

SPECTRAL CHEBESHEV COLLOCATION METHOD WITH ARTIFICIAL VISCOSITY FOR SIMULATIONS OF DFB-LASER DYNAMICS

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To analyze the system of laser equations, a numerical technique based on a combination of the Chebyshev spectral method with artificial viscosity and the forth order Runge-Kutta method are represented. On the example of the amplitude-frequency response of the DFB structure simulations, it is shown that the proposed technique significantly exceeds the effectiveness of standard difference method traditionally used for the considered class of problems.

Key words: Chebyshev Method; Artificial Viscosity; FDB-Lasers.

For simulations of the semiconductor DFB lasers dynamics taking into account the spatial hole burning effects, a system of two coupled first-order differential hyperbolic equations together with material ODE equations are used [1]:

$$\begin{aligned} \frac{1}{v_g} \frac{\partial E_{\pm}}{\partial t} \pm \frac{\partial E_{\pm}}{\partial z} &= G(N, E) (1 - i\alpha) E_{\pm} - \frac{\gamma}{2} E_{\pm} + i\kappa E_{\mp} + F_{\pm}(z, t), \\ \frac{dN}{dt} &= \frac{J}{e \cdot d} - BN^2 - CN^3 - \frac{v_g g_N (N - N_0) P}{1 + \varepsilon P} + F_N(z, t), \quad P = |E_+|^2 + |E_-|^2, \\ G(N, E) &= \frac{\Gamma g_N (N - N_0)}{2(1 + \varepsilon P)} \end{aligned} \quad (1)$$

Here E_{\pm} are complex envelopes of the light fields, v_g is the group velocity, N_0 is the carrier number at transparency g_n and δ are the amplification and the frequency detuning coefficients respectively, κ is the coupling coefficient J is the injection current, $F_{\pm}(z, t)$ represent the spontaneous emission noise (see ref. [1] for more details.).

For the numerical analysis of Eqs. (1), the finite difference methods of the characteristic type including splitting methods [1], TLLM (transmission - line laser model) [2] are used. As shown in [3], to solve stationary problems of counter propagating waves interaction of the form (1), the use of the spectral Chebyshev methods seems to be very effective.

To construct the spectral method for solving the non-stationary problem (1), we used the approximation of spatial derivatives using the Chebyshev spectral differentiation matrix, similarly to [3]. As a result, the original partial differential equations (1) are reduced to the Cauchy problem for a system of ordinary differential equations of dimension $2N$ where N is the number of spatial grid nodes. For the numerical analysis of the resulting system, the Runge-Kutta method of 4th accuracy order was used. Additionally, an artificial viscosity term in the form of second spatial derivative with the small parameter, $\nu \partial^2 E_{\pm} / \partial z^2$ was included in the right hand side of the field equations (1).

To show advantages of the proposed approach the following simplified coupling wave equations model with artificial viscosity where used

$$\begin{aligned} \frac{\partial E_{\pm}}{\partial t} \pm \frac{\partial E_{\pm}}{\partial z} &= \nu \frac{\partial^2 E_{\pm}}{\partial z^2} + i\kappa E_{\mp}, \quad z \in [-L/2, L/2], \quad t \in [0, T], \\ E_{\pm}(z, 0) &= 0, \quad E_{+}(-L, t) = \delta(t). \end{aligned} \quad (2)$$

For the accuracy estimation, the exact frequency dependence of the reflection coefficient [3] was used

$$\frac{|E_{-}(\Delta f, -L/2)|}{|E_{+}(\Delta f, -L/2)|} = R(\Delta f) = \frac{\sinh(\alpha)}{\sqrt{\cosh(\alpha^2 - \eta^2)}}, \quad \alpha = \sqrt{\kappa^2 - \Delta f^2}, \quad \eta = \Delta f / \kappa, \quad (3)$$

Simulation results for the problem (2), (3) using the finite difference and the spectral Chebyshev methods are presented in Fig. 1. The calculation time is defined the spatial grid size which is varied in the limits from N=13 to N=41 for the spectral methods and from N=25 to 1601 for the finite difference method.

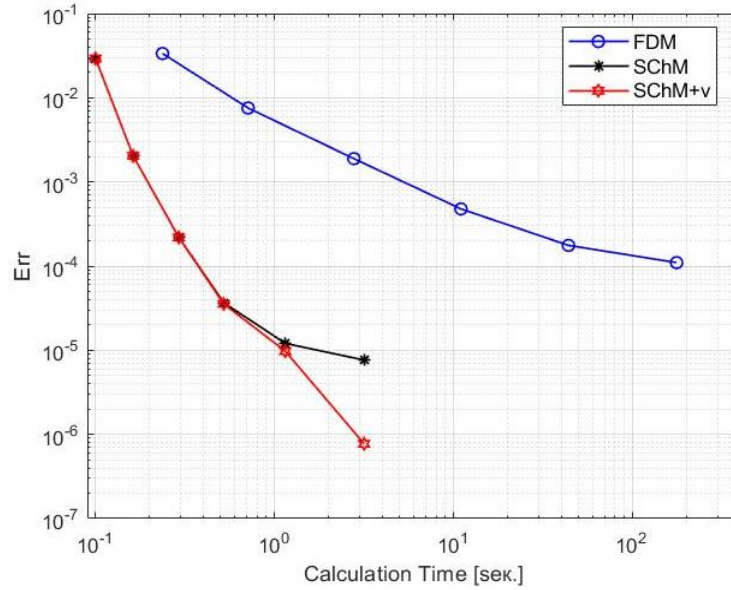


Fig. 1. Dependence of the accuracy on the calculation time

It can be seen in Fig. 1, the finite-difference methods provides the specified accuracy with sufficiently greater calculation time (about to hundred factor) in comparison with the spectral Chebyshev method. Moreover, introducing the artificial viscosity improve the spectral method accuracy about then times on the fine numerical grid.

References

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