described by J. S. Russell in 1884. The mathematical theory of solitary waves was created by Korteweg and De Vries almost half a century later. The new theory caused a significant stir in the scientific community. Indeed, as follows from the famous equation of Korteweg-De Vries (KdV), the soliton profile has an asymptotic $f(x) \sim \mathrm{sech}^2 x$ for small amplitudes. This means that the soliton remains nonlinear for arbitrarily small amplitudes and does not turn into a linear wave. Further studies have shown that solitons are general phenomenon of nature that describes the properties of nonlinear ion-acoustic waves, magneto-acoustic waves, electric currents in nonlinear transmission lines, and much more. A large number of scientific papers have been devoted to the study of soliton properties, but the physics of nonlinear waves and solitons is far from complete.

The goal of this work is to study the ability of solitary waves to transport matter [1]. On the one side, a soliton is a wave. As expected, material waves do not carry matter (they transfer momentum and energy). It is known that this statement is true only for linear waves of infinitely small amplitude. However, for finite amplitudes waves (even harmonic ones), nonlinear effects lead to the emergence of non-zero drift of matter. This phenomenon was predicted in 1847 by Stokes and was named after him (Stokes drift). As is known, the drift speed for a harmonic wave of small but finite amplitude is proportional to its square. Subsequently, the phenomenon of Stokes drift was repeatedly observed in practice for waves on the water surface, acoustic waves, etc. In the Stokes drift situations, the particle motion represented by a superposition of drift and oscillatory motions. Decrease in the wave amplitude leads to a linearization of the wave process and subsequent rapid (quadratic) decrease in the drift component. In this way, for small amplitude harmonic waves, this nonlinear phenomenon is usually neglected.

In the case of solitons, the nonlinearity cannot be neglected. It is shown theoretically that the unidirectional transport of matter (over a finite distance in the direction of soliton motion) is a fundamental property of KdV solitons. It is also shown that the matter transport cannot be neglected as the wave amplitude decreases (in contrast to the Stokes drift), because the magnitude of the transport decreases in proportion to the square root of the soliton amplitude. Due to the universality of the KdV equation, we expect generalization of our results to a wide range of nonlinear problems.

[1] Phys. Plasmas. 2023. V.30, P.112302.

Half-cycle dissipative solitons in resonant media

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When dealing with a subcycle pulse propagation in a resonant medium, common approximations, such as the two-level model, become invalid due to ultrabroad pulse spectrum. Therefore multiple energy levels in the medium have to be properly considered. We develop the higher-order sudden-perturbation approach to derive the general nonlinear equations for the propagation of subcycle pulses in a multi-level medium. Using these equations, we demonstrate the existense of stable half-cycle dissipative solitons in non-equilibrium media with multiple resonant transitions.

Kosambi-Cartan-Chern geometric invariants, and the structure of the radial differential equations for a Dirac particle in the Newman-Unti-Tamburino space-time

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Applying the methods of differential geometry and technics of Kosambi-Cartan-Chern invariants, we study the radial equations for a spin ½ particle in the Newman-Unti-Tamburino space-time. Starting with the system of two differential equations for massless Dirac particle, we calculate the deviation curvature tensor P^i_j associated with Jacobi stability of the dynamical system. We proof that the real parts of its eigenvalues are positive near the horizon and at infinity, which corresponds to divergence of a pencil of geodesics near these singular points. We construct an effective Lagrangian function associated with this dynamical system. For massive Dirac particle,