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Article

"Tracking rule" and generalization of special relativity

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Abstract

We recall that a consistent description of the Thomas precession and Thomas–Wigner rotation is impossible without introducing a "tracking rule" into the structure of the special theory of relativity (STR), as we have shown in our publications (A.L. Kholmetskii and T. Yarman. Eur. Phys. J. Plus **132**, 400 (2017); A.L. Kholmetskii, O.V. Missevitch, T. Yarman, and M. Arik. Europhys. Lett. **129**, 3006 (2020)). The purely phenomenological origin of this rule in the framework of STR allows assuming the existence of a more general theory of empty space–time than STR, where the "tracking rule" is intrinsically incorporated into its structure. We find a possible way of developing such a generalized theory of empty space–time, where the "tracking rule" naturally arises, and propose an experimental scheme for its verification.

Key words: special theory of relativity, Thomas–Wigner rotation, Thomas precession, tracking rule

1. Introduction

As is known, the Thomas precession of the electron spin has been introduced in ref. [1] in the framework of the semiclassical model of a hydrogenlike atom to explain the emergence of the multiplier ½ ("Thomas half") in the expression for the spin–orbit interaction in atoms. Historically, it played an important role in the validation of the entire hypothesis on the electron spin [2].

Later, the problem of spin–orbit interaction in the hydrogenlike atoms was successfully solved through the quantum electron theory of Dirac [3], which though did not diminish the significance of the Thomas precession, especially in its applications to macro-scale phenomena (see, e.g., [4–6]).

Since that time, numerous papers and books have been published on the Thomas precession and its implications, both on the micro- and macro-scales, and an extended review of these works can be found, e.g., in ref. [7].

Recently, we have analyzed some important features of the Thomas–Wigner rotation and Thomas precession [8–14] and, in particular, emphasized [12] that the known expression for the frequency of Thomas precession ω_T of the axis of a point-like gyroscope (e.g., the spin of a classical electron), given as

$$\omega_T = \left(1 - \frac{1}{\gamma}\right)\frac{\boldsymbol{v} \times \dot{\boldsymbol{v}}}{v^2} \tag{1}$$

is valid only if the velocity $\boldsymbol{v}$ of the co-moving electron's frame K_e in the laboratory frame K corresponds to the rotation-free Lorentz transformation between K and K_e at any time moment. Here $\dot{\boldsymbol{v}}$ stands for the acceleration of the electron, and γ is its Lorentz factor, defined in the frame K. Otherwise, the Lorentz transformation of the frequency (1) between different inertial observers is not fulfilled.

What is more, we concluded that a relativistically consistent description of the Thomas precession is possible only in the case when the rotation-free Lorentz transformation setting between the laboratory frame K and the co-moving electron's frame $K_e(t)$ at an initial time moment $t = 0$ is preserved at any $t > 0$ during the subsequent motion of the electron along any curved path.

This "tracking rule" [13], in particular, means that for an electron moving along a curvilinear trajectory and associated with its proper frames $K_e(t)$ and $K_e(t + dt)$ at the considered time moments t and $t + dt$, respectively, the frequency of the Thomas precession of its spin should be calculated through a sequence of Lorentz transformations $K \rightarrow K_e(t)$, $K \rightarrow K_e(t + dt)$, but not through a set of subsequent transformations $K \rightarrow K_e(t) \rightarrow K_e(t + dt)$, as, for example, was adopted in the historical paper by Thomas [1] and some other familiar papers and books (see, e.g., [15]).

For the convenience of the readers, in Section 2, we refer to the "tracking rule" [12, 13] and emphasize that its introduction into STR on a phenomenological basis does, in general, contradict the common opinion about STR as the most fundamental physics theory.

Indeed, the establishment of the "tracking rule" in the determination of the frequency of Thomas precession [13] signifies the existence of a more fundamental theory of empty space–time than STR, where this rule, being introduced into STR as a phenomenological one, should logically follow from the basic points of this general theory.

In Section 3, we explore the ways to develop such a generalized theory named "Manifested relativity theory" (MRT), which, like STR, completely agrees with all experimental data collected up to date in the physics of empty space–time; at the same time, it always ensures the Lorentz transformation for the frequency of Thomas precession between different inertial observers and naturally explains the physical origin of the "tracking rule".

In Section 4, we discuss the principal implications of MRT and propose the principal scheme for its experimental test. Finally, we conclude in Section 5.

2. Thomas–Wigner rotation and the "tracking rule"

The "tracking rule" has been found in ref. [13] when analyzing the well-known configuration historically used in the derivation of the Thomas precession [1], where a semi-classical electron e with spin $\mathbf{s}$ orbits around an immovable heavy nucleus with charge Ze. For this configuration, Thomas pointed out that successive Lorentz transformations from the rest frame of the nucleus K to the frame $K_e(t)$ co-moving with the electron at the considered moment t and then from $K_e(t)$ to $K_e(t + dt)$ entail a spatial rotation of the coordinate axes of the system $K_e(t + dt)$ with respect to the system K (later called the Thomas–Wigner rotation). The time derivative of this angle yields a frequency equal to half the frequency of the Larmor precession ω_L of the electron's spin. Therefore, the measured spin–orbit interval in the hydrogen atom should be twice smaller than the value associated with the frequency of the Larmor precession of the electron spin, in full agreement with the experimental data.

Here, we especially point out that Thomas [1] used a sequence of rotation-free transformations $K\rightarrow K_e(t)\rightarrow K_e(t + dt)$. At the same time, in some other publications (e.g., in the textbook [16]), the Thomas precession is analyzed in another situation, where the rotation-free Lorentz transformations are applied according to the scheme $K\rightarrow K_e(t)$, $K\rightarrow K_e(t + dt)$, where the frames $K_e(t)$ and $K_e(t + dt)$ are no longer related by the rotation-free Lorentz transformation, unlike the adoption by Thomas [1].

This finding motivated us to analyze the available publications on the Thomas precession, which can indeed be divided into two groups:

- the first group, where, following Thomas, a sequence of rotation-free Lorentz transformations $K\rightarrow K_e(t)\rightarrow K_e(t + dt)$ is applied, which corresponds to the Thomas–Wigner rotation between the systems K and $K_e(t + dt)$ (see, e.g., refs. 1, 2, 5, 21, 23–26 of [13]); and
- the second group, where another sequence of rotation-free Lorentz transformations $K\rightarrow K_e(t)$, $K\rightarrow K_e(t + dt)$ is adopted, which corresponds to the Thomas–Wigner rotation between the systems $K_e(t)$ and $K_e(t + dt)$ (e.g., refs. 22, 27–29 of [13]).

One should note that none of the authors of both groups of publications presented any comments with respect to the choice of the applied sequence of Lorentz transformations; the absence of such comments can perhaps be explained by the same value of the frequency of Thomas precession for both sequences of transformations, as seen by a laboratory observer.

However, for an inertial observer instantaneously co-moving with the electron, the calculated precession frequency of the electron's spin occurs to be sensitive to the choice of the sequences of Lorentz transformations mentioned above.

In particular, one can see that at the choice by Thomas, corresponding to the sequence of rotation-free Lorentz transformations $K\rightarrow K_e(t)\rightarrow K_e(t + dt)$, an observer in $K_e(t)$ sees a fixed spatial orientation of the axes of the frame $K_e(t + dt)$ at any t and measures the ordinary Larmor precession of the electron spin. Simultaneously, an observer in $K_e(t + dt)$ sees the rotation of the axes of the laboratory frame K at half of the Larmor frequency and, hence, from his viewpoint, the decrease of the frequency of the Thomas precession of the electron spin by two times in the laboratory frame K represents an apparent effect for a laboratory observer, resulting from the precession of the coordinate axes of his frame K [13].

It is obvious that this explanation is wrong from a physical viewpoint, since "the Thomas half" in the expression for the precession of the electron's spin is measured just in the laboratory frame.

In contrast, with an alternative choice of successive transformations $K\rightarrow K_e(t)$, $K\rightarrow K_e(t + dt)$, an observer in the electron's co-moving frame $K_e(t)$ sees a fixed spatial orientation of the axes of the laboratory frame K_0 and a spatial rotation of the axes of the frame $K_e(t + dt)$ at half the Larmor frequency of the electron spin, so that both observers in the laboratory frame K and in the frame $K_e(t)$ do agree with respect to the value of the frequency of rotation of the electron spin, which is twice smaller than the Larmor frequency [13].

Thus, we conclude that only the choice of successive transformations $K\rightarrow K_e(t)$, $K\rightarrow K_e(t + dt)$ according to the "tracking rule" provides a physically consistent explanation of the Thomas precession both in the laboratory frame K and in the electron's co-moving frame $K_e(t)$.

Moreover, we have shown [13] that the "tracking rule" has a general character and emphasized its application to the Bargmann–Michel–Telegdi equation [17], where it represents a necessary condition for a consistent description of a particle with spin, no matter that this fact was not emphasized by the authors of [17].

Next, it is important to indicate an important implication of the "tracking rule", which, in fact, requires that the velocity $\boldsymbol{v}$ in the expression for the frequency of Thomas precession (1) should always be associated with the rotation-free Lorentz transformation. As we have shown in ref. [12], this simultaneously represents the necessary and sufficient condition to fulfill a relativistic transformation of the Thomas precession frequency between different inertial frames.

Here we notice a recent comment by Lambere [18] on our paper [13], where the author argued against the "tracking rule" and claimed that both alternative sequences of the Lorentz transformation $K_0\rightarrow K_e(t)\rightarrow K_e(t + dt)$ and $K_0\rightarrow K_e(t)$, $K_0\rightarrow K_e(t + dt)$ are equally applicable to the analysis of Thomas precession. Furthermore, Lambere also argued against our assertion that the velocity $\boldsymbol{v}$ in the expression for the frequency of Thomas precession (1) should always be associated with a rotation-free Lorentz transformation, which, in fact, is closely related to the "tracking rule".

However, in our Reply [19] to the Comment [18], we have shown that the claim by Lambare against the unique application of the "tracking rule" contains errors related to his ignorance of the difference between the rest (non-inertial) electron frame K_e and the set of inertial frames $K_e(t)$ co-moving with the electron at different time moments t. In addition, we have shown that the argumentation by Lambare against our assertion that the velocity $\boldsymbol{v}$ in eq. (1) is always associated with a rotation-free transformation is also erroneous, because in his consideration of successive Lorentz transformations between three inertial reference frames with non-collinear relative velocities, Lambare ignored the Thomas–Wigner rotation of their coordinate axes [18], which made his conclusions erroneous.

Moreover, now we highlight the general character of the "tracking rule", which is applicable not only to the motional equation of the electron spin but also to any macroscopic gyroscopes. In this respect, we indicated [13] the possibility of developing new astrophysics methods based on the "tracking rule", which could determine the most fundamental modes of the motion of the Earth in the universe.

It is even more important that the "tracking rule" asserts that, in general, we are no longer free to set a sequence of rotation-free Lorentz transformations between different inertial frames according to our arbitrary choice, as was commonly implied. Namely, dealing with two inertial frames K_1 and K_2 and establishing the Lorentz transformation between them,[1] we have, in general, to address the pre-history of their motion in the past, to judge the possibility of applying (or not applying) the rotation-free Lorentz transformation between them, and, correspondingly, to calculate the Thomas precession frequency of any gyroscope located in K_2.

This obviously falls outside the facilities of the standard mathematical apparatus of STR and indicates the possible existence of a more general theory than STR, where the "tracking rule" is naturally incorporated into its structure.

It seems that the explicit formulation of this problem, which we approached in our previous publications [8, 9, 12, 13], has never been done in the scientific literature before, and the challenge of the possible existence of a more general theory than STR—where the "tracking rule" would be explained—looks indeed inspiring.

Such a generalized theory, like STR, should be based on the same general properties of homogeneity and isotropy of empty space–time and agree with all experimental facts collected to date in the physics of empty space–time. At the same time, its wider framework should provide an unambiguous explanation of the "tracking rule". A way to construct such a general theory is presented in the next section.

3. Generalization of postulative basis of STR in the "manifested relativity theory"

As is well known, the postulates of STR directly lead to the Minkowskian metric of empty space–time, and now we highlight the exclusive property of the Minkowskian geometry, where we can always achieve the coincidence of any measured values with their true ("physical") magnitudes using optimal measurement procedures, which should satisfy the requirements of definiteness, reversibility, and transitivity (see, e.g., [20]). As is also well known, Einstein and other relativists often used this property of Minkowskian geometry in explanation of familiar relativistic effects in different inertial reference frames, and, in fact, the adopted identity of "physical" and "measured" four-vectors in any inertial reference frame can also be considered as the basic postulate of STR.

In view of this property of the Minkowskian geometry, it becomes clear that any generalization of STR in the framework of the principle of general covariance should go beyond the above postulate, where we can admit that even in empty space–time, for an observer in an arbitrary inertial frame, "physical" and "measured" values may differ from each other and obey different space–time transformations.

At the same time, to ensure the compatibility of such a generalized theory with all the available experimental facts collected to date in the physics of empty space–time, one should construct this generalized theory in such a way where the measured space–time coordinates and their functions are always subject to Lorentz transformations in empty space–time, regardless of the choice of transformations in the empty physical space–time, which thus becomes non-observable.

In such a case, a crucial question arises: can we prescribe real meaning to such physical space–time in an arbitrary inertial reference frame, even if it cannot be directly accessed?

The general answer to this question, which will be substantiated below, is as follows: even under the impossibility to directly measure the physical space–time four-vectors, the recognition of their existence eliminates any arbitrariness in the choice of sequence of rotation-free Lorentz transformations between different inertial frames. Moreover, as we will show below, a negation of the postulate of STR about the identity of physical and measured values and the adoption of special properties for the physical space–time can provide unambiguous rules with regard to the order of implementation of successive Lorentz transformations for measured space–time four-vectors between arbitrary inertial reference frames.

Before clarifying these points, we have, first of all, to solve a purely technical problem and *determine special properties of the empty physical space–time, which ensure Lorentz transforms for the measured space–time four-vectors.*

In mathematical language, this problem can be formulated as follows.

[1] Hereinafter, speaking about the possibility to establish rotation-free Lorentz transformations between two inertial reference frames, we always imply the case of non-collinear velocities of K_1 and K_2 in the frame of observation K_0 and, by default, exclude the trivial case of collinear velocities of K_1 and K_2 in K_0, where special Lorentz transformations commutate with each other and do not include additional spatial rotation.

Consider an arbitrary inertial frame, where we suppose that the empty physical space–time, which is characterized by the metric tensor $g_{\mu\nu}$ ($\mu,\nu = 0\ldots3$), does differ, in general, from Minkowskian space–time, and the expression for the space–time interval generally reads as

$$ds^2 = g_{\mu\nu} dx_{\text{ph}}{}^{\mu} dx_{\text{ph}}{}^{\nu} \tag{2}$$

where $x_{\text{ph}}{}^{\mu}$ is a physical four-vector. We also assume that the physical four-vectors belong to a set of "admissible", where the known inequalities

$$g_{00} > 0,\; g_{\alpha\beta} dx^{\alpha} dx^{\beta} < 0 \; (\alpha = 1,\, 2,\, 3) \tag{3}$$

are fulfilled. As is known, inequalities (3) mean that the frame of observation is realizable in nature [15].

Next, we demand that the empty physical space–time satisfy the principles of homogeneity and isotropy. The isotropy principle, in particular, means the existence of at least one inertial reference frame K_0 (excluding trivial rotations and translations of space), where the "physical" (x'^{μ}_{ph}) and "measured" (x'^{μ}_{m}) four-vectors coincide with each other and with the Minkowskian four-vector x'^{μ}_{L}, i.e.,

$$x'^{\mu}_{\text{ph}} \doteq x'^{\mu}_{\text{m}} \doteq x'^{\mu}_{\text{L}} \tag{4}$$

Hereinafter, the primed space–time coordinates belong to the frame K_0, while the unprimed coordinates belong to an arbitrary inertial frame K.

Now, we look for the specific properties of the physical space–time, which would ensure the Lorentz transformation

$$(x_{\text{m}})_{\mu} = L_{\mu\nu}(\boldsymbol{v})\left(x'_{\text{m}}\right)^{\nu} \tag{5}$$

for the measured space–time coordinates between the frame K_0 and an arbitrary inertial reference frame K, moving in K_0 with a constant velocity $\boldsymbol{v}$.

Solving this problem, we first notice that space–time homogeneity requires a linear form of the relationship x_{m} and x_{ph} in the frame K,

$$(x_{\text{m}})_{\mu} = M_{\mu\nu}(\boldsymbol{v})\left(x_{\text{ph}}{}^{\nu}\right) \tag{6}$$

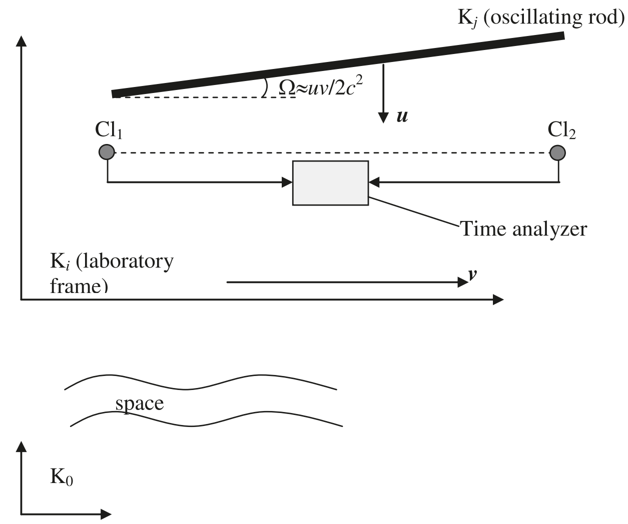
where the coefficients $M_{\mu\nu}$ depend only on the velocity $\boldsymbol{v}$ of K in K_0 and do not depend on the space–time coordinates.

At this point, it is worth noticing that in STR, the matrix **M** (which we named the "manifest matrix" for reasons clarified below) is tacitly adopted to be equal to the unit matrix due to the identity of the measured and physical values in any inertial reference frame. In this case, equality (4) can be immediately extended to all inertial reference frames, which thus become equivalent to each other.

Therefore, eq. (6) can indeed be considered as a necessary step towards the generalization of STR, where we assume that, in general, **M** is not represented by a unit matrix, and its coefficients may depend on the velocity $\boldsymbol{v}$ of K in K_0.

A theory with such a non-unit matrix $\mathbf{M}(\boldsymbol{v})$ can be named the "manifested relativity theory" (MRT), and our next goal is to determine its physical meaning and principal implications.

Fig. 1. Schematic of a possible experiment for testing MRT via observation of the Thomas–Wigner rotation of a moving rod in the laboratory (rod oscillates just to get repetitive measurements).



In this way, one has to first emphasize that the assumed dependence of the coefficients of the matrix **M** on the velocity $\boldsymbol{v}$ of K in K_0 signifies that the physical four-vector x^{μ}_{ph} cannot be directly accessible through measurements carried out in the frame K. Indeed, the reverse claim would imply that the physical four-vectors x^{μ}_{ph} and the measured four-vectors x^{μ}_{m} could be represented through each other in a linear form with constant coefficients independent of the velocity $\boldsymbol{v}$ of K in any other inertial frame. However, this result would contradict eq. (6), where the coefficients of the manifest matrix **M** are assumed to be velocity-dependent.

Nevertheless, even if direct access to the physical four-vectors x^{μ}_{ph} in an arbitrary inertial frame K is impossible, we will show below that the most fundamental problem—to describe the space–time kinematics of the outer world using the measurement tools available only in frame K—remains, as in STR, unambiguously solvable with application of the "tracking rule", which, as we will show below, is naturally incorporated into the mathematical structure of MRT.

Namely, we will show in Section 4 that the Lorentz transformation between two arbitrary inertial frames K_i and K_j is implemented as the sequence of the two rotation-free Lorentz transformations via the frame K_0 (i.e., $K_i \to K_0 \to K_j$), which can be utilized as a tool for the determination of the physical four-vectors in these frames and thus for experimental verification of MRT. A particular realization of this algorithm will be presented below in Fig. 1. This makes the physical content of this theory no poorer than the physical content of STR, at least on a principal level.

For further progress, one has to determine the properties of the matrix **M**, which, being different from the unit matrix, would nevertheless always ensure the coincidence of the measured four-vectors x^{μ}_{m} and the Minkowskian four-vectors

x_{L}^{μ} in an arbitrary inertial reference frame K, regardless of the choice of admissible transformations in physical space–time x_{ph}^{μ} between different inertial frames. For such a matrix **M**, one can indeed claim that all known experimental data in the physics of empty space–time, supporting STR, can also be considered in support of MRT.

Exploring the general properties of the matrix **M**, we explicitly introduce the metric tensor of the physical space–time $(g_{\mathrm{ph}})_{\mu\nu}$, so that the space–time interval in physical coordinates x_{ph} reads as

$$ds^2 = \left(g_{\mathrm{ph}}\right)_{\mu\nu} dx_{\mathrm{ph}}^{\mu} dx_{\mathrm{ph}}^{\nu} \tag{7}$$

Combining (6) and (7), we express the space–time interval through the measured space–time coordinates and demand that the geometry of space–time for the measured four-vectors be Minkowskian, i.e.,

$$ds^2 = \left(g_{\mathrm{ph}}\right)_{\mu\nu} \left(M^{-1}\right)_{\eta}^{\mu} dx_{\mathrm{m}}^{\eta} \left(M^{-1}\right)_{\eta}^{\nu} dx_{\mathrm{m}}^{\eta} = E_{\mu\nu} x_{\mathrm{m}}^{\mu} x_{\mathrm{m}}^{\nu} \tag{8}$$

where $E_{\mu\nu}$ is the metric tensor of the Minkowskian space–time.

Thus, our next problem is to determine the properties of the matrix **M**, which would ensure equality (8).

We find the solution to (8) as

$$\left(g_{\mathrm{ph}}\right)_{\mu\nu} \left(M^{-1}\right)_{\eta}^{\mu} \left(M^{-1}\right)_{\sigma}^{\nu} = E_{\eta\sigma} \tag{9}$$

which leads to the following expression for the metric of physical space time:

$$\left(g_{\mathrm{ph}}\right)_{\mu\nu} = M_{\mu\eta} M_{\nu}^{\eta} \tag{10}$$

Further on, we notice that the relationship between physical coordinates x_{ph} and Minkowskian four-vectors x_{L} can be written, by analogy with the known expression of the general theory of relativity, as [15]

$$dx_{\mathrm{L}}{}^{0} = \sqrt{g_{00}} dx_{\mathrm{ph}}{}^{0} + \frac{g_{0i} dx_{\mathrm{ph}}{}^{i}}{\sqrt{g_{00}}} \tag{11}$$

$$\Sigma \mathrm{d}x_{\mathrm{L}i}^{2} = \left(-g_{ij} + \frac{g_{0i} g_{0j}}{g_{00}}\right) \mathrm{d}x_{\mathrm{ph}}{}^{i} \mathrm{d}x_{\mathrm{ph}}{}^{j} \tag{12}$$

where $i, j = 1 \ldots 3$.

Substituting (10) into (11), (12), and using equality $M_j^i = E^{ik} M_{kj}$, we obtain a relationship between the four-vectors x_{L} andx_{ph} as follows:

$$\mathrm{d}x_{\mathrm{L}0} = \sqrt{(M_{00})^2 - \sum_i (M_{i0})^2} \mathrm{d}x_{\mathrm{ph}}^{0} + \frac{\left(M_{00} M_{i0} - M_{j0} M_{ji}\right) \mathrm{d}x_{\mathrm{ph}}^{i}}{\sqrt{(M_{00})^2 - \sum_i (M_{i0})^2}}, \tag{13}$$

$$\Sigma (dx_{Li})^2 = \left[-M_{\mu i} M_j^{\mu} + \frac{\left(M_{00} M_{0i} - M_k M_{ki}\right)\left(M_{00} M_{0j} - M_{k0} M_{kj}\right)}{(M_{00})^2 - \sum_i (M_{i0})^2} \right] \times dx_{\mathrm{ph}}^{i} dx_{\mathrm{ph}}^{j} \tag{14}$$

Next, we point out that the transformation (6) belongs to the type of transformations acting within the same inertial frame of references, i.e.,

$$x'^{0} = x'^{0}\left(x^{i}\right), \quad x'^{i} = x'^{i}\left(x^{j}\right)$$

and the second equality signifies that

$$M_{i0} = 0 \tag{15}$$

at any i. Thus, substituting (15) into (13) and (14), and using eq. (6), one gets:

$$(\mathrm{d}x_{\mathrm{L}})_0 = M_{00} \mathrm{d}x_{\mathrm{ph}}^{0} + M_{0i} \mathrm{d}x_{\mathrm{ph}}^{i} = (\mathrm{d}x_{\mathrm{m}})_0 \tag{16a}$$

$$\Sigma (\mathrm{d}x_{Li})^2 = \left[-M_{\mu i} M_j^{\mu} + M_{0i} M_{0j}\right] \left(\mathrm{d}x_{\mathrm{ph}}\right)^{i} \left(\mathrm{d}x_{\mathrm{ph}}\right)^{j} = M_{ki} M_{kj} \mathrm{d}x_{ph}^{i} \mathrm{d}x_{ph}^{j} = \Sigma (\mathrm{d}x_{\mathrm{m}i})^2 \tag{16b}$$

The obtained eqs. (16a) and (16b) indicate that for any linear relationship (6) between x_{m} and x_{ph}, as required by space–time homogeneity, complemented by constraint (4), as required by space–time isotropy, *the measured space–time coordinates x_{m} always coincide with the Minkowskian coordinates x_{L} and therefore do obey the Lorentz transformation, regardless of the particular choice of admissible space–time transformations in physical space–time*, which thus cannot be directly observed.

More specifically, we are free, in general, to assume any admissible linear transformation of the empty physical space–time between any inertial reference frames realizable in nature; nevertheless, regardless of our particular choice, the measured geometry of such an inertial reference frame will always manifest itself as Minkowskian, where the spatial and temporal interval obey the Lorentz transformation—and this fact explains the name "manifest matrix" for the matrix **M** in eq. (6).

Nevertheless, even in this situation, the properties of the physical space–time can be indirectly manifested in the Lorentz transformations between two arbitrary inertial frames K_i and K_j, which in MRT are implemented through a sequence of rotation-free transformations $K_i \rightarrow K_0 \rightarrow K_j$. Then the corresponding Thomas–Wigner rotation of the axes of K_i and K_j (considered in STR only as a purely kinematical effect resulting from the general group properties of Lorentz transformations and not requiring its physical interpretation) acquires a direct physical meaning in MRT in terms of physical space–time, as we will show below with the particular configuration in Fig. 1 for the proposed experimental test of MRT in Section 4.

What is more, one can realize that the sequence of rotation-free transformations established in MRT between two arbitrary inertial frames as $K_i \rightarrow K_0 \rightarrow K_j$ naturally explains the "tracking rule", which we disclosed in ref. [13] under calculation of the Thomas precession frequency in eq. (1). Therefore, unlike STR, this rule is naturally incorporated into the mathematical structure of MRT.

Next, we explicitly determine the matrix **M** for a particular choice of admissible space–time transformations in physical space–time with constraint (3) on its metric coefficients.

Being free in the choice of such an admissible transformation in the physical space–time, it is natural to consider first the simplest case of the admissible Galilean transformation **G** between the frame K_0 defined by eq. (4) and an arbitrary inertial frame K, moving in K_0 at a constant velocity $\boldsymbol{v}$, i.e.,

$$\left(x_{\mathrm{ph}}\right)_{\mu} = G_{\mu\nu}\left(\boldsymbol{v}\right) x'^{\nu}_{\mathrm{ph}} \tag{17}$$

here the primed coordinates belong to the frame K_0, and the non-vanishing coefficients of the matrix of the Galilean transformation are defined by the equalities

$$G_{00} = G_{11} = G_{22} = G_{33} = 1,\ G_{i0} = -v_i \tag{18}$$

Note that constraints (3) and (4) ensure the finite light velocity c in the frame K_0 independently of the choice of transformations in the physical space–time.

Addressing eq. (4) for the frame K_0, and introducing the matrix of Lorentz transformation $L_{\mu\nu}$ between the measured space–time four-vectors x_{m} in different inertial frames, we obtain the following expression for the matrix $M_{\mu\nu}$ in frame K:

$$M_{\mu\nu}\left(\boldsymbol{v}\right) = \left(G^{-1}\right)^{\eta}_{\mu}\left(\boldsymbol{v}\right) L_{\eta\nu}\left(\boldsymbol{v}\right) \tag{19}$$

Combining (18), (19), and using the known form of the Lorentz transformation matrix **L** (see, e.g., [15]), we arrive at an explicit presentation of the manifest matrix **M** in the frame K as a function of its velocity $\boldsymbol{v}$ in K_0:

$$\begin{aligned}&\left(M^{-1}\right)_{00} = \gamma,\ M_{i0} = 0,\ \left(M^{-1}\right)_{0i} = \gamma\frac{v_i}{c^2},\\ &\left(M^{-1}\right)_{ij} = \delta_{ij} + \frac{v_i v_j}{c^2}\left(1 - \frac{1}{\gamma}\right)\end{aligned} \tag{20}$$

where δ_{ij} is the Kronecker symbol.

Substituting (20) into (6), and taking into account the obvious equality $x_{\mathrm{L}} = x_{\mathrm{ph}}$ at $\boldsymbol{v} = 0$, resulting from eq. (4), we obtain the following expressions for the spatial and temporal intervals in the frame K as functions of its velocity $\boldsymbol{v}$ in K_0:

$$\boldsymbol{r}_{\mathrm{ph}}\left(\boldsymbol{v}\right) = \boldsymbol{r}_{\mathrm{ph}}\left(v = 0\right) + \frac{\boldsymbol{v}}{v^2}\left(\boldsymbol{r}_{\mathrm{ph}}\left(v = 0\right)\cdot\boldsymbol{v}\right)\left(\frac{1}{\gamma} - 1\right) \tag{21}$$

$$t_{\mathrm{ph}}\left(\boldsymbol{v}\right) = \gamma t_{\mathrm{ph}}\left(v = 0\right) + \gamma\frac{\left(\boldsymbol{r}_{\mathrm{ph}}\left(v = 0\right)\cdot\boldsymbol{v}\right)}{c^2} \tag{22}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor.

Formally, eqs. (21) and (22) look analogous to equations of STR; however, the physical meaning of these equations in both theories is different.

Namely, in STR, the value $\boldsymbol{r}(v = 0)$ in (21) determines the length and spatial orientation of the rod, measured in *its rest frame*, whereas $\boldsymbol{r}(\boldsymbol{v})$ determines the length and spatial orientation of the same rod, measured in *another inertial frame*, wherein the rod moves with a constant velocity $\boldsymbol{v}$.

In contrast, in MRT, the $\boldsymbol{r}_{\mathrm{ph}}(\boldsymbol{v})$ and $\boldsymbol{r}_{\mathrm{ph}}(v = 0)$ in (21) represent the physical lengths of the rod, defined simultaneously *in all* inertial frames of observations (including the frame K_0 defined by eq. (4)); $\boldsymbol{r}_{\mathrm{ph}}(v)$ corresponds to the case when this rod *moves* in the frame K_0 with a constant velocity $\boldsymbol{v}$, while $\boldsymbol{r}_{\mathrm{ph}}(v = 0)$ corresponds to the case where the rod *rests* in frame K_0.

To clarify the physical meaning of these equations, we first address eq. (21), which yields the following relationships:

$$\left(\boldsymbol{r}_{\mathrm{ph}}\left(\boldsymbol{v}\right)\cdot\boldsymbol{v}\right) = \frac{\left(\boldsymbol{r}_{\mathrm{ph}}\left(v = 0\right)\cdot\boldsymbol{v}\right)}{\gamma} \tag{23}$$

$$\left(\boldsymbol{r}_{\mathrm{ph}}\left(\boldsymbol{v}\right)\times\boldsymbol{v}\right) = \left(\boldsymbol{r}_{\mathrm{ph}}\left(v = 0\right)\times\boldsymbol{v}\right) \tag{24}$$

Equations (23) and (24) show that the component of vector $\boldsymbol{r}_{\mathrm{ph}}$, which is collinear to $\boldsymbol{v}$, is contracted by γ times, while the component of $\boldsymbol{r}_{\mathrm{ph}}$, which is orthogonal to $\boldsymbol{v}$, remains unchanged. Hence, when we recognize the frame K_0 defined by equality (4) as a preferable one, we reveal that eqs. (23) and (24) describe the well-known Fitzgerald-Lorentz contraction hypothesis of the Lorentz ether theory (see, e.g., [15]).

At the same time, it was well understood by Lorentz and his followers that the contraction of the scale in the frame K as a function of its velocity $\boldsymbol{v}$ in K_0 is non-observable due to the proportional contraction of the unit scale in K.

Furthermore, addressing eq. (22), we obtain for the time interval in a fixed spatial point ($r_{\mathrm{rh}} = 0$) of an arbitrary inertial frame K:

$$t_{\mathrm{ph}}\left(\boldsymbol{v}\right) = \gamma t_{\mathrm{ph}}\left(v = 0\right) \tag{25}$$

This equation again looks similar in form to the corresponding equation of STR; however, its physical meaning is different. Namely, MRT eq. (25) describes the effect of a dilation of physical time for a clock moving in the frame K_0 with the constant velocity $\boldsymbol{v}$, and this effect takes place for all inertial observers, including the observer in K_0. Here, the physical value $t_{\mathrm{ph}}(v = 0)$ corresponds to the case when the clock is at rest in frame K_0.

We remind you that the absolute dilation of time (25) was also adopted in the late version of the Lorentz ether theory [15]. At the same time, the effect (25), like the effect of contraction of scale (23), (24) in the physical space–time, is also unobservable in any inertial frame K due to proportional dilation of time for the standard clock located in K.

Next, comparing the rate of two clocks separated by a distance $\boldsymbol{r}_{\mathrm{ph}}$ and introducing the standard Einstein procedure for their synchronization [15], we have to keep in mind that the adoption of the Galilean space–time transformation (17) in the physical space–time leads to the Galilean law of velocity composition between the frames K and K_0 for the physical light speed, i.e.,

$$\boldsymbol{c}_{\mathrm{ph}}\left(\boldsymbol{v}\right) = \boldsymbol{c} - \boldsymbol{v} \tag{26}$$

Therefore, the application of the Einstein method of synchronization of two spatially separated clocks of an arbitrary inertial frame K (or any other admissible synchronization method in K that meets the requirements of definiteness, reversibility, and transitivity [20]) leads to some systematic "error" of such synchronization; however, one can straightforwardly show that it is exactly balanced by the second term on the rhs of eq. (22). As a result, the light velocity anisotropy in the physical space–time of the frame K becomes non-observable, and the measured light speed is always equal to c.

Thus, in the framework of MRT, we seem to confirm the known assertion [15] that no experimental tools exist that could discriminate between STR and the Lorentz ether theory.

However, as we will see below, this claim is valid only in the particular case of rotation-free Lorentz transformations, when the relative velocities of all inertial frames under consideration are collinear to each other. As is known, in this case, successive Lorentz transformations commute with each other, and the "tracking rule" becomes unimportant.

Since the latter case was implied in most experimental attempts to detect the "ether wind speed", the application of the Lorentz ether postulates to explain the "null" results of all such experiments has always been successful [15].

Moreover, due to the general character of eqs. (16a) and (16b), the latter claim is extended to any admissible space–time transformations in the physical space–time (which replace the Galilean matrix in eq. (17)), given that successive Lorentz transformations for the measured space–time four-vectors are carried out at collinear velocities between different inertial frames.

Thus, we conclude that any experimental test of MRT should be based on a motional diagram of the inertial reference frames under consideration, where their velocities in K_0 are not collinear with each other and the corresponding Lorentz transformations are not rotation-free.

At the same time, let us prove that the Lorentz transformation for the Minkowskian four-vectors x_L from an arbitrary inertial frame K_i to the frame K_0 should always be rotation-free to ensure equality (4) in K_0.

To prove the latter claim, we use the freedom of choice of admissible transformations in physical space–time, and choose such a transformation that constitutes a sub-group without spatial rotations, e.g., the admissible Galilean transformations (17) considered above. Then, one can see that the coincidence (4) of the physical x_{ph} and the measured ($x_m \equiv x_L$) space–time four-vectors in the frame K_0 is implemented only in the case when the Lorentz transformations for the measured spatial and temporal intervals between frames K_i and K_0 are also rotation-free. This is what we wanted to prove.

Due to the arbitrary choice of frame K_i, we obtain that for any other inertial frame K_j, we again have to demand the rotation-free Lorentz transforms between the Minkowskian four-vectors x_L in K_j and K_0. Therefore, we conclude that the rotation-free Lorentz transformation from K_i to K_j for measured space–time coordinates should be carried out as a sequence of two transformations implemented through the frame K_0, i.e., $\mathbf{L}(K_i \to K_0)\mathbf{L}(K_0 \to K_j)$. Hence, according to the general properties of the Lorentz group, the direct Lorentz transformation from K_i to K_j is no longer rotation-free but is accompanied by the Thomas–Wigner rotation of the coordinate axes of K_i and K_j at angle [15]

$$\Omega \approx |\boldsymbol{v}_i \times \boldsymbol{v}_j|/2c^2 \tag{27}$$

(ignoring terms of order c^{-3} and lower), where $\boldsymbol{v}_i$ and $\boldsymbol{v}_j$ denote the corresponding velocities of K_i and K_j in K_0.

Thus, the frame K_0 defined by (4) represents a single inertial reference frame (excluding as before any trivial translations and rotations in space), which ensures a rotation-free Lorentz transformation to any arbitrary inertial frame K_i, moving at a constant velocity in K_0, whereas a Lorentz transformation between two arbitrary inertial frames K_i to K_j is accompanied by a Thomas–Wigner rotation of their coordinate axes by the angle (27), depending on the velocities of these frames in K_0.

In fact, with respect to the measured four-vectors $x_m \equiv x_L$ (i.e., with respect to all observations in empty space–time), this is a single difference between the kinematics of MRT and STR, where the sequence of rotation-free inertial reference frames can be set arbitrarily.

In the next section, we discuss some consequences resulting from this difference between STR and MRT.

4. Basic implications of manifested relativity theory and possibility of its experimental test

First of all, we emphasize the principal result of MRT:

– As soon as we adopt the linear relationship (6) between the physical x_{ph} and the measured x_m four-vectors in any inertial frame K, as required by the space–time homogeneity, and demand the equality (4) at least for one inertial reference frame K_0, as required by the space–time isotropy, we always obtain Lorentz transformations for the measured space and time intervals (see eqs. (16a) and (16b)), regardless of the particular choice of transformation in the physical space–time between different inertial frames.

Such a particular choice does affect the specific form of the manifest matrix **M**, and we have shown above that in the illustrative case of admissible Galilean transformations (17) for physical four-vectors x_{ph}, the matrix **M** is defined by eq. (19) with an explicit presentation of its coefficients via (20). At the same time, the dependence of the matrix **M** on the choice of admissible space–time transformations in the physical space–time does not seem to be so significant, since the physical space–time is anyway obscured from direct observations due to eqs. (16a) and (16b). Therefore, the actual problem goes to comparing the predictions of STR and MRT for the measured space–time four-vectors x_m, which always obey the Lorentz transformation, regardless of the choice of transformation for x_{ph}.

In this respect, we highlight the fundamental feature of MRT, which does not allow an arbitrary choice of rotation-free Lorentz transformations for the measured four-vectors x_m between two arbitrary inertial frames and determines a unique inertial frame K_0, defined by eq. (4), which ensures a rotation-free Lorentz transformation for x_m to any arbitrary inertial frame K_i.

One can realize that this result straightforwardly leads to the "tracking rule", which defines the sequence of Lorentz transformation between three inertial reference frames, moving with non-collinear velocities.

By such a way, we arrive at a result of the principal importance: the "tracking rule", introduced in STR as a phenomenological one, acquires its natural explanation in MRT; moreover, the introduction of this rule concurrently eliminates any ambiguities in the determination of the frequency of Thomas precession (1), where the velocity parameter should always be associated with the rotation-free Lorentz transformation.

On a conceptual level, it is even more important to stress that the derivation of the "tracking rule" in MRT does harmonize two facts with each other, which, from a historical viewpoint, have so far been considered strongly antagonistic: the proven validity of the Lorentz transformation for any measured spatial and temporal intervals in the empty space between inertial frames and the existence of a preferable inertial frame K_0 defined by eq. (4).

At this point, one has to specially emphasize that the frame K_0 is defined *exclusively* through the rotation-free Lorentz transformation to any other inertial reference frame and is not associated with any kind of "ether". Rather, one can imagine K_0 as the very first inertial frame, as soon as space and time were created just after the Big Bang. Then, due to the implementation of the "tracking rule", the rotation-free Lorentz transformation was kept between K_0 and any other inertial frames associated with any objects created during the evolution of the universe. If this scenario was actually realized, then it seems natural to identify K_0 with the frame of isotropy of the cosmic relic radiation K_c. In such a way, in the framework of the MRT, we find fundamentally important properties of the cosmological frame K_c:

- It is the sole inertial frame wherein equality (4) is implemented;
- It is the sole inertial frame that is related to all other inertial frames in the universe by rotation-free Lorentz transformations.

The latter property of the cosmological frame can, at least in principle, be subjected to experimental testing. Indeed, taking two inertial frames K_i and K_j, which are related to K_c through the rotation-free Lorentz transformations, we find that a direct Lorentz transformation between K_i and K_j should be accompanied by a Thomas–Wigner rotation due to the general properties of the Lorentz group, and thus, one may hope to measure, at least in principle, the angle of Thomas–Wigner rotation (27) in a properly designed experimental scheme. We especially emphasize that such measurements can be carried out using only "internal" measurement procedures, without any communication with the outer world, with an unambiguous determination of the velocity of the frames K_i and K_j in the cosmological frame K_c.

To be more specific, we may introduce in the laboratory frame K_i an elongated macroscopic object (rod) with its proper frame K_j, which moves in K_i with a velocity $\boldsymbol{u}$ (see Fig. 1), and thus we can observe, at least in principle, the rotation of the elongated object at the angle

$$\Omega \approx |\boldsymbol{v} \times \boldsymbol{u}|/2c^2 \tag{28}$$

with respect to the axes of K_i by measuring the time difference between the moments of arrival of short signals to the time analyzer from two clocks Cl_1 and Cl_2, which sharply strike the opposite ends of the oscillating object.

Then, assuming the velocity v of K_i in the cosmological frame K_0 equal to $10^{-3}c$ (typical velocity of Galaxy objects), and taking the velocity of K_j in K_i $u \approx 1$ m/s, we obtain the angle $\Omega \approx 10^{-12}$, which is indeed a tiny value for any moving macroscopic object. At the same time, by realizing repeated measurements of the time difference, we can expect a correlation of such measurements with the daily self-rotation and the annual orbital rotation of Earth due to a variation in the direction of $\boldsymbol{v}$, which, thus, could simplify the data processing procedure for evaluating its numerical value.

Next, we find it important to emphasize three principal points related to this experimental proposal.

First, the scheme of the experiment in Fig. 1 represents a particular demonstration of the "tracking rule", where we must adopt that the frames K_i, K_j, which we have attached with the resting clocks and the oscillating rod, correspondingly, are not related to each other through rotation-free Lorentz transformations, but rather through a sequence $K_i \rightarrow K_0 \rightarrow Kj$, as is required by the "tracking rule". This indeed looks at odds with the "natural" choice of the direct rotation-free transformation $K_i \rightarrow Kj$, as it would be implied in STR for this configuration.

Second, this experimental scheme demonstrates that MRT and STR can indeed be distinguished from each other at the level of measured quantities, such as the nonzero indications of the time analyzer in Fig. 1, which could vary synchronously over days and years with the motion of the Earth in the cosmological frame K_0. Here it should be especially emphasized that such a nonzero indication of the time analyzer in Fig. 1 does not contradict the mathematical structure of STR, where, in general, the choice of a sequence of Lorentz transformations $K_i \rightarrow K_0 \rightarrow Kj$ is also possible. However, the preservation of the same sequence of transformations $K_i \rightarrow K_0 \rightarrow Kj$ during the daily self-rotation of the Earth and its annual revolution around the Sun looks somewhat artificial in STR, where the implementation of the "tracking rule" is not mandatory. On the contrary, in MRT, the indicated sequence of transformations $K_i \rightarrow K_0 \rightarrow Kj$ is unambiguous and directly stems from the requirement of the "tracking rule".

Third, even under the impossibility to directly measure the "physical" space–time four-vectors x_{ph}, the expected dependence (28) of the measured angle Ω on the velocity $\boldsymbol{v}$ of the Earth in the cosmological frame K_0 still admits its interpretation in terms of the physical space–time. Let us show how this can be done with the simplest choice of an admissible Galilean transformation (17).

Indeed, consider the case where the velocity v of Earth in the cosmological frame K_0 lies along the axis x. Hence, one can find that the contraction of the length of the oscillating rod along the velocity $\boldsymbol{v}$ (see eqs. (23) and (24)) induces its rotation with respect to the axis x by the angle $\Omega \approx -uv/2c^2$ to the accuracy c^{-2}. Taking into account the anisotropy of the physical light velocity (26) along the axis x of the laboratory frame K_i, we obtain the indication of the time analyzer as

$$\Delta t = \frac{L\sqrt{1 - v^2/c^2}\Omega}{u} + \frac{L\sqrt{1 - v^2/c^2}}{c - v} - \frac{L\sqrt{1 - v^2/c^2}}{c - v} \approx \frac{Lv}{2c^2} \tag{29}$$

to the accuracy c^{-2}.

What is more, having repeated measurements of Δt at different spatial orientations of the setup in Fig. 1—which in terrestrial conditions is naturally achieved due to the daily rotation of the Earth—we can determine through eq. (29) the value and spatial orientation of the vector $\boldsymbol{v}$ at any fixed time moment, which describes the motion of the laboratory frame K_i in the frame K_0. Further on, using equality (4) for the frame K_0, we get the physical spatial and temporal intervals in the laboratory frame, even if they cannot be directly measured in this frame.

In such a way, the problem of describing the space–time kinematics of any object in an arbitrary inertial frame, using only the measurement tools available in this frame, occurs in MRT and is fully solvable.

At the moment, such measurements lie on the edge of modern experimental capabilities, and the proposed design of the experiment in Fig. 1 serves only to demonstrate the principal possibility of measuring the velocity of the Earth in the cosmological frame K_c through the "tracking rule". The actual experimental scheme for such measurements can be substantially different, although the general idea—to determine the velocity $\boldsymbol{v}$ of the laboratory frame K_i in the cosmological frame K_c by measuring the Thomas–Wigner rotation between the axes of K_i and another inertial frame K_j, moving with a finite velocity $\boldsymbol{u}$ in K_i—remains the same. As an example of such a realistic experimental scheme, we mention the paper [21], where the proposed experimental idea—to measure the geometry of a rapidly spinning disc in the Lorentz ether theory—also perfectly fits for testing the "tracking rule", which implies the sequence of Lorentz transformations $K \rightarrow K_c \rightarrow K_a$, where K denotes the laboratory frame K, and the frame K_a is attached to a resonant absorber at the edge of a spinning rotor. As shown in ref. [21], such a choice of successive Lorentz transformations leads to the relative energy shift between an emitted and an absorbed resonant line of about 10^{-14} (at $v \approx 10^{-3}c$) which can be reliably measured using the Mössbauer effect on a rotating resonant absorber irradiated by resonant gamma-quanta of the appropriate synchrotron radiation.

5. Conclusion

Our previous [12, 13] and present analysis of the Thomas–Wigner rotation and Thomas precession allow revealing the "tracking rule", which should be fulfilled to ensure a consistent description of these relativistic effects.

We further emphasized that the "tracking rule" is introduced into STR on a phenomenological basis [13], which allows assuming the existence of a more general theory than STR, where this rule must be naturally incorporated into its structure.

We argued that a single reasonable (and, perhaps, the only possible) way to develop such a general theory of empty space is to assume that the physical space–time four-vectors x_{ph} do not necessarily coincide with the measured space–time four-vectors x_{m}, but are related through a non-unit matrix **M** (see (6)). At the same time, to ensure the compatibility of such a theory with numerous experimental facts confirming the validity of the Lorentz transformations for the measured space–time four-vectors, we have, first of all, to determine the conditions that lead to the Lorentz transformations for x_{m}. Among these conditions, we assumed the isotropy of the empty space–time reflected in eq. (4), and the homogeneity of the empty space–time reflected in eq. (6) through the independence of the coefficients of the matrix **M** from the space–time coordinates.

Then, we find that the adopted eqs. (4) and (6), along with the natural constraint (15)—reflecting the fact that transformation (6) is carried out within the same frame of references—are sufficient to prove that the measured space–time coordinates x_{m} of empty space–time always manifest through the four-vectors x_{L}, which obey the Lorentz transformation between different inertial frames, while the physical space–time four-vectors x_{ph} are, in general, not directly observable. Correspondingly, we named the matrix **M**, linked the physical x_{ph}, measured $x_{\text{m}} \equiv x_{\text{L}}$ four-vectors as a manifest matrix, and called our approach "manifested relativity theory" (MRT).

Next, we have shown that in the MRT, the measured space and time intervals of inertial frame K_0, defined by eq. (4), should be linked with the measured space and time intervals of any other inertial reference frames K_i through the rotation-free Lorentz transformation. Therefore, considering the Lorentz transformation between two arbitrary inertial reference frames K_i and K_j, we find that the rotation-free Lorentz transformations between these frames should be carried out through the sequence $K_i \rightarrow K_0 \rightarrow K_j$. Therefore, according to the general properties of the Lorentz group, a direct transformation from K_i to K_j should include a Thomas–Wigner rotation in the case where the velocities of K_i in K_0 and K_j in K_0 are not collinear to each other, and such a rotation, at least in principle, represents a measurable effect in the frames K_i and K_j. This finding allows, at least in principle, to determine the velocity of K_i in K_0 in the Gedanken experiment, schematically presented in Fig. 1, and in a real experiment for measurement of the Mössbauer effect on a rotating resonant absorber irradiated with resonant gamma-quanta of the synchrotron beam [21].

Thus, both theories—STR and MRT—can indeed be distinguished at the experimental level, and this is the most important outcome resulting from our generalization of special relativity.

We further emphasize that in MRT, the preferable frame K_0 is defined only by the fact of the rotation-free Lorentz transformation of the measured space and time intervals to any other inertial reference frame and does not require to imagine any "ether". Therefore, no specific characteristic of the "ether" should be involved in space–time transformations, which thus in the general case remain ten-parametric (i.e., four initial space–time coordinates, three rotational angles, and three components of a relative velocity between two inertial frames), which ensures the general covariance of the space–time kinematics of an empty space [15]. Rather, one can assume K_0 as the very first inertial frame in the universe, representing a frame of isotropy of the cosmic relic radiation K_c, and the fact of the rotation-free Lorentz transformation to any other inertial frame can be linked with the validity of the "tracking rule" entering into force along with the creation of space–time, thanks to the Big Bang.

Thus, exemption from the vague hypothesis of the "world ether" in determination of the properties of the preferable

Table 1. Comparison of STR and MRT in terms of key aspects of space–time.

STR	MRT
The physical and measured space–time four-vectors coincide with each other in all inertial reference frames	The physical and measured space–time four-vectors, in general, are not identical to each other in an arbitrary inertial reference frame
All inertial reference frames are equivalent to each other	There is a single inertial reference frame K_0 (except for rotations and translations in space), where the physical and measured space–time four-vectors are identical to each other
Spatial and temporal intervals in different inertial frames K_i, K_j obey the Lorentz transformations at any i, j	For any admissible choice of transformations in physical space–time, the measured spatial and temporal intervals in the difference inertial frames K_i, K_j obey the Lorentz transformations at any i, j
Rotation-free Lorentz transformation can be set between two arbitrary inertial reference frames K_i and K_j	Rotation-free Lorentz transformations for the measured space–time four-vectors are carried out between the frames K_0, K_i and K_0, K_j. The direct Lorentz transformation between K_i and K_j is no longer rotation-free and includes the Thomas–Wigner rotation of the spatial axes of these frames
"Tracking rule" is introduced on a phenomenological basis	"Tracking rule" is always fulfilled in the rotation-free Lorentz transformations $K_0 \rightarrow K_i$, $K_0 \rightarrow K_j$ for any i, j
Thomas–Wigner rotation represents a purely kinematical effect, which does not require its explanation	Thomas–Wigner rotation between the axes of K_i and K_j under rotation-free Lorentz transformations $K_0 \rightarrow K_i$, $K_0 \rightarrow K_j$ admits its interpretation in terms of the physical space–time, even if it is not accessed in frames K_i and K_j

frame K_0 defined by eq. (4) once again confirms the correctness of the basic claim of MRT about the possibility to describe the space–time kinematics of the outer world in a given inertial frame K using only measurement instruments of K. From this angle of view, the experimental scheme in Fig. 1 can be considered as a particular demonstration of the validity of this general assertion.

Finally, we notice that, with respect to any ordinary practical applications, the tiny Thomas–Wigner rotation angle (27) between terrestrial inertial frames K_i and K_j is hardly observable in the vast majority of problems.

Thus, in ordinary laboratory practice, we can well ignore the effect (27) and apply rotation-free Lorenz transformations between the chosen inertial frames according to our own arbitrary choice, as is customarily done in STR.

Finally, in Table 1, we compare STR and MRT in terms of the key aspects of space–time discussed above.

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