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# Tunable Liquid Crystal Twisted-Planar Fresnel Lens for Vortex Topology Determination

*Elena Melnikova, Yekatsiaryna Pantsialeyeva, Dmitry Gorbach, Alexei Tolstik, and Alina Karabchevsky**

**The development of straightforward and effective methods to determine the topological charge of phase singular beams is crucial for broadening the applications of optical vortices. Here, a straightforward and elegant method for determining the topological charge of an optical vortex using an innovative switchable achromatic nematic liquid crystal Fresnel lens is proposed. The approach allows for the unambiguous determination of both the absolute value and the sign of the topological charge of a singular beam across a wide range of the visible spectrum, as demonstrated both experimentally and theoretically. The research outcomes are promising for applications in optical information transmission systems, cryptography, and quantum computing.**

## 1. Introduction

Research focused on developing methods for determining the topological charge of vortex laser beams, which carry orbital angular momentum, is a highly active and cutting-edge area in the field of contemporary singular optics, spanning both free-space and on-chip optical systems[1] due to their active application in information transmission systems and communication,[2–4] quantum communication,[5,6] microscopy,[7] astronomy,[8,9] manipulation of micro-objects,[10,11] atmospheric sounding systems under turbulence conditions,[12] etc.

Phase vortex beams have a spiral wavefront,[13] which can be represented in the form of $U(x, y) = f(r)e^{i\ell\theta}$.[14] The phase multiplier $e^{i\ell\theta}$ contains an azimuthally varying phase $\ell\theta$, where $\theta$ is the azimuthal coordinate relative to the center of the beam, $\ell$ is the topological charge of the optical vortex. The direction of rotation of the helical wavefront of the phase singular beam is determined by the sign of the topological charge. The amplitude multiplier $f(r)$ can be a Laguerre–Gaussian mode,[15] in the form of $r^m$,[12] some others.[16] Regardless of the form in which the multiplier responsible for the wave's amplitude is written, the intensity in the core of the optical vortex ($r = 0$) takes zero value, and the phase multiplier is not defined.

During a complete detour around the singularity point, the phase changes by a multiple of $2\pi$ equal to $2\pi\ell$, where the number of multiplicities $\ell$ is the topological charge of the light beam. The sign of the topological charge can take both positive and negative values, depending on the direction of the wavefront rotation (clockwise or counterclockwise). If there are several points of phase singularity in the beam, then the topological charge of the beam is equal to the sum of the topological charges of each singularity.

Currently, many experimental methods exist to determine the value and sign of the topological charge of optical vortices.[6,17–47] Most are based on interferometric[6,17–20,22,23,25–34] and diffraction[21,24,35–47] approaches.

One of the most effective and technically simple ways to analyze the topology of singular-phase beams is to directly study the interference pattern obtained by combining an optical vortex with a plane or spherical wave. One of the main disadvantages of interference methods is the need for a reference (coherent signal wave), which makes it impossible, for example, to analyze a signal coming from free space. The interference pattern is also disturbed due to defects and aberrations of optical elements and sensitivity to external influences (vibrations, heat fluctuations, etc.), which requires the interferometer to be constantly adjusted. Recently, interference methods for determining the phase topology of vortex beams without using a reference wave have attracted the attention of researchers.[17]

The measurement of the topological charge by diffraction methods involves the separation of radiation into several beams, followed by analyzing the spatial distribution of intensity maxima and minima. Diffraction methods for determining topological charge are simple and do not require a reference wave, however, they often need special optical elements and most of them have

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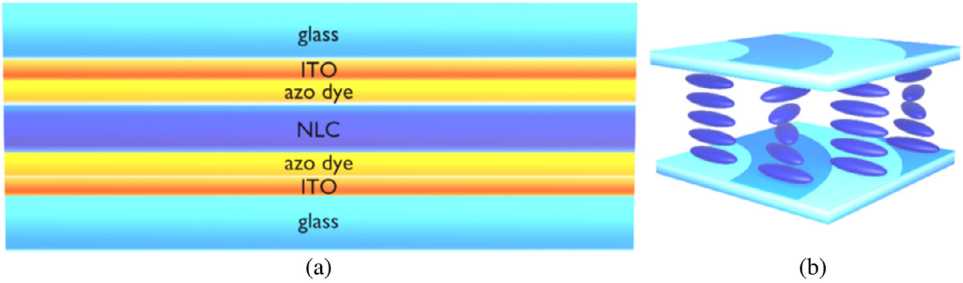


**Figure 1.** a) Schematic diagram of NLC element. b) The orientation distribution of the LC director in the twist-planar NLC structure.

an upper threshold of topological charge values that can be determined with their help.[36,45,47] There are also methods by which it is impossible to determine the sign of a topological charge.[45] Among the diffraction methods, it is worth noting the method of determining the charge by introducing astigmatism into the optical vortex using a cylindrical lens.[46] With its help, the sign and absolute value of the topological charge $\ell = \pm 100$ were experimentally determined.

Here, a new method for unambiguously determining a topological charge's sign and absolute value is proposed and implemented. The process is based on the diffraction of a phase singular beam on an electrically controlled liquid crystal twist-planar Fresnel lens followed by interference of 0 (reference) and ±1 (signal) diffraction orders. A simple and cheap technology has been developed for creating a diffraction liquid crystal device using photo orientation of a liquid crystal using an azo dye AtA-2 sensitive in the blue region of the spectrum.[48] The experimental results obtained are in agreement with the analytical predictions.

## 2. Results

### 2.1. Properties of the Electrically Switchable Twist-Planar NLC Fresnel Lens

The diffraction nematic liquid crystal (NLC) element presented in the work (**Figure 1a**) consists of two glued glass substrates with a capillary gap of thickness $d = 20$ µm. The inner surfaces of the substrates contain a transparent homogeneous conductive indium tin oxide layer (ITO), which allows electrical control of the orientation of the liquid crystal director. The capillary gap was filled with a nematic positive NLC VIN-7 (threshold voltage $U_t = 1.1$ V, optical threshold $U_{op} = 1.5$ V). Periodic orientation of the LC director (Figure 1b) is necessary to ensure the initial modulation of the refractive index. Which can be created by the photoinduced anisotropy of the surface of the orienting coating. In early works,[49–51] twist and planar domains in the LC layer were formed using polymer layers sensitive in the UV region of the spectrum and azo dyes sensitive in the blue region of the spectrum as orientants. Note that the sensitivity of orientants in the blue region of the spectrum simplifies the manufacturing technology of LC elements. In the article,[50] the required activation dose to create an LC element based on the polyimide PI-3744 dye was 5 J cm$^{-2}$. In our work, the AtA-2 azo dye, sensitive in the range 450–520 nm,[48] with an activation dose lower in order of magnitude (0.3 J cm$^{-2}$ to be precise)[52] of dye layer was applied to an electrically conductive coating (the thickness of the photoorientant is ≈30 nm) to orient the LC director. The initial twist-planar topology of the orientation of the director of the LC diffraction element (Figure 1b) represents periodically alternating twist and planar microdomains in the nematic layer (details of the fabrication in the Supporting Information).

The external electric field on the microstructured element determines the optical properties of the NLC of the Fresnel zone plate (**Figure 2**) and the depth of anisotropy modulation ($\Delta n(U)$) (see the Supporting Information). In the range of control voltage values of 0 – 1.5 V, a microstructured anisotropic twist-planar NLC Fresnel lens can be considered as two independent thin amplitude gratings with a rectangular stroke profile (diffraction efficiency of 20% in the entire optical range).[53] When an electric voltage is applied above the threshold value, the Mogen mode[54] is disrupted due to the reorientation of the NLC director along the lines of force (Fredericks transition),[55,56] which is accompanied by the transformation of a binary amplitude thin lattice into a thin phase diffraction grating. The dependence of the diffraction

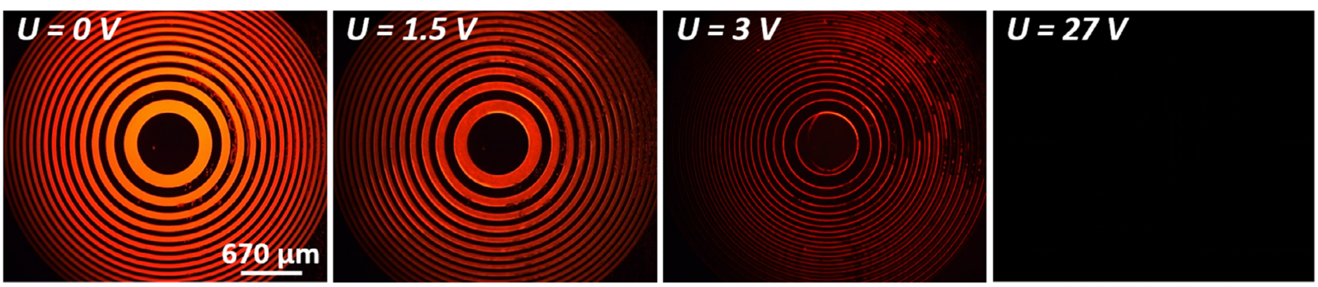


**Figure 2.** Micrographs of an electrically controlled diffraction NLC element in crossed polarizers at different control voltage values ($U$): from 0 to 27 V (from left to right).


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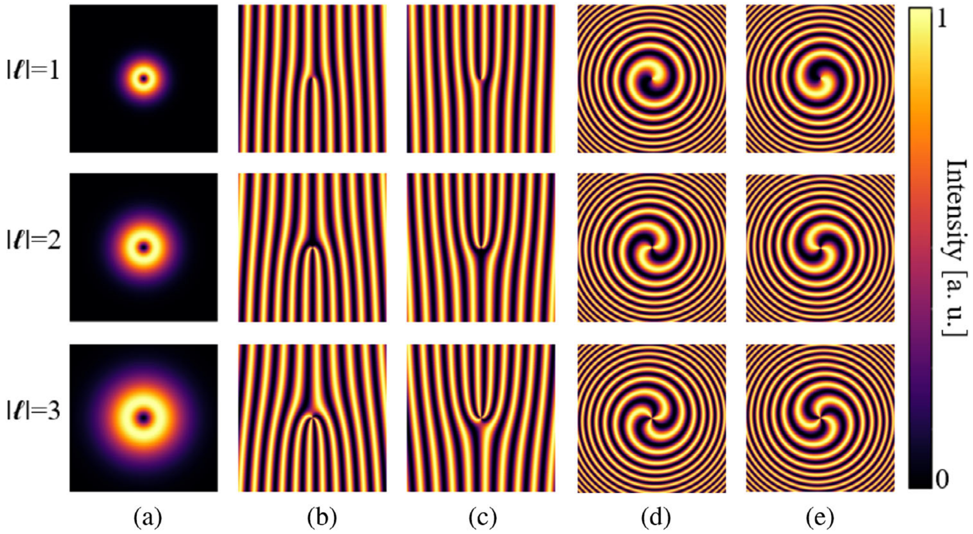


**Figure 3.** Simulation results. Calculated traditional optical phase vortices with topological charge indicated $|\ell|$=1,2,3: a) intensity distribution; b) and c) interference patterns of addition of an optical vortex with a plane; d) and e) a spherical wave; Note (b) and (d) shows negative topological charge, while (c) and (e) show – positive topological charge.

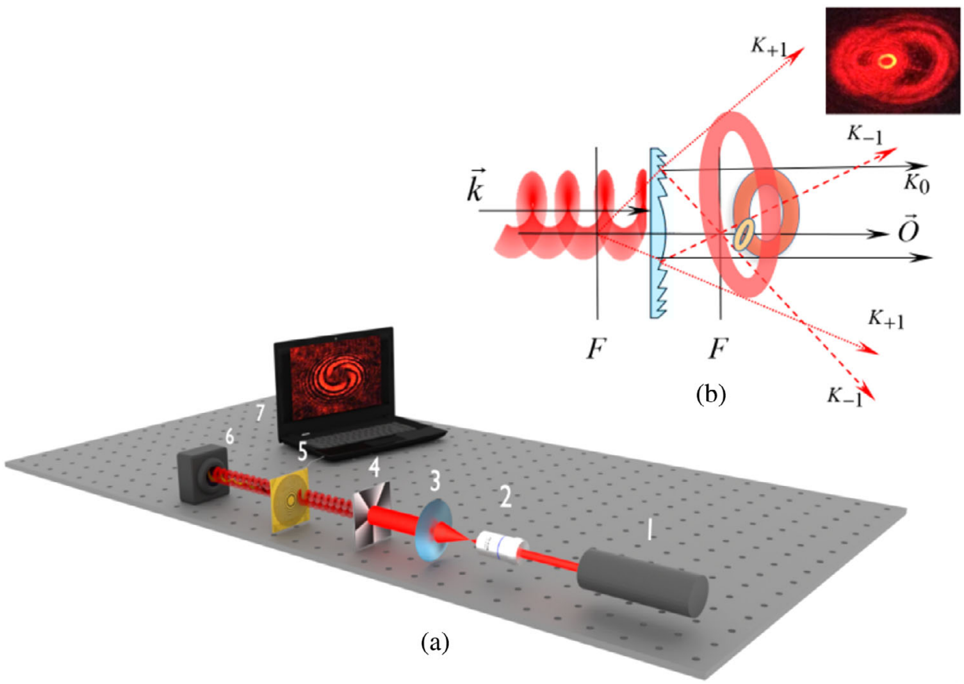


**Figure 4.** a) The experimental setup for determining the topological charge of an optical vortex, where 1 is a He–Ne laser ($\lambda$ = 632.8 nm); 2 is a 20X lens; 3 is a lens; 4 is a vortex phase plate; 5 is a Fresnel NLC lens; 6 is a CCD camera; 7 is a computer. b) Analysis and principle of measuring TC of vortex beams with twist-planar Fresnel lens. The overlap in the space of radiation passing in the –1 order direction and the ring of an optical vortex diffracted in the 0 order direction.

efficiency on the voltage applied to the cell has a local maximum of 30% at a voltage value of the order of 3 V,[53] followed by a monotonic decrease. At a voltage amplitude of 25–27 V, the director's orientation in the entire NLC layer becomes homeotropic and the light beam spreads along the axis of the birefringent crystal with almost no loss, since the diffraction structure in the mesogenic layer disappears.

### 2.2. Determination of the Topological Charge of an Optical Vortex

**Figure 3** shows examples of a widely used method for determining the topology of vortex phase beams by analyzing the interference pattern formed by coherent addition of optical vortices with a plane or spherical reference wave. When a singular phase beam is combined with a plane coherent wave, the formation of


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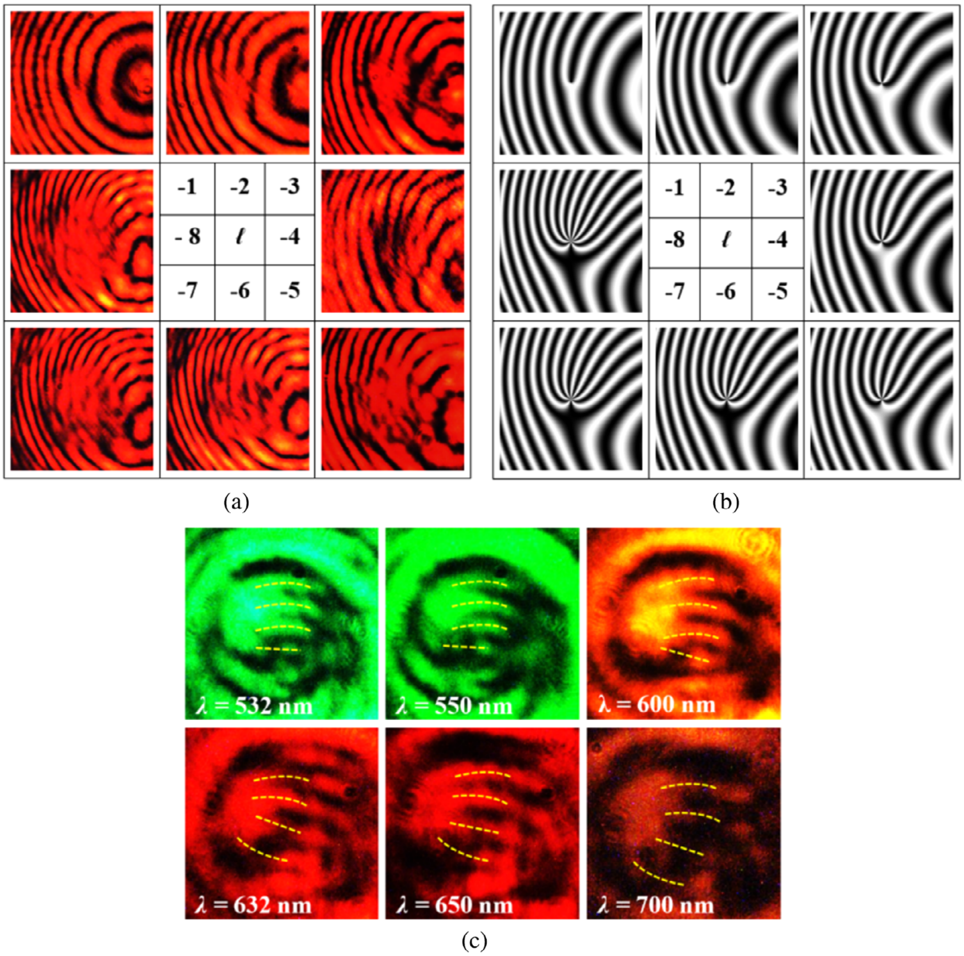


**Figure 5.** Experimental results. Interference patterns of the "core" of an optical vortex with a topological charge $\ell$ passing in the –1 order direction and the "ring" of an optical vortex diffracted in the 0 order direction (a) experimental results, b) theoretical results, c) experimentally obtained interference patterns for determining the topological charge $\ell$ = - 4 of an optical vortex in the spectral range from 532 to 700 nm). The inset in the center $\ell$ labels the topological charge of the experimental subplot images accordingly.

branching interference fringes known as a "fork" is observed. The value of the topological charge is determined by the number of fork branches. The sign is determined by the orientation of the fork in the interference pattern: the downward orientation corresponds to the negative sign of the topological charge, and the upward one corresponds to the positive one. The analysis of the interference pattern of an optical vortex with a coherent spherical wave is a simpler method for determining the magnitude and sign of the topological charge, since it does not require control over the mutual orientation of the wave vectors of interfering beams. In this case, the interference pattern is a spiral, with the number of branches uniquely determining the value of the topological charge of the optical vortex. The direction of the spiral twist in the interference pattern uniquely determines the sign of the topological charge of the phase singular beam: clockwise – positive, counterclockwise – negative. This method is suitable only for investigating the topology of phase beams formed inside one of the arms of the interferometer. We propose an interference method that will allow us to analyze optical vortices coming from outside.

The concept and principle of measuring the topological charge of vortex beams by Fresnel lens are illustrated in **Figure 4**. The vortex beam falls on the Fresnel zone plate at an offset position from the center of the lens Figure 4b. When diffraction of an optical vortex on a Fresnel lens in the directions of +1 and –1 orders of diffraction, divergent and converging waves with a topological charge equal to the charge of the original wave are formed, while the forward wave completely preserves its phase topology. Such a diffraction geometry of the optical vortex on the element makes it possible to realize in space an effective coherent addition of the vortex core (radiation propagating in the direction of –1 order of diffraction - a converging wave) and the "ring" of the vortex diffracted in the direction of 0 order. Thus, when an optical vortex passes through a Fresnel zone plate in such a geometry, a converging signal wave (an optical vortex is a –1–order diffraction wave) and a coherent quasi-plane reference wave (a ring of an optical vortex propagating in the direction of the 0th diffraction order) are formed.

The work proposes using an electrically controlled liquid crystal structure as a Fresnel lens. The developed structure makes it


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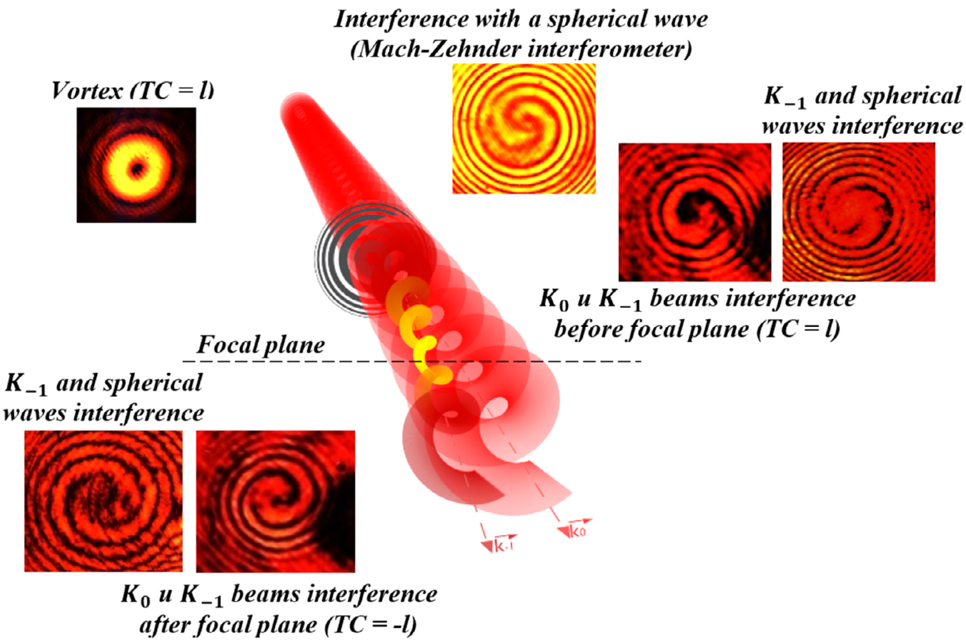


**Figure 6.** Analysis and principle of measuring TC of vortex beams with twist-planar Fresnel lens. The overlap in space and the interference pattern of radiation passing in the –1 order direction and the ring of an optical vortex diffracted in the 0 order direction.

possible to evaluate the topology of the wavefront of a laser beam in real-time. It does not require changing the circuit to determine the topological charge.

An experimental demonstration of the determination of the topological charge of a singular light beam using the developed element was carried out on the installation shown in Figure 4a. After passing the collimating system *2, 3*, the radiation of the He-Ne laser *1* propagated through the phase plate *4* to form a singular light beam. The phase vortex passed through the NLC Fresnel lens *5* at an offset position from the center, behind the focal plane of which the charge-coupled device (CCD) camera *6* was located. Information about the intensity distribution in the interference pattern was displayed on the computer's screen connected to it *7*.

To test the proposed method, the topological charges of phase vortices were measured experimentally (**Figure** 5a) and simulated theoretically (Figure 5b). Based on the interference pattern containing a characteristic "twisted fork", it is possible to accurately determine the topological charge of the optical vortex. The number of forks and the direction of the twist of the "twisted fork" allows you to uniquely determine the charge. The calculations were carried out without taking into account the amplitude component of the intensity, which would turn to 0 in the center of the optical vortex and complicate the determination of the topological charge.

Figure 5c shows interference patterns in the spectral range from 532 to 700 nm obtained using laser radiation with a parametric frequency converter. The conducted studies of a phase singular beam with a topological charge $\ell = -4$ illustrate the effective operation of the proposed LC element in the visible range. The deterioration in image quality is explained by the sensitivity of the CCD camera.

**Figure** 6 shows a scheme for the propagation of beams of 0 and –1 diffraction orders and experimental results for determining the topological charge of an optical vortex propagating in the direction of –1 diffraction order (converging wave), based on the proposed method performed before and after the focal plane of the Fresnel lens. As can be seen, before the optical element's focal plane, the vortex's charge sign entering the element is preserved, after it is inverted.

As mentioned earlier, the wavefront of an optical vortex has a helical shape, while the phase distribution $\phi$ is given by a variable function $\ell\theta$, where $\theta$ is the polar angle. The transverse phase gradient of this vortex is circulating, and near its core, it has a very large magnitude, which leads to uncertainty of the wave phase and, consequently, zero amplitude in its core. At the periphery of the beam (away from the core), the gradient has an insignificant value, and the wavefront can be considered quasi-flat. Thus, the coherent addition of the vortex core (–1 diffraction order) $E^{-1}(r, \theta)$ with the peripheral part of the optical vortex $E^{0}(r, \theta)$ ($0^{\text{th}}$ diffraction pattern) makes it possible to form an interference pattern characterizing the topological charge of the beam $E^{-1}(r, \theta)$. Thereby, the considered interference method eliminates the need for an additional reference wave and allows direct analysis of the phase topology of the vortex without an interferometer.

## 3. Conclusion

An interference method for determining the topological charge of a phase singular light beam based on optical vortex diffraction on an electrically controlled liquid crystal twist-planar Fresnel lens has been developed. Topological charges from $\ell = \pm 1$ to $\pm 8$ of optical vortices have been determined experimentally and theoretically. An experimental approbation of the proposed method for pulsed laser radiation of nanosecond duration in a wide range of the visible part of the spectrum has been carried out. The developed twist-planar NLC element allows you to con-


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trol the intensity of focused radiation and turn the lens on and off using an external low-power voltage. The maximum focusing capacity of the LC element corresponds to a voltage of ≈3 V.

The proposed method makes it possible to unambiguously determine the sign and magnitude of the topological charge of phase singular beams. An easy-to-manufacture and miniature NLC Fresnel lens is required for its implementation, thanks to which reference and signal waves are simultaneously formed. When determining the topology of the vortex wavefront, no additional elements are required since the external electrical control of the LC element allows the diffraction structure to be turned on/off. The method can find applications in optical cryptographic systems that perform quantum computing and information transfer.

## 4. Experimental Section

*Fabrication of the Electrically Controlled Twist-Planar Fresnel Lens*: The concept of the NLC Fresnel lens is schematically shown in Figure 1 and the preparation procedure in the Supporting Information. A photo of the NLC zone plate is also shown in the Supporting Information. Glass substrates with a transparent electrically conductive ITO coating with a resistance of 100 Ohm square$^{-1}$ and a thickness of 1.1 mm were used for the fabrication of the LC element (INTEGRAL OJSC, Minsk, Belarus). The cutting of glass substrates with a size of 30 × 20 mm was carried out using a diamond roller cutter (Bohle AG).

The surfaces of the glass substrates were cleaned in an ultrasonic bath in three stages: once in an aqueous (distilled water) surfactant solution at a temperature of $T = 45$ °C for 15 min; then twice in distilled water at a temperature of $T = 45$ °C for 15 min. To remove dust particles, the surfaces of the substrates were wiped with a clean cotton cloth moistened with isopropyl alcohol and blown with a jet of compressed air (compressor CJSC Remeza, Minsk, Belarus). A UV cleaning system was used to clean the substrates from potential traces of biological contamination (Photo Surface Processor PL16-110D, SEN LIGHTS Corp., JP).

A layer of azo dye ATA-2[57] synthesized at the Institute of Chemistry of New Materials of the National Academy of Sciences of Belarus was applied to the cleaned surfaces of electrically conductive layers of glass substrates as a film thickness of 25–30 nm by centrifugation (concentration of 1% solution in dimethylformamide, application conditions: rotation speed 3000 rpm for 40 s). The solution was dried up at a temperature of 120 °C for 10 min. Azo dye AtA-2 has high adhesion energy to liquid crystal molecules, thermal and photostability.[48] One of the substrates was exposed in two stages:[52,58,59] 1 - homogeneous illumination by linearly polarized LED radiation at a wavelength of $\lambda = 450$ nm with a power density of $I = 15$ MW cm$^{-2}$, exposure time $t = 20$ s (the radiation dose was 300 MJ cm$^{-2}$); 2 - repeated exposure the substrate through an amplitude mask (Fresnel zone plate) was produced by linearly polarized radiation with an orthogonal direction of the polarization vector (relative to its direction with homogeneous illumination). The reversibility of the photoinduced surface anisotropy process made it possible to create alternating annular Fresnel zones on the substrate with an orthogonal orientation direction of the LC director. Only the first stage of manufacturing was used to set a uniform direction of surface anisotropy over the entire surface of the second substrate.

The prepared glass substrates were placed in a laminar flow cabinet equipped with an air circulation and purification system, where the NLC cell was assembled. Calibrated polymer spacers-spheres with a diameter of 20 µm - were used to form a homogeneous capillary gap during the assembly of the LCD cell. The substrates were glued with UV glue (Norland NOA 81, Edmund Optics), which was cured to a durable solid polymer under the influence of ultraviolet light at a wavelength of $\lambda = 365$ nm at an irradiation dose of $D = 2$ J cm$^{-2}$.

The cell was filled with a nematic liquid crystal by the capillary method under the condition of its isotropic phase at a temperature of 65 °C (drying cabinet laboratory furnaces SNOL 58/350, AB UMEGA-GROUP Litva).

The amplitude glass mask (are shown in Supporting Information) is a Fresnel zone plate used for photoactivation of the orienting coating (AtA-2) made at JSC Integral (Minsk, Belarus) based on lithographic technology according to the design of amplitude transparency prepared in advance in the Klayout editor, the transmission profile of which corresponded to the Fresnel lens: the radius of the first zone $r_1 = 333$ µm, focal length $f = 17.6$ cm for $\lambda = 0.633$ nm.

*An Alternating Sinusoidal Pulse Voltage*: with frequency 1 kHz was applied to the NLC element from the Arbitrary Waveform Generator B-332 (LANFOR Company, St. Petersburg, Russia).

*Polarisation Microscopy*: Polarizing photographs of the Fresnel LC lens (see Figure 2 and the Supporting Information) at different values of the amplitude of the control voltage are recorded using a Video complex based on the polarizing microscope (Optoelectronic Systems OJSC, Minsk, Republic of Belarus) with a digital camera: the resolution of 2560 × 1920 pixels.

To form an optical vortex with a given topological charge, a spiral phase plate (SP) VPP-m633 (manufactured by RPC Photonics, Inc.) was used, which allows the formation of vortices with a topological charge from $\ell = 1$ to $\ell = 8$. Vortices with charges $\ell = 1 - 8$ were realized in the experiment. Linearly polarized radiation with a wavelength $\lambda = 632.8$ nm was generated by He–Ne laser LGK 7665 P18 (LASOS Lasertechnik GmbH). The initial beam diameter was 1.0 mm of 18 mW radiation power.

An optical system, including a 20x microlens and a focusing lens ($f = 20$ cm), was used to match the size of the laser beam with the aperture of the NLC Fresnel lens (3.7 mm). A gradient neutral filter 14CNDF-50-90/1-1 (Standa Ltd) carried out intensity control.

A Beamstar CCD Laser Beam Profiler (OPHIR Optronics Ltd.) was used to control the beam size before and after passing the NLC lens. It was located in the lens's focal plane and operated in the spectral range 320–1100 nm, with a sensitive area size of 4.6 mm × 6.2 mm. Information about the intensity distribution was displayed on a computer screen (shown in Supporting Information).

*Interference Patterns*: (Figures 5a,c and 6) were recorded using a digital video camera FLIRCHAMELEON CMLN-13S2M-CS (FLIR Integrated Imaging Solutions, Canada).

The spectral range of the element's operability was studied using YAG radiation: Nd laser with parametric frequency converter LT-2215 (LOTIS TII). Pulse duration $t = 15$ ns, repetition frequency $\nu = 10$ Hz, pulse energy $E = 40$ mJ, the possible range of radiation tuning 420–2300 nm. The study was conducted in the range of 532–700 nm. Radiation with a shorter wavelength was not used due to the increasing absorption of the azo dye, which could deteriorate the quality of the element structure. The radiation with a longer wavelength went beyond the digital camera's sensitivity. The deterioration in image quality at a wavelength of 700 nm is due to a decrease in the sensitivity of the CCD camera.

*Numerical Simulation (Figures 5b)*: Interference patterns of the vortex "ring" formed in the direction of the $0^{th}$ order of diffraction during the passage of an optical vortex through a liquid crystal Fresnel band plate with a singular beam diffracted in the –1 order direction (divergent beam) were analytically calculated based on a program created in the Python programming language.

The amplitude of the electromagnetic field $E^0(r, \theta)$ propagating optical vortex in the direction of 0 of the diffraction order of a phase singular beam can be represented as follows:[60,61]

$$E_0(r, \Theta) = A_0 (\frac{r}{r_0})^{|\ell|} exp(-\frac{r^2}{2r_0^2} + i\ell\Theta_0 + ikz) \tag{1}$$

where $r = \sqrt{x^2 + y^2}$, $x$, $y$, $z$ are the coordinates of a point, $r_0$ - radius of a singular beam (the origin of the coordinate system is in the center of the beam), $A_0$ is the initial amplitude, $\Theta = arctg(y/x)$ is the azimuth angle, $\ell$ is the topological charge of the light beam.


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The amplitude of the beam $E^{-1}(r, \theta)$ in the direction −1 of the diffraction order:

$$E_{-1}(r,\Theta) = \frac{A_{-1}}{r}\left(\frac{r}{r_{-1}}\right)^{|\ell|} exp\left(-\frac{r^2}{2r_{-1}^2} + i\ell\Theta_{-1} + ikr\right) \quad (2)$$

where $r = \sqrt{x^2 + y^2 + z^2}$, $r_{-1}$ - radius of a singular beam (the origin of the coordinate system is in the center of the $0^{th}$ order of diffraction), $\Theta = arctg(y/x + a)$ is the azimuth angle, $a$ is the displacement of the center of the beam of −1 order relative to the center of the coordinate system, $A_{-1}$ is the initial amplitude, $\ell$ – the topological charge of the light beam.

## Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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## Conflict of Interest

The authors declare no conflict of interest.

## Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

## Keywords

electrically controlled Fresnel lens, nematic liquid crystal, optical vortex, photoorientation, topological charge

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