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Modeling the Energy Characteristics of Ratchets by a Game-Theory Approach

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The hopping diffusion model is often used to describe the motion of Brownian ratchets. On the other hand, hopping diffusion is well modeled by Parrondo's paradoxical game method. In this paper, this method is used to simulate the energy characteristics of ratchets. It is known that the most efficient ratchet models are those in which the periodic potential profile can block the backflow of particles and fluctuates for half a period. Therefore, we have considered a one-dimensional hopping diffusion model with two nonequivalent nodes in an elementary cell, the hops of a Brownian particle between which were specified by two sets of transition probabilities. These sets of probabilities corresponded to potential profiles of the desired shape, which periodically shifted relative to each other by half a period. The time dependencies of the work done by the particle against the load force (output energy) and the energy transferred to the particle when switching potentials (input energy) of the system were calculated. The ratchet efficiency (the ratio of output energy to input energy) was calculated as a function of the load force at the moments of potential switching. This value ceased to depend on the time when the process became steady. The simulation results showed that the selected sets of transition probabilities ensure high efficiency of the considered ratchets up to 70%. In this case, the dependence of the efficiency on the load force is a nonmonotonic function, the course of which is in good agreement with the known theoretical data.

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1. Introduction

The ratchet mechanism, which leads to the directed motion of nanoparticles in asymmetric systems in the absence of concentration gradients and average applied forces, can function only under nonequilibrium conditions in the presence of energy supply to the system [1, 2]. Due to this, in the description of ratchet systems, a special role belongs to modeling not only the average velocity of the ratchet, which is frequently referred to as its main characteristic, but also the energy characteristics, input and output energies. Calculations of them make it possible to evaluate the ratchet efficiency – the ratio of input to output energies. In early theoretical works, the efficiency of the ratchet mechanism was assumed to be rather low and not exceeding a few percent [3, 4]. On the other hand, molecular (protein) motors in cells in some cases demonstrate high efficiency up to 100 percent [5, 6]. It was shown in [7, 8] that a high efficiency of a ratchet system can be expected in cases of potential profiles being fluctuated with shifts in their extrema. The first analytical model of an efficient Brownian motor having high efficiency was described in [9].

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It was a Brownian motor with an asymmetric potential profile undergoing random half-period shifts; its analogue with a hopping mechanism for overcoming potential barriers of a double-well potential profile was presented in [10]. The ratchet efficiency is determined by the expended energy E_{in} (energy input) and the useful work E_{out} (energy output) performed by the ratchet against an additionally introduced load force F [4,11,12]. Generally, for cyclic processes (inherent to deterministic ratchets), these quantities are determined per the period of the process, τ. When the load force F is introduced, the periodic potential energy profile $V(x,t)$ should be replaced by the total potential energy $U(x,t) = V(x,t) + Fx$. Here, the positive sign before Fx corresponds to the direction of the load-force vector against the direction of the ratchet motion, and the value of the force F is entered into $U(x,t)$ as the modulus of this vector. Useful work is defined as the product of the load force and the average displacement $\langle x \rangle$ of the ratchet particle, that is

$$E_{\text{out}} \equiv F \langle x \rangle . \tag{1}$$

The expended energy in the theory of Brownian motors is defined as the average energy transferred to a particle due to a change in its potential energy

$$E_{\text{in}} \equiv \int_0^{\tau} dt \int_0^{L} dx \frac{\partial U(x,t)}{\partial t} \rho(x,t). \tag{2}$$

Using Eqs. (1) and (2), the efficiency of converting the expended energy into useful work is easily calculated as

$$\eta \equiv E_{\text{out}}/E_{\text{in}}. \tag{3}$$

The average velocity of the motor directed motion $\langle v \rangle = d\langle x \rangle / dt$ is a monotonically decreasing function of the load force F [3, 9, 10, 12, 13]. It takes the largest value at $F = 0$ and vanishes at some value $F = F_s$ corresponding to the ratchet stopping point. The value of F_s is an important characteristic of the motor, associated with its ability to resist the load. According to Eqs. (1) - (3), the efficiency η is proportional to the product $F_l \langle v \rangle$, and therefore it vanishes both at $F = 0$ and at $F = F_s$. This means that the efficiency is a non-monotonic function of the load force, which takes on the maximum value $\eta_{\max}$ at a certain value of F. Based on the analysis of various models of pulsating ratchets [1, 4, 7–9, 11, 13–15], where the directed motion of a nanoparticle appears in a time-dependent periodic potential, the necessary and sufficient conditions for high efficiency ($\eta_{\max} \to 1$) have been formulated [12, 14–16]: 1) adiabatic (slow or fast) change of potential relief in time; 2) shift (continuous or abrupt) of potential extrema; 3) the presence of an effective mechanism for rectifying nonequilibrium fluctuations at large (exceeding the thermal energy) amplitudes of the potential profile and, in addition, a certain asymmetric shape of this profile in the adiabatically fast case. In this paper, to describe the energy characteristics of Brownian ratchets, we use the hopping diffusion model, which can be effectively investigated by Parrondo's paradoxical games approach [17]. The development of this approach for adiabatic ratchet systems with a discrete change in variables was presented in [18, 19]. The characteristics of highly efficient ratchet systems were modelled for hopping diffusion between the nodes of a 1D periodic structure, with a unit cell having two nonequivalent nodes in it. In Ref. [15], just for such a model, corresponding to a double-well potential profile fluctuating by half a period, the high efficiency of the ratchet operation under conditions of high potential barriers was obtained based on the general theory operating with continuous-time kinetic equations. In contrast to the approach of Ref. [15], it is convenient to perform numerical simulation of hopping diffusion with a discrete change in the time variable, since it allows one to operate with sets of probabilities of particle hops between neighboring nodes with subsequent averaging of the resulting trajectories [18]. Moreover, the proposed modeling can be much more simple in accounting the tunnelling processes when describing quantum ratchets.

2. Antisymmetric kinetic model of a ratchet with continuous time variation

It was shown in Ref. [15] that the simplest kinetic model that can be used to describe the ratchet effect is constructed through considering two states A and B with two reaction channels α and β (Fig. 1a). The ratchet effect arises at stochastic or deterministic dichotomous fluctuations of transition rate constants $\alpha^{\pm}_{\rm AB}$, $\alpha^{\pm}_{\rm BA}$, $\beta^{\pm}_{\rm AB}$ and $\beta^{\pm}_{\rm BA}$, which are denoted by indices $''+''$ and $''-''$ (Fig. 1b). This model corresponds to the hopping model of particle motion in a periodic double-well potential, which undergoes dichotomous fluctuations in the values of wells and barriers (Fig. 2).

If fluctuations can be considered as shifts of the double-well potential relief by half a period, then such a model is referred to as antisymmetric, and the transition rates satisfy the relations:

$$\alpha_{AB} \equiv \alpha^{-}_{AB} = \beta^{+}_{BA}, \quad \beta_{AB} \equiv \beta^{-}_{AB} = \alpha^{+}_{BA},$$
$$\alpha_{BA} \equiv \alpha^{-}_{BA} = \beta^{+}_{AB}, \quad \beta_{BA} \equiv \beta^{-}_{BA} = \alpha^{+}_{AB}. \quad (4)$$

The ratchet effect is characterized by the current:

$$J = J_f + \frac{\beta_{\rm AB} + \beta_{\rm BA} - \alpha_{\rm AB} - \alpha_{\rm BA}}{\Sigma} J_\gamma, \quad J_\gamma = \frac{1}{\tau}\frac{\xi_{\rm B} - \xi_{\rm A}}{\Sigma}\varphi, \quad J_f = \frac{\alpha_{\rm AB}\beta_{\rm BA} - \alpha_{\rm BA}\beta_{\rm AB}}{\Sigma},$$
$$\varphi = \begin{cases} \tanh(\Sigma/\Gamma), & \text{determ,} \\ \Sigma/(\Gamma+\Sigma), & \text{stoch,} \end{cases} \quad \xi_{\rm A} = \alpha_{\rm AB} + \beta_{\rm AB}, \quad \xi_{\rm B} = \alpha_{\rm BA} + \beta_{\rm BA}, \quad \Sigma = \xi_{\rm A} + \xi_{\rm B}. \quad (5)$$

Here, the dependence of the current on the period τ of the deterministic process or the average period of the stochastic process τ, equal to $4/\Gamma$, where Γ is the inverse correlation time, is determined by the function $\varphi(\tau)$. For both processes the value of $\varphi(\tau)$ increases linearly with growth in τ as $\Sigma\tau/4$ at $\Sigma\tau << 1$ and tends to unity at $\Sigma\tau >> 1$.

For the Arrhenius temperature dependence of the rate constants for overcoming potential barriers, we have the following relations:

$$\alpha_{\rm AB} = k_0 p^r_1, \quad \beta_{\rm AB} = k_0 p^l_1,$$
$$\beta_{\rm BA} = k_0 p^r_2, \quad \alpha_{\rm BA} = k_0 p^l_2, \quad (6)$$

where k_0 is a coefficient that has the dimension of inverse time and determines the time scale of the process, $p^r_{1,2}$ is the probability of transition from nodes 1 or 2 to neighbouring nodes to the right, and $p^l_{1,2}$ is the probability of transition from nodes 1 or 2 to the left. These values are expressed in terms of the potential relief parameters, $u = u^-_{\rm B} - u^-_A$, $v = v^-_\beta - (u^-_A + u^-_{\rm B})/2$, $V = v^-_\alpha - (u^-_A + u^-_{\rm B})/2$, $f = FL/(4k_BT)$ (see Fig. 2), measured in units

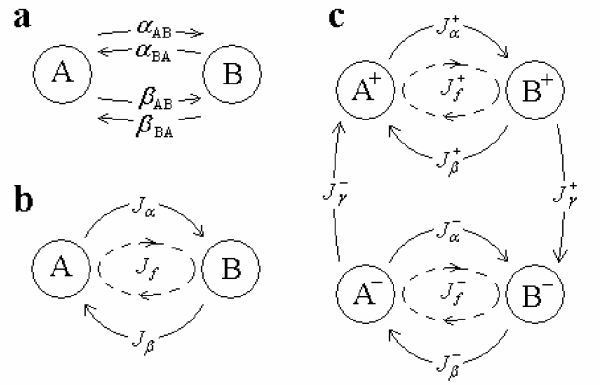

FIG. 1. The simplest system that allows motion along a closed trajectory (circulation) consists of two states A and B with two reaction channels α and β , characterized by rate constants (a) or corresponding currents J_α. and J_β (b). Scheme of currents arising for two sets of states $\rm A^{\pm}$ and $\rm B^{\pm}$, which are deterministically or stochastically switched (c), which leads to the currents $J^{\pm}_\gamma$.

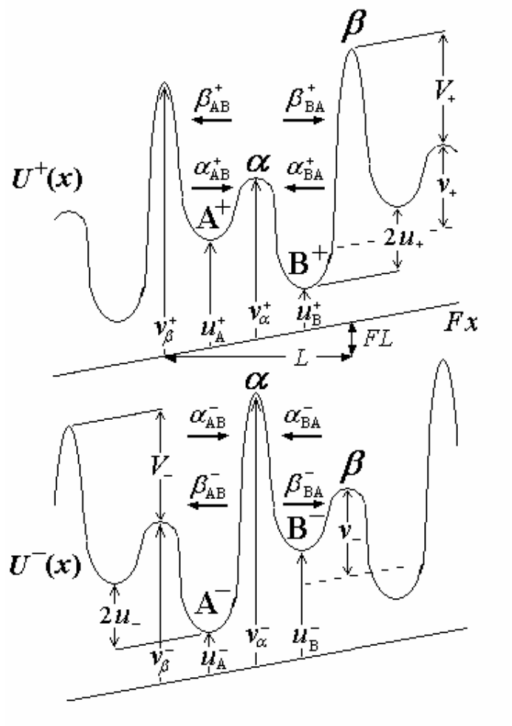

FIG. 2. Two periodic profiles of the potential energy $U^{\pm}(x)$ fluctuating for half a period in the field of the load force $-F$ ($F > 0$). For both profiles, the depths of the wells $A^{\pm}$, $B^{\pm}$ and the heights of the barriers $\alpha^{\pm}$, $\beta^{\pm}$ are given, as well as the corresponding characteristics of the profiles and the rate constants for overcoming the barriers.

of thermal energy, as follows:

$$p_1^r = \exp(-V-u-f), \quad p_1^l = \exp(-v-u+f),$$
$$p_2^r = \exp(-v+u-f), \quad p_2^l = \exp(-V+u+f). \tag{7}$$

In our antisymmetric model, the efficiency η of converting the expended energy into useful work, determined by formula (3), can be calculated by the formula:

$$\eta = \frac{f}{u}\frac{J}{J_\gamma} = \frac{f}{u}\left(\frac{\beta_{AB}+\beta_{BA}-\alpha_{AB}-\alpha_{BA}}{\Sigma} + \frac{J_f}{J_\gamma}\right). \tag{8}$$

Since the transition rate constants α_{AB} and α_{BA} (corresponding to the reaction channel α^- in the state $''-''$) and the numerator of the expression for the current J_f in Eqs. (5) ($\alpha_{AB}\beta_{BA} - \alpha_{BA}\beta_{AB} = -2k_0^2 e^{-V-v}\sinh(2f)$) are proportional to e^{-V} and hence exponentially small for large values of the larger barrier V, then $(\beta_{AB} + \beta_{BA} - \alpha_{AB} - \alpha_{BA})/\Sigma \to 1$, $J_f/J_\gamma \to 0$, and the efficiency tends to unity as . This can be directly verified through the analysis of the expression

$$\eta = \frac{f}{u}\left[\frac{\cosh(u-f) - e^{-V+v}\cosh(u+f)}{\cosh(u-f) + e^{-V+v}\cosh(u+f)} - \frac{2\zeta}{\varphi}\frac{e^{-V+v}\sinh(2f)}{\sinh(u-f) + e^{-V+v}\sinh(u+f)}\right],$$
$$\Sigma/\Gamma = \zeta[\cosh(u-f) + e^{-V+v}\cosh(u+f)], \quad \zeta = (k_0\tau/2)e^{-v}. \tag{9}$$

obtained by direct substitution of Eqs. (6) and (7) into Eq. (8). Note that, at large values of the barrier V, the contribution to efficiency, which depends on the period of the dichotomous process τ and on whether this process is deterministic or stochastic, is also exponentially small.

3. Simulation of ratchet characteristics with discrete time variation

Let $q = 0, 1, 2, \ldots$ be the number of time intervals of duration $\Delta t \approx k_0^{-1}$ between the attempts of a particle to move from one node of the 1D structure to a neighbouring one, characterized by the probabilities (7). We will

assume that at the initial moment of time the particle was located in an even node 2 in a state characterized by a potential profile $U^-(x)$ (see Fig. 2), so that the probabilities of transitions to node 1 are determined by the quantities $p_2^{r,l}$ according to Eq. (7). After g time intervals of the duration Δt, the state with the profile $U^-(x)$ is replaced by an antisymmetric state with the profile $U^+(x)$, in which nodes 1 and 2 are interchanged, and the corresponding transition probabilities (7) have the inverted indices 1 and 2; it is this inversion that implements the shift of the double-well profile by half a period. Generally, the lifetime of each antisymmetric state is assumed to be equal to the half-period $\tau/2$, therefore $\Delta t = \tau/(2g)$, and the particle jumps will occur at the time instants

$$t = q\tau/(2g). \tag{10}$$

Assuming $\Delta t = k_0^{-1}$, we obtain an obvious relation for the quantity g: $g = k_0\tau/2$. Note, that, for the system under consideration, its characteristic relaxation time τ_{rel} to the equilibrium state is determined by the quantity $(\alpha_{AB}^- + \alpha_{BA}^- + \beta_{AB}^- + \beta_{BA}^-)^{-1}$. Since the coefficient k_0 is equal to $(\Delta t)^{-1}$ in order of magnitude, then

$$\tau_{rel}/\Delta t = [2e^{-v}\cosh(u-f) + 2e^{-V}\cosh(u+f)]^{-1}. \tag{11}$$

From Eq. (11) we can estimate the minimum number of hops $q_{\min} = \tau_{rel}/\Delta t$ for the system to relax to a given potential profile. Thus, for $g > q_{\min}$, the system has time to relax while, for $g < q_{\min}$, does not. The motion of the particle along the node chain will be characterized by an integer variable $l = 0, \pm1, \pm2, ...$, whose odd and even values correspond to nonequivalent nodes 1 and 2. Since the period of the elementary cell is L, then the length of each step made by the particle and determined by the distance between neighbouring potential wells, is equal to $L/2$; then, the corresponding displacement coordinate is $x = lL/2$. Therefore, the trajectory of the particle hopping motion is described by the function $x(t) = l(q)L/2$, where is determined by Eq. (10). The average over the set of trajectories is then

$$\langle x(t)\rangle = \frac{L}{2}\langle l(q)\rangle. \tag{12}$$

Since the product FL in units of thermal energy is represented as $F = 4f/L$, Eq. (1) becomes

$$E_{\text{out}} \equiv 2f\langle l(q)\rangle. \tag{13}$$

The main development presented in this paper is the method of accounting for the energy of the ratchet. We introduced a discrete variable, $w = 0, \pm1, \pm2, ...$, which is needed to implement a hopping numerical model for calculating (simulating) the energy characteristics of a ratchet. Since the states are switched (the potentials are shifted by half a period) when q multiples g (see Eq. (10)), and just at these time steps the variable can change by $+1$ or -1 depending on the parity of the node and on which state the transition occurs from. The algorithm for these changes is defined as follows. From Fig. 2 one can see that, in the state with the potential profile $U^-(x)$, the well A^- corresponding to node 1 is deeper, so that the vertical transition to the well A^+ corresponding to node 2 is accompanied by the acquisition of energy $2u$, which is described by the increase w by unity. On the contrary, the vertical transition from well B^- to well B^+ (transition from node 2 to node 1) is accompanied by the loss of energy $2u$ and is described by the decrease by unity. If transitions between wells occur from the state with the potential profile $U^+(x)$ to the state with the potential profile $U^-(x)$, then the direction of changes of the value of the random variable w is reversed. Thus, the variable w can be represented as a function of an integer argument $[q/g]$: It is constant for q non-multiples of g and changes by unity for q multiples of g. Introducing the average value $\langle w(q)\rangle$ over an ensemble of trajectories, we represent the average energy transferred to a particle due to a change in its potential energy in the form:

$$E_{\text{in}} = 2u\langle w(q)\rangle. \tag{14}$$

Then the energy conversion efficiency is modeled by the formula

$$\eta = \frac{f}{u}\frac{\langle l(q)\rangle}{\langle w(q)\rangle} \tag{15}$$

with sufficiently large values of q. Greater values of efficiency can be expected at high barriers V blocking the reverse motion of the particle. The simplest simulation of hopping diffusion, easily implemented in software, corresponds to the symmetric case $p_1^r = p_1^l = p_2^r = p_2^l = 1/2$. If the initial state of the particle is given by the values $q = 0$ and $l = 0$, then at $q = 1$, the particle with equal probability goes to the point $l = 1$ or $l = -1$. Then the dependence $l(q)$ will be a broken line (trajectory) starting at zero and deviating from the horizontal line in both directions. We are interested in the dependence $\langle l(q)\rangle$ averaged over an ensemble of a large number of trajectories. The above symmetric case is characterized by zero mean, $\langle l(q)\rangle = 0$, while the standard deviation $\langle l^2(q)\rangle$ is proportional to q, as it should be for the free diffusion, in which the standard deviation of the particle is proportional to the elapsed time. Consider the asymmetric case. Let in a state with an odd value of the discrete coordinate l (location node) of a particle, the probability of the event $l \to l + 1$ is p_1^r, of the event $l \to l - 1$ is p_1^l, and the probability for the particle coordinate to remain unchanged is $1 - p_1^r - p_1^l$. In the even node l, the values of similar quantities p_2^r and p_2^l will be considered different from the values p_1^r and p_1^l in the case of odd l. The detailed balance condition which keeps the mean value $\langle l(q)\rangle$ constant is formulated as $p_1^r p_2^r = p_1^l p_2^l$. To illustrate and test the computer implementation of the model, we choose the values used in [18] for $f = 0$ and satisfying the detailed balance condition:

$$p_1^r = 0.2,\ p_1^l = 0.3,\ p_2^r = 0.54,\ p_2^l = 0.36. \tag{16}$$

Since $p_1^r \neq p_1^l$, we have $p_2^r \neq p_2^l$. The geometric interpretation of the considered asymmetric case follows from an analogy with the scheme of motion of a Brownian particle in an asymmetric double-well periodic potential profile (see Fig. 1 in the absence of the slopes at $F = 0$). According to the Arrhenius law, the probabilities of transitions from one potential well to another are proportional to the exponential function of the ratio of the potential barrier to thermal energy taken with the opposite sign (see Eq. (7) at $f = 0$). The discussed asymmetric case is then modeled as follows. Let $l(0) = 0$. Since the initial value l is an even number, the particle displacements are determined by the second set of probabilities p_2^r and p_2^l. Since $p_2^r + p_2^l = 0.9 < 1$, then there is a non-zero probability that the position of the particle at the next moment of time will not change, remaining still zero. With this outcome of a random trial, the second set of probabilities will again be used; similar results will occur until a random event is realized, namely, the transition of a particle to a node with an odd number. After this, the displacements of the particle will be determined by the set of probabilities of the first type with parameters p_1^r and p_1^l. Depending on the result, that is whether the particle stays at the odd node or moves to the even one, the first or second set of probabilities will be used again, and so on. The described algorithm is easy to realize by a programming language. At each step q, the value of a basic random variable uniformly distributed over the interval $[0, 1]$ is generated; it is checked for hitting the interval $[0, p^r]$, $[p^r, p^r + p^l]$ or $[p^r + p^l, 1]$. In the first case, the particle is shifted forward by one node with the addition of unity to the current value of l ($l(q) := l(q - 1) + 1$), in the second case it is shifted back by one node with subtraction from l by unity ($l(q) := l(q - 1) - 1$), in the third case, the value l does not change ($l(q) := l(q - 1)$). An array of discrete values for all steps is stored. The procedure being repeated K times, K trajectories $l(q)$ are obtained, which are then averaged to obtain the desired average trajectory $\langle l(q)\rangle$. As noted above, the detailed balance condition ensures the constancy of the mean value $\langle l(q)\rangle$. At $f \neq 0$, the detailed balance condition is violated, since according to Eq. (7):

$$p_1^r p_2^r - p_1^l p_2^l = -2e^{-V-v}\sinh(2f) < 0. \tag{17}$$

This leads to the particle shift to the left and the decrease of the average value $\langle l(q)\rangle$ with the increase of the discrete time q. The ratchet effect is able to shift the average values $\langle l(q)\rangle$ with the change of q even if the detailed balance condition is met. For that, it suffices to invert indices 1 and 2 of the probabilities in Eq. (7) every steps of the time variable q. Such an inversion corresponds to transitions between antisymmetric states with potentials shifted by half a period. The set of parameters (16) at $f = 0$ corresponds to the potential relief with

$$u = 0.2939, \quad v = 0.9101, \quad V = 1.3155. \qquad (18)$$

For the set of values (18), the results of modeling the behaviour of $\langle l(q)\rangle$ and $\langle w(q)\rangle$ are shown in Fig. 3 for different loads f and in Fig. 4 for $f = 0$. The characteristic relaxation time for the parameters (18) is $\tau_{rel}/\Delta t \approx 0.7$, so the thermodynamic equilibrium will be established during the lifetime of each potential profile $g = 6$. This leads to the following regularities.

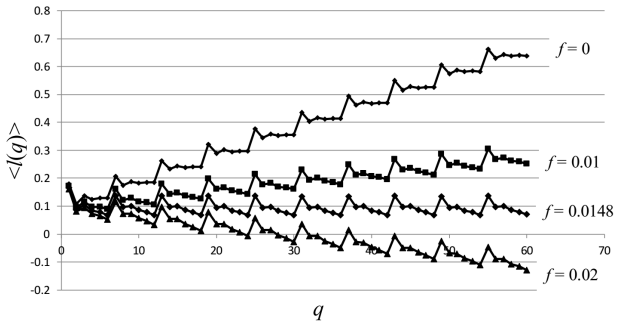

FIG. 3. Dependencies of the average values $\langle l(q)\rangle$ of the chain node occupancy on discrete time q, calculated for the set of parameters (18) with $g = 6$ by numerical simulation with averaging over 3 million trajectories.

The average displacements $\langle l(q)\rangle$ experience jumps at the moments of potential profile shift by half a period, and then for time intervals shorter than the lifetime, they either remain constant at $f = 0$ or decrease with increasing q at $f > 0$, since the direction of f is opposite to the direction of particle motion. For $f < 0.0148$, the values of the

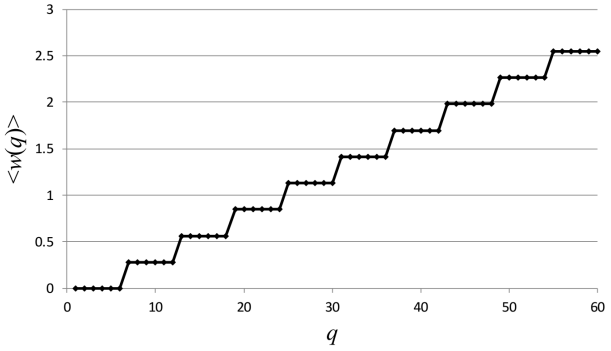

FIG. 4. Dependence of the average values $\langle w(q)\rangle$ of the accumulative energy variable, which determines the energy input into the system, on the discrete time q, calculated for the set of parameters (18) with $g = 6$ at $f = 0$ by numerical simulation with averaging over 3 million trajectories.

function $\langle l(q)\rangle$, additionally averaged over half-period, increase and the ratchet effect dominates the load force, while for $f > 0.0148$, the load force becomes dominant. Therefore, for the chosen parameters, the value $f_s \approx 0.0148$ is an estimate of the force corresponding to the ratchet stopping point. The value of the efficiency (11) with $f = 0.01$ turns out to be very small, $\eta \approx 0.002$. This result follows from the smallness of the potential barriers (18): they are of the order of the thermal energy and cannot provide the reverse motion blocking which is necessary for high efficiency in the discussed case. To simulate the operation of a highly efficient ratchet with deterministic fluctuations of the potential for half a period and discrete time, we will use parameters from Ref. [10], where the same potential were chosen for the theoretical description of a stochastic ratchet:

$$u = 4.5, \quad v = 5.5, \quad V = 14.5. \qquad (19)$$

Figures 5 and 6 show the dependencies of the mean value of the particle displacement $\langle l(q)\rangle$ and the energy transferred to the particle $\langle w(q)\rangle$ as functions of the number of steps q for various values of f. Since, for the set (19), the energy barriers are high compared to the thermal energy, the probabilities for the particles to overcome

these barriers are rather small, so that the characteristic relaxation time $\tau_{rel}/\Delta t$ will be 32 at $f = 2.5$ and 71 at $f = 3.5$, and the lifetime of each potential profile, $g = 6$, is too small for the thermodynamic equilibrium to be established in these profiles. As a result, the average displacements $\langle l(q)\rangle$ treats the potential profile shifts worse and, therefore, are smoother functions of q than those corresponding small barriers (18) (see Fig. 4). From Eq. (17), the deviation of the system from the detailed balance condition is the smaller, the higher the potential barriers. Therefore, the decrease of the values of $\langle l(q)\rangle$ with increasing q at $f > 0$ is not observed. Since the particle does not have time to get into a deeper well during the lifetime of the potential, the transferred energy oscillates after the potential switching; subsequently, the oscillations decay and the decay is faster, the shorter the characteristic relaxation time (see Fig. 5).

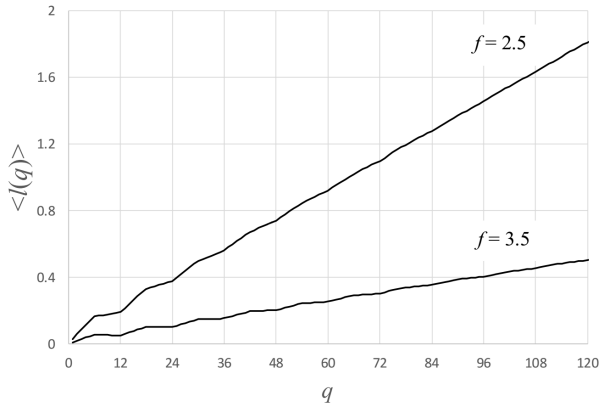

FIG. 5. Dependencies of average values $\langle l(q)\rangle$ of the chain node occupancy on discrete time q, calculated for the set of parameters (19) with $g = 6$ by numerical simulation with averaging over 50 million trajectories.

In Fig. 7, the dependence of the efficiency (15) on the load force for the parameters (19), obtained as a result of simulation with a discrete time change, is compared with the theoretical dependence (9), obtained with a continuous change in time. The stopping-point force is $f_s \approx 4.16$, and the maximum efficiency value reaches

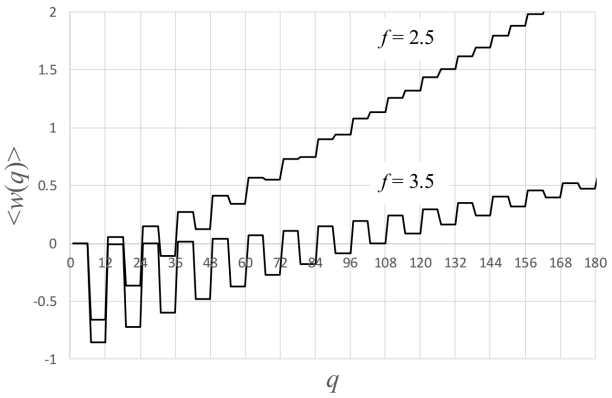

FIG. 6. Dependencies of the average values $\langle w(q)\rangle$ of the accumulative energy variable, which determines the energy input into the system, on the discrete time q, calculated for the set of parameters (19) with $g = 6$ by numerical simulation with averaging over 50 million trajectories.

0.71 at $f \approx 3.3$. The analytical dependence η on f (9) with a continuous change in time reproduces well the simulation results, except for the maximum region: the maximum value of the efficiency reaches 0.605 at $f \approx 3.2$. The theoretical value of the stopping-point force reproduces the simulation result $f_s \approx 4.16$ with good accuracy.

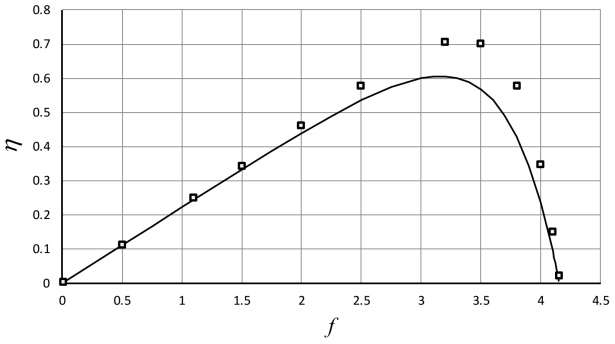

FIG. 7. Efficiency of the ratchet operating due to dichotomous shifts of the double-well potential profile with the parameters (19) for half a period as a function of the load force. The markers correspond to discrete-time simulations at $g = 6$ and averaged over 50 million trajectories. The curve is calculated by Eq. (9) obtained with the continuous change in time with $k_0\tau = 2g = 12$.

4. Discussion and conclusions

In this paper, the method of Parrondo's paradoxical games with discrete time variation is used to simulate the energy characteristics of ratchets that operate due to hopping diffusion in a periodic double-well potential profile fluctuating for half a period. The time dependences of the work performed by the particle against the load force (output energy) and the energy transferred to the particle when switching potentials (input energy) have been calculated. We showed that the average displacements of a Brownian particle experience jumps at the moments of shifts of the potential profiles, and then, for time intervals shorter than the lifetime of the potential profiles, either remain constant if the load force is absent or decrease with discrete time otherwise. The average values of the energy entering the system change only at the moments of potential profile shifts. These changes are positive at low load forces, but they can also have alternating negative contributions with a general tendency to increase the input energy at high forces. The ratchet efficiency has been calculated as a function of the load force at the moments of potential switching. This value ceased to depend on the time when the process became steady. Two sets of parameters of the potential relief have been used, with energy barriers of the order of and exceeding the thermal energy. The simulation results showed that, in the second case, the ratchet efficiency can be high, up to 70.%, since the reverse particle current is blocked. The nonmonotonic function describing the dependence of the efficiency on the load force was compared with a similar function obtained for the antisymmetric kinetic ratchet model with a continuous change in time. The good agreement between the results makes the game theory approach promising in simulation of the energy characteristics of ratchets.

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