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# Development of the automatic procedures for spinor matrix element calculation with massive particles

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# Development of the automatic procedures for spinor matrix element calculation with massive particles

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**Abstract.** We present a new version the procedure for calculation helicity amplitudes with massive particles. First, we introduce a massive 4-momentum into higher dimensional space, where it can be treated as massless. Secondly, we apply the symmetry properties of gauge theories and get quite simple matrix elements in terms of the field strength.

## 1. Introduction

Calculation of matrix elements for processes involving massless particles is proved to be effective using (Weyl) spinors [1, 2]. Especially elegant results can be obtained for massless gauge theories, such as QED and QCD [3]. In the case of massive fermions some successfull results were obtained [4, 5, 6].

In the SANC [7] framework we develop alternative procedure form matrix elements with massive particles. We believe that unconstrained parametrization of massless momentum with a Weyl spinor in one of the ingredients for expressions to be simple. In order to generalize these properties we have to embed massive 4-momentum into higher dimensional space where it can be considered as massless. Formalism for spinors in $d = 6$ dimensions is well developed[8, 9] and relations with familiar Dirac spinors can easily be established.

Another source of matrix element simplification is symmetry properties of gauge theories. Common consideration shows that covariant form is most natural for symmetric object. Polarization vector of gauge boson is not a covariant object but field strength bivector is. So we expect matrix elements to look simpler when expressed in terms of field strength. Gauge-invariant expressions can be obtained by fixing some gauge. Axial gauge is proved to be convenient in massless case so we wish to generalize it for massive vectors.

## 2. Polarization vector and field strength

Field strength bivector is an antisymmetric tensor and can naturally be expressed as an element of Clifford algebra of Dirac matirces by contracting with $\gamma^{[\mu}\gamma^{\nu]} = \gamma^\mu \wedge \gamma^\nu$.

Let's consider photon with 4-momentum $k^2 = 0$ and polarization vector $\varepsilon$. Maxwell bivector (contracted with Dirac matrices) is

$$\mathbf{F} \equiv F_{\mu\nu}\gamma^\mu\gamma^\nu = \not{k} \wedge \not{\varepsilon}$$

Maxwell equation becomes $\not{k}\mathbf{F} = 0$. It is also evident that gauge transfomation $\varepsilon \to \varepsilon + Ck$ lives bivector $\mathbf{F}$ unaffected.

Axial gauge can be defined by additional condition $\varepsilon \cdot g = 0$ with some (massive) vector $g$. Solving it together with $\varepsilon \cdot k = 0$ we obtain polarization vector in axial gauge

$$\not{\varepsilon} = \frac{\langle \not{g}\mathbf{F} \rangle_1}{g \cdot k}, \qquad \langle A \rangle_1 \equiv \overline{\mathrm{Tr}}[A\gamma^\mu]\gamma_\mu, \qquad \not{\varepsilon}(g_1) - \not{\varepsilon}(g_2) = -\frac{\overline{\mathrm{Tr}}[\not{g}_1\not{g}_2\mathbf{F}]}{(g_1 \cdot k)(g_2 \cdot k)}\not{k}, \qquad \overline{\mathrm{Tr}} = \frac{1}{4}\,\mathrm{Tr}\,.$$

Changing vector $g$ leads to gauge transformation.

As an example we obtain matrix element $\mathcal{A} = 2\sqrt{2}(\mathcal{A}^3 + \mathcal{A}^4 + \mathcal{A}^5)$ for process $e^+(p_1) + e^-(p_2) + \gamma(p_3) + \gamma(p_4) + \gamma(p_5) \to 0$ by choosing $g_3 = g_4 = g_5 = p_2$

$$\mathcal{A} = -\frac{\mathrm{Tr}[\not{p}_1\not{p}_2\mathbf{F}_3]}{z_{13}z_{23}z_{24}z_{25}}\bar{v}_1\Big\{\langle \mathbf{F}_4\not{p}_2\mathbf{F}_5\rangle_1 - \not{p}_2\langle \mathbf{F}_4\mathbf{F}_5\rangle_{0,4}\Big\}u_2 + \frac{\bar{v}_1\mathbf{F}_3\Big\{\langle \mathbf{F}_4\not{p}_2\mathbf{F}_5\rangle_1 - \not{p}_2\langle \mathbf{F}_4\mathbf{F}_5\rangle_{0,4}\Big\}u_2}{z_{13}z_{25}z_{24}}$$

Where $z_{ij} = 2p_i \cdot p_j$ and $\langle A \rangle_{0,4} \equiv \overline{\mathrm{Tr}}[A] + \overline{\mathrm{Tr}}[A\gamma_5]\gamma_5$.

It is also possible to calculate "amplitude" for $e^+(p_1) + e^-(p_2) + Z(p_3) + \gamma^*(p_4) \to 0$ with off-shell photon

$$\mathcal{A} = -\frac{\mathrm{Tr}[\not{P}_1\not{P}_2\mathbf{F}_4]}{Z_{14}Z_{24}}\bar{v}_1 e_3 u_2 + \frac{\bar{v}_1\mathbf{F}_4\not{e}_3 u_2}{Z_{14}} + \frac{\bar{v}_1\not{e}_3\mathbf{F}_4 u_2}{Z_{24}}$$

Where $P_1 = p_1 + \frac{p_4}{2}$, $P_2 = p_2 + \frac{p_4}{2}$, $\mathbf{F}_4 = \not{p}_4 \wedge \not{e}_4$, $Z_{14} = 2P_1 \cdot p_4$, $Z_{24} = 2P_2 \cdot p_4$ and $P_1 + P_2 + p_3 = 0$.

## 3. Spinors in 6-dimensions

To make reations with 4-dimensional Dirac spinors as simple as possible we choose metric signature like this $g^{MN} = \mathrm{diag}[g^{\mu\nu}, 1, -1]$. According to [10] we have to introduce dotted Dirac indexes, and corresponding Dirac spinor metric.

$$\acute{\gamma}_{M\alpha}{}^{\dot{\beta}} = \{\gamma_\mu, \gamma_5, +1\}, \qquad \epsilon^{\alpha\dot{\beta}} = \epsilon^{\dot{\beta}\alpha} = \epsilon_{\alpha\dot{\beta}} = \epsilon_{\dot{\beta}\alpha} = \begin{bmatrix} \epsilon^{AB} & 0 \\ 0 & \epsilon_{\dot{A}\dot{B}} \end{bmatrix}$$
$$\grave{\gamma}_{M\dot{\alpha}}{}^{\beta} = \{\gamma_\mu, \gamma_5, -1\},$$

Note, that in $d = 5$ our choice $\acute{\gamma}^6{}_\alpha{}^{\dot{\beta}} =$ "1" allows undistinguish dotted and undotted Dirac spinor indexes. In this case, for example, we can express (Levi-Civita) totally antisymmetric spinor in terms of Dirac spinor metric $\epsilon^{\alpha\beta\gamma\delta} = 3\epsilon^{[\alpha\beta}\epsilon^{\gamma\delta]} = \epsilon^{\alpha\beta}\epsilon^{\gamma\delta} - \epsilon^{\alpha\gamma}\epsilon^{\beta\delta} + \epsilon^{\alpha\delta}\epsilon^{\beta\gamma}$ with $\epsilon^{1234} = 1$.

Spinors in $d = 6$ are $4 \times 2$ matrices, because there are SU(2) little group degree of freedom. We introduced indexless notation, and use spinor metric to upper and lower indexes.

$$|u\rangle\!\rangle = u_\alpha{}^a = \begin{pmatrix} u_A{}^a \\ u^{\dot{A}a} \end{pmatrix} = \begin{pmatrix} |u^a\rangle \\ |u^a] \end{pmatrix}, \qquad \langle\!\langle u| = u_a{}^\alpha = \begin{pmatrix} u_a{}^A & -u_{a\dot{A}} \end{pmatrix} = \begin{pmatrix} \langle u_a| & -[u_a| \end{pmatrix}$$

Every massless (in 6-dimensional sense) can be parametrized by a spinor $\acute{p} \equiv |u\rangle\!\rangle\langle\!\langle u| = |u^a\rangle\!\rangle\langle\!\langle u_a|$. It is also usefull to introduce a dual spinors $\grave{p} = |u^{\dot{a}}]\!][\![u_{\dot{a}}|$. They satisfy orthogonality properties $\grave{p}|u^c\rangle\!\rangle \equiv 0 \Rightarrow [\![u_{\dot{a}}|u^b\rangle\!\rangle = 0$. It is easy to prove the relation between spinor products, which are $2 \times 2$ matrices with little-group indexes and scalar product $\langle\!\langle p|\grave{q}|p\rangle\!\rangle = \langle\!\langle p|q]\!][\![q|p\rangle\!\rangle = 2\acute{p} \cdot \grave{q}$.

Maxwell bivector can compactly be expressed in terms of spinors

$$\acute{\acute{F}}{}^a_{\dot{a}} = \sqrt{2}|k^a\rangle\!\rangle \otimes [\![k_{\dot{a}}|, \qquad \grave{\grave{F}}{}^{\dot{a}}_a = \sqrt{2}|k^{\dot{a}}]\!] \otimes \langle\!\langle k_a|$$

As an example we present an amplitude for $e^+(p_1) + e^-(p_2) + Z(p_3) + \gamma(p_4) \to 0$

$$\frac{\mathcal{A}}{\sqrt{2}} = -\frac{1}{\langle 1|4\rrbracket}\langle 1|2\rrbracket\frac{1}{\langle 4|2\rrbracket}\otimes\langle 1|e_3|2\rangle + \langle 1|e_3|4\rangle\otimes\frac{1}{\langle 2|4\rrbracket} + \frac{1}{\llbracket 4|1\rangle}\otimes\langle 4|e_3|2\rangle$$

In similar way amplitude for $e^+(p_1) + e^-(p_2) + \gamma(p_3) + \gamma(p_4) \to 0$ can be obtained

$$\mathcal{A}/2 = -\frac{1}{\langle 4|1|3\rangle}\otimes\Big(\langle 1|3\rrbracket\otimes\llbracket 4|2\rangle - \langle 1|4\rrbracket\otimes\llbracket 3|2\rangle\Big) + \frac{1}{\llbracket 4|1\rangle}\llbracket 4|3\rangle\otimes\llbracket 3|4\rangle\otimes\frac{1}{\langle 2|4\rrbracket} + \frac{1}{\llbracket 3|1\rangle}\llbracket 3|4\rangle\otimes\llbracket 4|3\rangle\otimes\frac{1}{\langle 2|3\rrbracket}$$

As an illustration of the numerical results we present a plot for pseudorapitity distribution of $\tau$-lepton in process $e^+e^- \to \tau^+\tau^-\gamma$ , obtained with ReneSANCe Monte-Carlo generator and compared with `WHIZARD` (dots) for all polarizations.

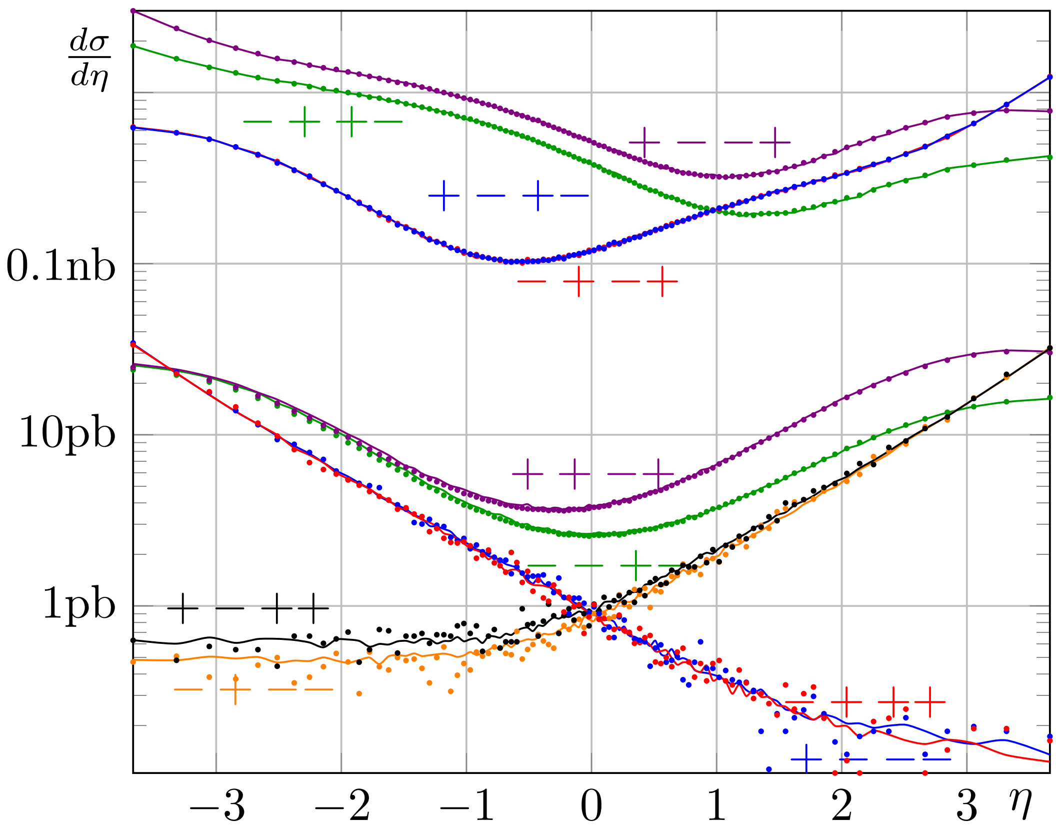


**Figure 1.** Pseudorapitity distribution of $\tau$-lepton in process $e^+e^- \to \tau^+\tau^-\gamma$ at $\sqrt{s} = 500$GeV, obtained with ReneSANCe and `WHIZARD` (dots) for all polarizations of external particles.

## 4. Conclusions

Applying extended set of Clifford-algebra operations we obtained *explicitly gauge-invariant form* of amplitudes for some processes. Expressions contain only field strength bivector and relations to scalar QED and photon power expansion become transparent. We propose a generalized form of axial-type gauge which allows massive gauge-fixing vectors. Simplification of "amplitude" with off-shell photons is also possible. Spinor formalism in $d = 6$ dimensions is applied to obtain modular form of amplitude. It is implemented as `C++17` library and allows pseudo-mass term $\mu\gamma_5$ in Dirac equation, which can be useful to deal with 1-loop integrands.

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