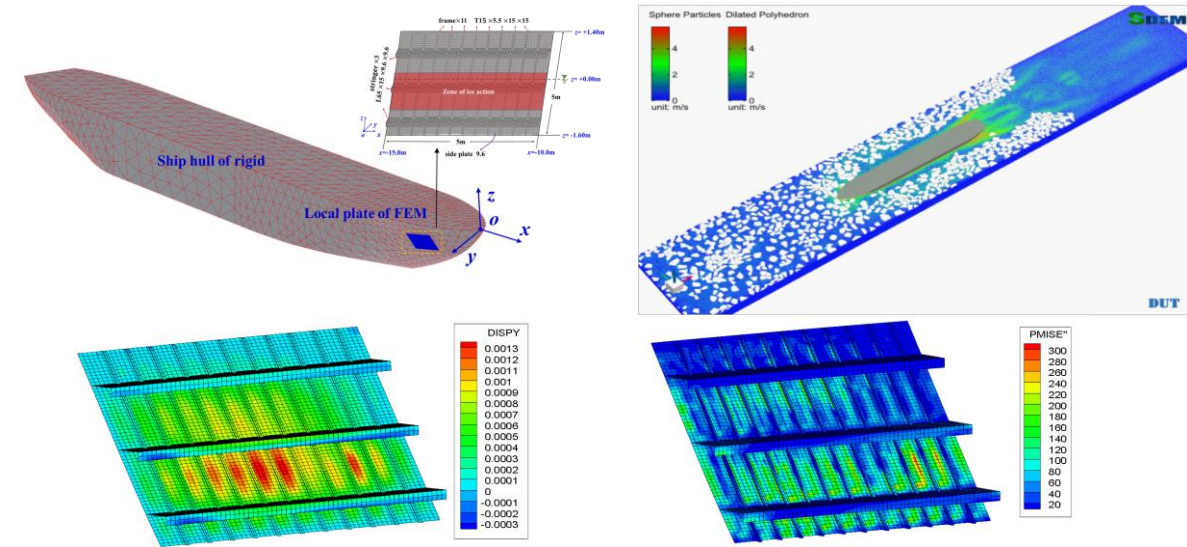


**Figure 4. Simulation of a multi-habitat ship running from water to ice (a) ship model, (b) process of multi-habitat ship running from water to ice (c) von Mises stress distribution of wear layer of ship (d) ice impact load in z direction**

#### 3.2 Case2: Simulation of ice-gonging ships navigated in the broken ice

The SPH-DEM-FEM coupling model is used to simulate the navigation of ships in the ice breaking area. The ice-gonging ship model is mainly divided into two parts: the rigid hull structure constructed by triangular mesh and the local grillage finite element model composed of plate and shell elements. The local grillage structure is located at the waterline height, the port side of the hull and near the shoulder, which is the main position for the collision between the ship and the ice floe. The Voronoi cutting algorithm is used to generate the ice crushing area with a density of 40%, and the broken ice element is polyhedral DEM. The water is calculated by SPH. The whole simulation process and results are shown in Figure 5.





**Figure 5. Simulation of ice-gonging ships navigated in the broken ice (a) model of ice-gonging ship (b) process of ice-gonging ships navigated in the broken ice (c) displacement distribution of y direction (d) von Mises stress distribution**

#### 4. Conclusions

This paper develops a novel coupled SPH-DEM-FEM model for FPSI. In the coupled model, the SPH was employed to simulate the fluid flow, the DEM was employed to model the sea ice and the FEM was used to model the structural deformation. In FSI of SPH, the repulsive force model is used to calculate the force between SPH particles and discrete elements, finite element structures. In particles-FEM coupling, the contact force between the particles and the finite element is equivalent to the element node based on the shape function difference technique. Finally, the proposed model is employed to investigate a multi-habitat ship running from water to ice and ice-gonging ships navigated in the broken ice.

#### Acknowledgments

This study is supported by the National Natural Science Foundation of China (Grant Nos. 42176241, 52192693) and the Ministry of Industry and Information Technology of China (2021-342).

#### References

1. Chen C., Shi W., Shen Y., et al. A multi-resolution SPH-FEM method for fluid-structure interactions[J]. Computer Methods in Applied Mechanics and Engineering, 2022,401:115659.
2. Li B., Wang C., Li Y., et al. Dynamic Response Analysis of Retaining Dam under the Impact of Solid-Liquid Two-Phase Debris Flow Based on the Coupled SPH-DEM-FEM Method[J].Geofluids, 2020, 6635378.

3. Leonardi A., Wittel F., Mendoza M., et al. Particle-fluid-structure interaction for debris flow impact on flexible barriers[J]. *Computer-aided Civil and Infrastructure Engineering*, 2016, 31(5):323-333.
4. Liu C., Liang L., A coupled SPH–DEM–FEM approach for modeling of debris flow impacts on flexible barriers[J]. *Arabian Journal of Geosciences*, 2022, 15(5):1-18.
5. Larsson M J P. A novel approach for modelling of physical interactions between slurry, grinding media and mill structure in wet stirred media mills[J]. *Minerals Engineering*, 2020, 148:106180.
6. Adhav P., Besseron X., Peters B. Development of 6-way CFD-DEM-FEM momentum coupling interface using partitioned coupling approach[J]. *Results in Engineering*, 2024, 22, 102214.
7. Long X, Liu S , Ji S. Discrete element modelling of relationship between ice breaking length and ice load on conical structure[J]. *Ocean Engineering*, 2020, 201(1):107152.
8. Liu L., Ji S. Dilated-polyhedron-based DEM analysis of the ice resistance on ship hulls in escort operations in level ice[J]. *Marine Structures*, 2021, 80:103092.
9. Liu L., Wu J., Ji S., DEM-SPH coupling method for the interaction between irregularly shaped granular materials and fluids[J]. *Powder Technology*: 2022,400: 117249.