Quantal Theory of Gravity (QTG): Essential points and implications

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"Quantal Theory of Gravity" (QTG) is a new undertaking that describes the behavior of classical and quantum particles in a gravitational field – and, in fact, any field the object at hand interacts with – on the basis of the law of energy conservation. QTG successfully combines metric and dynamical methodologies via a conjoint quantum mechanical formulation. Accordingly, a wave-like test object consisting of a quantal part and a corpuscular part – whose energies are initially identical and which start as concentric – must get torn apart when it engages gravity; and such a test object should then be treated separately as a two-entity problem. We further show that the said problem can be reduced to a single-entity problem. This straightforwardly delivers a new quantal equation of motion, and points to a novel metric expression of space-time wherefrom one can reverse-engineer all of the classical findings of the past century. Said feature constitutes one of the principal novelties in this contribution. Thus, QTG and the General Theory of Relativity (GTR) yield, within the measurement precision, identical results for classical problems, except singularities, through though totally different means. What is more, QTG separately explains the propagation of projectile-like objects such as high-energy γ-quanta (which thus do not behave wave-like); in which case, QTG predicts the nullification of gravitational attraction. This constitutes another principal novelty of QTG, the manuscript at hand brings up (which is backed by a recent experiment). Finally, we show how GTR could have so successfully coped with the known classically measured results only (and amazingly though) as a consequence of the quantal application of QTG and its single-entity approach. That constitutes the final and most cardinal novelty we herein bring to attention.

Keywords: Quantal Theory of Gravity (QTG), General Theory of Relativity (GTR), Yarman's Approach, gravitational bending, Schwarzchild metric, Yilmaz metric, non-bending of γ-quanta, nullification of gravitation, Gharibyan Experiment

1. Introduction

The current undertaking, which we hereafter call Quantal Theory of Gravity (QTG), deals with massive quantum objects in the approximation of a classical gravitational field. It is principally based on the law of energy conservation and is in full symbiosis with Quantum Mechanics (QM). It has no precedent, which justifies why we do not herein feel the need to delve into quantum theories of gravity only trying to circumvent the difficulties about advancing General Theory of Relativity (GTR), along with efforts spent to quantize it. Moreover, we find it important to emphasize that QTG is a full and working alternative to GTR, and not merely a complementary theory put together to salvage GTR.

Section 2 lays out the fundamental features of QTG.

In Section 3, we frame a series of quantum mechanical theorems led to by the pioneering Yarman's Approach (YA) [1]. A principal and novel aspect of QTG is that a gravitating test object gets torn apart, where the *quantal part* and the *corpuscular part* extend off-center and where such a formalism should then be treated separately. Nevertheless, we relegate this two-entity problem for a future work.

All the same, we show in Section 4 that the whole task can be reduced to a single-entity conceptualization; we handle it via fixing the motion of the test object altogether to the motion of its corpuscular part, wherefrom we derive the equation of motion for the *quantal* (*wave-like*) case. We show that, in the case of weak gravity, said solution agrees with all of the centennial space-time findings attributed to the success of GTR. However, in the limit of a strong gravitational field, the predictions of GTR and QTG turn out to be very much different; in particular, *QTG excludes singularities.*

Section 5 features two familiar classical problems: i) the gravitational deflection of visible light, and ii) the precession of the perihelion of a planet, where we show that both theories agree with each other within the achieved measurement precision.

Further, in Section 6, we present an ad hoc metric expression of QTG wherefrom one can independently derive all of the known results so far thought to validate GTR.

In Section 7, we outline an attempt towards the quantum mechanical deployment of QTG by starting with the expression of the law of energy conservation and the de Broglie relationship written in *quantum mechanical operator* form, which can be reduced under weak gravity to a Schrödinger-type Equation written for gravity.

We conclude in Section 8 by underlining the remarkable coincidence of the confirmed predictions of GTR [2] and QTG, but only with regards to our *wave-like case* and within the upcoming single-entity approximation, whilst pointing out the existence of a unique phenomenon – *i.e.*, the practically null deflection of high-energy *γ*-quanta behaving *projectile-like (and not wave-like)* in the presence of gravity [19]. This latter result totally falls outside the scope of GTR, which should thus be conclusively tested.

We remind that the words *quantal* and *wave-like* are used interchangeably in this paper. Likewise, we use the terms *projectile-like* and *corpuscle-like* interchangeably.

2. Fundamental aspects of QTG

QTG originates from YA [1, 3, 4], which is grounded upon the following postulate:

- *The rest mass – or the same, rest energy (if one supposes the velocity of light to be unity) – of an object interacting with a host body is less than its rest mass $m_{0\infty}$ measured in empty space; and this, as much as the static binding energy E_B it delineates in the given force field.*

This postulate expresses, in fact, the energy conservation law in the presence of gravity or any other non-radiative field. It is indeed so, given that the *classical potential energy associated with the "field"* can be imagined to be *not in the field itself, but* localized inside the object.[1] This holds true in the case where the gravitational field represents a function of the state,[2] which is always valid if one assumes gravitational waves to be neglected absent – thus, for the vast majority of problems. Our approach can naturally be extended to any non-radiative force field such as the bound electromagnetic (EM) field for electrically charged particles – which, in the absence of EM radiation, can indeed be described as a function of the present space-time coordinates [5].

[1] For this reason, we will particularly avoid the usage of the term "potential energy" in the text, and rely instead on the term "static binding energy"; meaning the energy one has to furnish to quasistatically lower a client entity from infinity to a given altitude above the center of the source body, or vice versa, to bring it from the location of concern, still quasistatically, back to infinity. The "static binding energy", unlike "potential energy", is always a positive quantity.

[2] What this means is that the "field" represents a function of the present space-time coordinates of the source body, and does not depend on the prehistory of its motion. It is akin, for instance, to what gets realized in classical electrodynamics with respect to a bound EM field.

Thus, the overall energy $E_{Overall}$ of a stationary closed system made of a test object of rest mass $m_{0\infty}$ *(defined at infinity)* bound to a *source body* of ponderable rest mass M must remain constant; and, in the case where M[[$m_{0\infty}$, we can write [3,4]

$$E_{Overall} = \text{¥}\, m_{0\infty}c^2\left(1 - E_B / m_{0\infty}c^2\right) = Constant\ , \tag{1}$$

where E_B is the *static binding energy* the test object acquires vis-à-vis the source body (with both being strictly at rest at the onset), c is the speed of light in vacuum, and ¥ is the Lorentz factor associated with the test object.

At a first glance, one tends to question whether the mass of a supposedly indivisible elementary particle such as the electron will decrease in gravity, too. It is useful to recall that the concepts of “mass” and “energy” are the same had c been taken to be unity. So, any “mass decrease” is synonymous with “internal energy diminution”, and it is precisely what happens when a test object is embedded in a gravitational field according to the present approach. There is effectively an easy way to check this out, and the measurements do in fact corroborate us. It is sufficient to replace the electron bound to an atomic nucleus with a muon. The bound-muon decay-rate is then retarded precisely due to the decrease of its rest mass in accordance with T. Yarman’s quantum mechanical theorems to be stated below [6].

We would hence like to point out that, in QTG, under the constant energy framed by Eq. (1), the test object’s rest mass $m(r)$ varies with the change of its distance r to the host object as assessed by the distant observer through the corresponding dependence of the static binding energy on r. This characteristic fundamentally distinguishes the present theory from Newtonian mechanics.

In order to determine the expression of $E_B(r)$, we adopt a gravitational force attraction expression resembling Newton’s law of gravitation in principle, but in a form novelly implying a *rest mass decrease* for the gravitating client object [1, 3, 4]; that and also a gravitational constant G as assessed by the distant observer, which is different from G_0 as measured locally like Cavendish did on Earth (as will be elaborated on soon):

$$F_{Distant\,Observer} = G\frac{Mm(r)}{r^2}. \tag{2a}$$

Thus, one may notice that we are no more within the classical Newtonian bounds. It is all the more so since even the $1/r^2$ dependence of the gravitational force is not borrowed from Newton’s law of gravitation, but can be shown to be a requirement imposed by T. Yarman’s quantum mechanical theorems which we will expound below [1].

We should emphasize that Eq. (2a) is nothing but the adoption of the YA postulate expressed by Eq. (1), which precisely means the test object of original rest mass $m_{0\infty}$ becomes a variable quantity depending on its spatial location with respect to the host. This, in turn, implies a corresponding variation of the space-time metric of the test object with the variation of its spatial coordinate r.

Eq. (2a) further manifestly signifies that the metric of space-time is different for a “distant observer” compared to what it will be for the “local observer” attached to the test mass $m_{0\infty}$. The “distant observer”, or “remote observer”, is attached to, as usual, a frame of reference situated at a far enough distance where there is practically no effect of gravity by the host. Conversely, the “local observer” is attached to the co-moving frame on the surface of the test object. Lengths and durations are naturally measured by the *local observer* situated on the surface of the test object as being different in comparison to what the *distant observer* measures [1]. We will see below how transformations can be made between the quantities measured by the local observer and those measured by the distant observer.

We emphasize that Eq. (2a) is written for the distant observer only with regards to interacting masses at rest – where r is the distance between M and $m(r)$ as assessed by the distant observer; with G not being a constant quantity when assessed by the remote observer, since the latter differs from the gravitational constant G_0 as measured by the local observer attached to the test object the way we will detail below.

Next, we define the distance r_0 of the test object to the host body as measured by the local observer attached to the test object; *e.g.*, he could do this via sending a signal unaffected in principle by gravity (see below) to the host and measuring the period of time it will take for the beam to go forth, bounce back, and return.

It is also worth noticing that, just as in GTR, the time rate of clocks located at different spatial points turn out to be different in general in the presence of gravity (see below). This means, in particular, that the distance r_0 measured by the local observer to the host differs, in general, from the distance r occurring between the same objects but as measured by the distant observer. (Take heed, nevertheless, of the fact that we use the wordings "host body" and "source body" synonymously, just like "test object" and "client object" stand for the same thing for us.)

The present approach accordingly furnishes an *invariant* expression between G, G_0, r and r_0 in terms of [1]

$$\frac{G}{r^2} = \frac{G_0}{r_0^2}. \tag{2b}$$

Based on this, we can re-write Eq. (2a) as

$$F_{Distant\,Observer} = G_0 \frac{Mm(r)}{r_0^2}. \tag{2c}$$

This is the gravitational force measured by the *distant observer* between the test object and the host, with both being at rest, and with r_0 assessed by the local observer attached to the test object.

One can moreover express Eq. (2c) as [1, 3, 4]

$$F_{Local\,Observer} = G_0 \frac{Mm(r)}{r^2}, \tag{2d}$$

which defines the gravitational force between the test object and the host (where both are again at rest) as assessed by the local observer attached to the test object, and with r referring to the distant observer.

Eq. (2d) specifies that, in the case where r is measured by the distant observer, we imply a force as assessed by the local observer, and when we have r_0 as measured by the local observer in Eq. (2c), we imply the same force but as assessed by the distant observer. We emphasize that in Eqs. (2a) – (2d) both masses M and $m(r)$ are at rest with respect to each other.

The force law stated by Eq. (2d) and written in view of the local observer attached to the test object allows the determination of the *static binding energy* taking place in Eq. (1). For this purpose, we displace the test mass $m(r)$ being at rest at r *quasistatically* from r to $r+dr$. The energy dE we deliver to it is

$$dE = F_{Local\,observer}\, dr. \tag{2e}$$

Unlike the common viewpoint, and in the absence of radiation losses, T. Yarman has postulated instead that [3, 4] the surplus dE must be stored inside of $m(r)$, thus becoming $m(r+dr)$; and if $m(r+dr)$ is now set to a free fall, the kinetic energy dK the said test object would acquire on the way from $r+dr$ to r ought to be compensated by the transformation of a minimal rest mass *dm(r)* out of its internal energy, so that $dK=dE=c^2dm(r)$. This, then owing to the law of energy conservation, entails the relationship [3, 4]

$$dm(r)c^2 = G_0 \frac{M\,m(r)}{r^2} dr\,. \tag{3}$$

The integration of Eq. (3) yields:

$$m(r)c^2 = m_{0\infty} c^2 e^{-\alpha(r)}, \tag{4}$$

where we posed

$$\alpha(r) = G_0 M / rc^2\,. \tag{5}$$

The static binding energy $E_B(r)$ according to Eq. (4) becomes

$$E_B(r) = m_{0\infty}c^2 - m(r)c^2 = m_{0\infty}c^2\left[1 - e^{-\alpha(r)}\right]. \tag{6}$$

In the corresponding expression for the static binding energy for the distant observer, the distance r in Eq. (5) must be expressed in terms of r_0.

Next, combining Eqs. (6) and (1), we obtain:

$$E_{Overall} = \gamma\, m_{0\infty} e^{-\alpha} = Constant\,. \tag{7}$$

Hence, the equation of motion for a test particle acquires the general form

$$\frac{d}{dt}\left(\gamma m_{0\infty} e^{-\alpha}\right) = 0\,, \tag{8}$$

which shows that its solution does not depend on the mass of the test particle.

Therefore, the postulate (1) is fully compatible with the weak equivalence principle (where the motion is, in effect, independent of the mass of the object under consideration). It is furthermore in full harmony with the outcome of Eötvös-type experiments [7].

What is more, YA also implies the validity of both local Lorentz invariance and local position invariance, which makes it compatible with the strong equivalence principle, too.

Furthermore, we wish to point out that we arrived at the motional equation Eq. (8) only on the basis of Eqs. (1) and (2d) without any explicit adoption determination of any space-time metric.

This already indicates that QTG does not belong to the category of purely metric theories of gravity, but rather combines both metric and dynamical characteristics.

Next, we emphasize that YA and its basic postulate (1) are applicable to any entity including a quantum of light, where the photon rest mass is assumed to be non-zero no matter how small it might be and how much it may lie below the limits of actual measurement capabilities [8-10]. Note that not assuming a photonic rest mass insurmountably remains, in general on the other hand, at odds with the wave-particle duality, as argued, e.g., in [12]. Thence, photons seem to must anyhow possess a finite rest mass and cannot ever exactly attain the asymptotic upper bound velocity *c*.

The present framework does not, as such, make any distinction between ordinary matter and a photon whether or not they behave wave-like or corpuscule-like.

3. T. Yarman's quantum mechanical theorems between mass-energy-frequency-length and their implications: Identification of the two-entity problem

As shown in [3, 4], the variation of the rest mass of any particle given by Eq. (4) induces a corresponding variation of its temporal unit T_0 and spatial unit L_0 (both measured in empty space and at rest) owing to the intrinsic quantum mechanical relationship of YA between the quantities "mass-energy-frequency-length".

Thus, consider a relativistic or a non-relativistic quantum mechanical description of a given object, depending on whichever is appropriate. Said description points to an *internal dynamic* (such as, *e.g*., that of a diatomic molecule) which consists of a regularly repetitive *"clock motion"* (such as, *e.g*., the vibration of the given molecule) realized in a *"clock space"* (such as, *e.g*., the average distance between the atoms of the molecule) during a *"unit period of time"* (such as, *e.g*., the inverse of the frequency of the vibrational motion of the molecule). The quantum mechanical description of concern is supposed to be based on K particles altogether. In this framework, the theorem stated below is fulfilled [3, 4].

Theorem 1: If the rest masses m_{k0} (k =1, …, K) constituting the object at hand are multiplied by the arbitrary number ξ, then the total *energy eigenvalue* E_0 associated with the clock motion of the given quantum object at rest in the field is increased as much – or the same, *the unit period of time* T_0 of the motion associated with this energy is decreased as much. The *characteristic length* L_0 to be associated with the space the clock motion takes place in is also decreased as much. In mathematical words, these results can be re-phrased as:

$$[(m_{k0}, k=1, \ldots, K)\rightarrow(\xi m_{k0}, k=1, \ldots, K)] \ \hat{\mathrm{G}} \ (E_0\rightarrow\P E_0),\ (T_0\rightarrow T_0/\P), \text{ and } (L_0\rightarrow L_0/\P). \qquad (9)$$

We note that this theorem, along with Eq. (4), already predicts the frequency shift of the electromagnetic radiation emitted by any excited state of the object at hand in the presence of gravity – *i.e.*, the gravitational redshift – regardless of the definition of the metric properties of space-time. Thus, we state the following theorem.

Theorem 2: The unit period of time T_0 of the clock of the test object, measured at infinity and at rest, retards quantum mechanically due to the mass deficiency it undergoes in gravity as much as the static binding energy it delineates within the field under consideration. Concurrently, its original space size L_0 in which it is installed stretches as much.

Apart from the foregoing elaborated cases, the relativistic mass of any wave-like behaving object in motion – *for instance, the relativistic mass m_i of a photon measured in vacuum* – gets increased during the gravitational fall, where the object speeds up following the relationship shaped by de Broglie [8]; *i.e.*,

$$m_i c^2 = hf_i,\ m_i = m_{0\infty}/(1-v_i^2/c^2)^{1/2}, \qquad (10a), (10b)$$

with h being the Planck constant, m_i being the *initial relativistic mass* of the test object, v_i being its initial velocity, f_i being its initial quantal frequency, and $m_{0\infty}$ being its rest mass before it engages gravity.

Let us stress that the de Broglie relationship (10a) has a general validity and is applicable to any wave-like behaving ordinary body, including evidently a wave-like behaving photon. Still, the test object at hand may not act wave-like in the medium of concern; in which case, we would not be able to start with Eq. (10a). *(This leads, in turn, to the nullification of gravitational attraction as we will elaborate below.)*

The philosophy de Broglie developed along with Eq. (10a), and the way he derived his wavelength relationship as $\lambda_B = h/p$, is summarized in Appendix A; p here is the momentum of the test object at hand.

The appearance of the rest mass $m_{0\infty}$ in Eq. (10b) may be taken to mean that this equation is principally written for the local observer.

In any event, Eqs. (10a) and (10b) can be extended to any test object characterized by the original rest mass $m_{0\infty}$ when it engages gravity with the instantaneous wave-like frequency f and the instantaneous velocity $v=v_{hf}$ at a given altitude and time, so as to allow one to write

$$m_{hf}c^2=hf,\ m_{hf}=¥\ m_{0\infty}. \qquad \text{(10c), (10d)}$$

Thereby, we state the next theorem.

Theorem 3: In a wave-like (quantal) case, given that the initial mass $m_{0\infty}$ decreases by $e^{-\alpha}$ based on Eq. (1), the quantity $(1-v_i^2/c^2)^{1/2}$ accordingly decreases as much. Thus, the initial relativistic mass $m_i=m_{0\infty}(1-v_i^2/c^2)^{-1/2}$ increases as much to become $m_{hf}=m_ie^{\alpha}$ at the given altitude. By the same token, the initial frequency f_i increases as much to become $f=f_ie^{\alpha}$; which makes that the corresponding wavelength $\lambda=\lambda_ie^{-\alpha}$ shortens as much.

We notice that Theorem 3 provides us with a straightforward explanation of the Pound & Rebka experiment [11] (see Section 4 for details). One can moreover note that the essence of Theorem 3 constitutes a concrete measurement of the de Broglie frequency's increase in motion [8] *(next to the relativistic decrease of the said frequency by the same amount)*, which happens to be an intriguing topic debated for a long time.

Next, using Eq. (4), we transform Eq. (10d) to the form

$$E_{Overall} = ¥\, m_{0\infty}c^2e^{-\sim} = m_{hf}c^2e^{-\sim} = m_ic^2 = Constant\ . \qquad \text{(10e)}$$

For an easier presentation, we propose to multiply the above equation by $m_{0\infty}$:

$$\left(¥\, m_{0\infty}\right)\left(m_{0\infty}e^{-\sim}\right) = m_{hf}m_{0\infty}e^{-\sim} = m_{0\infty}\times Constant\,/\,c^2 = Constant'\ . \qquad \text{(10f)}$$

Thus, we have two equations, (10c) and (10e), written for two unknown variables: i) f or γ, ii) and α. We can express them in terms of two distinct masses: i) the quantal mass $m_{hf}=hf/c^2$ operating *en bloc*, and ii) the corpuscular mass $m_{0\infty}e^{-\alpha}$ pertaining to the core component of the particle at hand. We can refer to the core as the *kernel* in the case of a photon [12].

Note that both the *quantal mass* $m_{hf}=hf/c^2$ and the *core rest mass* $m_{0\infty}e^{-\alpha}$ are equal to the proper rest mass $m_{0\infty}$ when they are measured in vacuum.

The quantal mass m_{hf} is, by its nature, the same as what a local observer would assess.

The mass $m_{hf}=hf/c^2$ is a wave-like quantity, which is thus to be considered effective locally if any quantum mechanical interaction between the test object and gravity were to arise.

The mass $\gamma m_{0\infty}e^{-\alpha}$, on the other hand, is measured as referred to the distant observer. The energies hf and $\gamma m_{0\infty}e^{-\alpha}c^2$ are clearly different from each other, and their motions will be described via different equations, as shall be seen below.

While hf and $\gamma m_{0\infty}e^{-\alpha}c^2$ are concentric in the absence of gravity, they get torn apart in gravity and extend off-center if tracked by the remote observer as elaborated on in Appendix B. Anyway, both masses in question are locally identical.

All the same, $m_{hf}=hf/c^2$ speeds up in a fall as viewed by the distant observer. The necessary energy for this is supplied by a minimal transformation of the core's rest mass $m_{0\infty}e^{-\alpha}$, which will recoil a fortiori as we will elaborate on further below. The two masses of concern are then no more concentric as assessed by the distant observer.

Take heed that the value of $m_{hf}=hf/c^2$ represents a proper value when assessed by the remote observer. Its fall velocity $v=v_{hf}$ is moreover the same for both the remote observer and the distant observer given that, under the present approach, periods of time (*i.e.*, durations) and lengths are transformed conformally in gravity [1].

Come what may, the mass $m_{hf}=hf/c^2$ will interact quantally in a wave-like regime, and in such a case, both the local observer and the remote observer will agree on the manner of its interaction.

While all this necessitates the treatment of the test object as a two-entity model, we shall nevertheless concentrate on treating it as a single-entity in this paper.

We remind that Theorem 3, following de Broglie's fundamental setup, refers solely to the wave-like (quantal) case, and is no longer applicable to an object interacting non-quantally with gravity. We will also consider this as an independent issue below.

We additionally like to emphasize that Theorem 1 demarcates a ubiquitous scaffolding which holds at every stage of the organization of matter [13].

Had the quantities T_0 and L_0 entering into Theorems 1 and 2 for some object with the rest mass $m_{0\infty}$ been embedded in a gravitational medium at an altitude r from the host body, then the corresponding quantities (at rest) would become $T(r)$ and $L(r)$ when assessed by a resting distant observer outside the interacting system, both of which would then be a function of E_B (see Eq. 6); *i.e.*,

$$T(r) = \frac{T_0}{1 - E_B(r)/m_{0\infty}c^2} = T_0 e^{\sim} , \tag{11a}$$

$$L(r) = \frac{L_0}{1 - E_B(r)/m_{0\infty}c^2} = L_0 e^{\sim} . \tag{11b}$$

Let us stress that the transformations (11a) and (11b) for the "local unit period of time" T_0 and the "local unit length" L_0 are to be written for the object at hand when it is considered *at rest* in gravity. Otherwise, and in case we have a quantal motion at hand, Theorem 3 must be exercised.

At this stage, we can relate the distance r between the host and the test object, as assessed by the distant observer, to the same distance r_0, but as measured by the local observer [1, 3, 4]:

$$r = r_0 e^{\alpha}. \tag{11c}$$

Thus, because the pace of the *local clock* is slower than that of the *twin clock* situated far away sitting next to the *distant observer*, the number of ticks registered by the *local clock* for a round trip signal sent to the host *(which is supposedly unaffected by gravity, and which is possible in cases when the signal behaves in a corpuscular fashion)* will be less than the same number of ticks registered but by the distant observer. The differential of Eq. (11c) yields [1, 3, 4]:

$$dr = dr_0 \frac{e^{\sim}}{1+\sim} , \tag{11d}$$

so that dr and dr_0 are practically the same in a weak gravity. Recall that Eqs. (11a) – (11d) are written in the static case – *i.e.*, where the objects are practically *at rest* with respect to each other. That is to say, they do not involve the peculiarities of Theorem 3.

The equation of motion of the test particle can, on the other hand, be directly determined through the differentiation of Eq. (7) [7, 14, 15].

In such a way, QTG successfully combines the properties of both metric and dynamical theories; and such a combination, as shown, ensures compatibility between gravitation and QM [4, 11, 14]. In other words, it is the quantum mechanical rest mass decrease in gravity (or any field the test object interacts with) which leads to the known gravitational redshift together with size stretching the way delineated by Theorems 1 and 2.

Recall that QTG can equally handle a wave-like behaving particle and a corpuscle-like behaving particle *(which we will deal with a little ahead).*

In the wave-like case, we anticipate the following: While, say, the wavepacket *hf* built on just the proper rest mass $m_{0\infty}$ [*cf.* Eqs. (10c) and (10d)] gets accelerated in a free fall, the mass of the core $m_{0\infty}e^{-\alpha}$ supplying the necesssary energy will a fortiori recoil (see further below); so *the object eventually gets torn apart.*

During the lagging behind of the gravitating core $m_{0\infty}e^{-\alpha}$ in comparison to hf/c^2 cruising just a bit ahead which, under the current simplified single-entity model, is the only constituent the distant observer can track, i) the wavelength λ remains to be associated with the wavepacket *hf*, and ii) T is ascribed to the locally fixed twin clock's mass $m_{0\infty}e^{-\alpha}$ when assessed by the remote observer.

Thus, hereinafter, we save the subscript "$_0$", or similarly "$_i$", to label proper quantities, which shall then be differently assessed by the remote observer.

Hence, when the proper energy wavepacket $hf_0=hf_i$ ticks as observed by a locally fixed observer, in order to cross a proper distance equal to the wavelength $\lambda_0=\lambda_i$, it would spend a proper period of time $T_0=T_i$ thereat as referred to a fixed local twin clock associated with the twin rest mass $m_{0\infty}$ situated at the given altitude in the possession of the said local observer.

According to such a conception, the two quantitites λ and T, based on the local underlying quantitites λ_0 and T_0, will therefore be transformed in different manners from the viewpoint of the distant observer.

In the case where, in a gravitational fall, hf_0 and $m_{0\infty}$ are supposed to move with the same velocity v_0 for the locally fixed observer, we are to write for the fall velocity v_{core} of the recoiling core $m_{0\infty}e^{-\alpha}$, the way assessed by the distant observer,

$$\neg_{core}=\frac{a(r)}{T(r)}=\frac{a_0 e^{-\sim}}{T_0 e^{\sim}}=\neg_0 e^{-2\sim} \tag{12}$$

in full accordance with Theorem 3.

A cross-check of this result, based on the momentum conservation between the wavepacket energy *hf* and the core's instantaneous rest energy $m_{0\infty}e^{-\alpha}c^2$, will be provided below.

4. The general setup of the problem pertaining to a wave-like regime and the equation of motion of the single-entity in a radially symmetric gravity

Addressing Eq. (8), and carrying out its differentiation, we obtain [3, 4]:

$$-\frac{G_0 M m_{0\infty} e^{-\sim}}{r^2 c^2}\sqrt{1-v_0^2/c^2}\,dr=\frac{v_0 dv_0}{c^2}\frac{m_{0\infty}e^{-\sim}}{\sqrt{1-v_0^2/c^2}}\,. \tag{13a}$$

Note that, here, v_0 pertains to the velocity of the test object, which is the same for both the quantal mass $m_{hf}=hf/c^2$ and the mass of the core $m_{0\infty}e^{-\alpha}$ of the test object when measured locally as expressed in Eq. (12). Moreover, these two masses are naturally identical *in vacuum* and *at rest* according to de Broglie's setting (Eq. (10a)).

The above equation leads to

$$-\frac{G_0 M}{r^2 c^2}\left(1-v_0^2/c^2\right)dr=\frac{v_0 dv_0}{c^2}\,. \tag{13b}$$

Take heed that this equation happens to feature a suppression factor in front of the Newtonian force term.

One can, in keeping with Theorem 3, re-arrange Eq. (13b) as follows to arrive at QTG's wave-like regime [3]:

$$-\frac{G_0 M\, m_i e^{\sim}}{r^2}dr=v_0 d\left(m_i e^{\sim} v_0\right). \tag{14a}$$

This makes the current undertaking compatible with centennial findings thought up to now to confirm Einstein's GTR in view of his adoption of the equality of the *gravitational mass* and the *inertial mass,* aside from the increase of *yet* the *quantal mass* in gravity under QTG [Eq. (10a)].

While Eq. (14a) is, so far, the same equation as (13b), still, it says that the initial quantal mass $m_i=hf_i/c^2$ increases exponentially in a gravitational fall – which, however, is ensured in our approach by the exponential decrease of the core's mass $m_{0\infty}$ as expressed by Eq. (13a). Any singularity is henceforth excluded in our case.

Theorem 3, on the other hand, makes certain that, in a gravitational fall, a *quantal mass increase* takes place together with a length contraction in full concordance with what GTR proposes; *i.e.*, conjoint mass increase and length contraction in gravity, together with time dilation in the radially symmetric metric. In QTG, though, the latter property is secured on the basis of Theorem 2.

In other words, no mass increase would take place in our quantal case without a conjoint mass decrease.

Take further heed that the process underlined via Eq. (14a) removes the supressing factor $(1-v_0^2/c^2)$ that appears in front of the gravitational force. Let us stress that this happens in our quantal case and shall not take place if the test object at hand does not behave quantally in gravity.

Eq. (14a) is therefore a key equation of QTG. Based on it, one can derive all the known experimental results considered up until today to confirm GTR. As a first example, in Subsection 4.1, we will show how the outcome of the familiar Pound & Rebka experiment [11] can be easily understood within the framework of our approach.

Then, in Subsection 4.2, we will derive the angular momentum conservation law.

Eq. (14a) shall further allow us to pin down the velocity v_{core} of the core of mass $m_{0\infty}e^{-\alpha}$ that gets recoiled in a gravitational fall as assessed by the remote observer.

4.1 Equation of motion of the quantal part *hf*: Pound & Rebka result

Let us remind that we consider the *locally measured quantal mass* $hf/c^2=m_{0\infty}(1-v^2/c^2)^{-1/2}$ and the *corpuscular mass* $m_{0\infty}e^{-\alpha}$ of the test object, which is assessed by the remote observer, as distinct quantities no matter whether or not they are both written based on the mass $m_{0\infty}$. They are not equal in the first place unless they are at rest and in vacuum. Hence

$$e^{\alpha}=m_{hf}/C, \tag{14b}$$

where C is a constant.

Next, combining Eqs. (14a) and (14b), we obtain:

$$-\frac{G_0 M\, m_{hf}}{r^2 c^2}dr = v_0 d\left(m_{hf} v_0\right), \tag{14c}$$

where v_0 is the velocity of the mass $hf_0/c^2=m_{0\infty}(1-v_0^2/c^2)^{-1/2}$ as gauged by the locally fixed observer – and, the way we conceptualized the setup at hand, it *has the same value for both the local observer and the remote observer* given that lengths and periods of time stretch by the same amount as viewed by the remote observer [*cf.* Theorem 2, and Eqs. (11a) and (11b)].

Thus, the instantaneous fall velocity $v=v_{hf}$ of the quantal mass hf/c^2 is the same as v_0, given that the lengths and the periods of time under the present approach effectively stretch by the same amount in gravity.

Next, we remind the relationship:

$$\frac{\boldsymbol{r}\cdot d\,\boldsymbol{r}}{rdr} = \frac{\boldsymbol{v}_0\cdot d\,\boldsymbol{v}_0}{v_0 dv_0}, \tag{14d}$$

and, combining Eqs. (14c) and (14d), as well as using the equality $v_{hf}=v_0$, we obtain

$$-\frac{G_0 M m_{hf}}{r^2}\frac{\boldsymbol{r}}{r}=\frac{d\left(\boldsymbol{v}_{hf}\, m_{hf}\right)}{dt}. \tag{14e}$$

Eq. (14e) describes the motion of the quantal mass $m_{hf}=hf/c^2$ in gravity [*cf.* Eqs. (10c) and (10d)], and it is precisely how we readily tapped the outcome of the Pound & Rebka experiment in [11]. The above equation implies the identity of the *gravitational mass* and the *inertial mass* just like it is asserted under GTR; but in QTG, this happens only when the test object behaves wave-like.

Also, Eq. (14e) is valid for both the local observer and the remote observer, except that, in view of the distant observer, r should ultimately be replaced by r_0e^{α} [*cf.* Eqs. (2c) and (11c)].

Further on, one has to recall that Eq. (14a) allows the treatment of the quantal mass hf/c^2 and the mass of the core $m_{0\infty}e^{-\alpha}$ as a single-entity. The full two-entity solution shall be handled elsewhere.

Using Eq. (3), we can present Eq. (14a) in the form

$$\left|dm\right|\frac{c^2}{v_0 e^{-2\sim}}=m_{0\infty}d\left(e^{\sim}v_0\right)\ , \tag{15a}$$

which expresses the momentum conservation pertaining to the wave-like case.

It can be written alternatively as

$$\left|dm\right|c^2=m_{0\infty}v_0e^{-2\sim}d\left(e^{\sim}v_0\right)\ . \tag{15b}$$

Reading Eq. (15a), we can conclude that the core of the local mass $m_{0\infty}$ of the said entity depletes the infinitesimal rest mass $|dm|$ through an infinitely short gravitational fall involving a de *Broglie-like phase propagation velocity* $c^2/v_0e^{-2\alpha}$ [8] (*cf.* Appendix A)

This governs the *exchange of energy* between the core $m_{0\infty}$ and the wavepacket hf_0, thus delivering an infinitesimal kick forward to hf_0. It further implies that the core a fortiori recoils in a gravitational fall and its cruise velocity v_{core} is

$$v_{core}=v_0e^{-2\alpha}. \tag{15c}$$

This result is the same as that of Eq. (12).

Briefly, this means that the core $m_{0\infty}e^{-\alpha}$, as assessed by the remote observer, must come to cruise behind the wavepacket hf, again as assessed by the remote observer.

We can now depart from Eq. (15b), or the same Eq. (14c), in treating the object at hand as a single-entity, where its motion is nailed to that of the core $m_{0\infty}e^{-\alpha}$. Thus, in the said model, the object made of the masses hf/c^2 and $m_{0\infty}e^{-\alpha}$ altogether moves with the velocity v_{core} as expressed by Eq. (15c). *(Otherwise, as conveyed, hf would move with the velocity* $v=v_0=v_{hf}$*.)*

4.2 Angular Momentum Conservation and Equation of Motion for a single-entity

Recalling that $dr/dt=v_{hf}$, and using the equivalent of c^2dm following Eq. (3), we get from Eq. (15a)

$$-\frac{G_0 M\, e^{\sim}}{r^2}\frac{\boldsymbol{r}}{r}=\frac{d(\boldsymbol{v}_0\, e^{\sim}\ )}{dt}. \tag{16}$$

It can be checked via Eq. (14b) that this is the same equation as Eq. (14e), except that we keep the term e^{α} instead of the Lorentz coefficient.

Eq. (16) is our equation of motion for a single-entity where i) v_0 at the rhs should yet be reverted to v_{core} via Eq. (12), and ii) r should yet be reverted to r_0 via Eq. (11c) as discussed in Section 2 given that we now work in the single-entity regime.

With regards to the analysis of Eq. (16), we shall first derive the angular momentum conservation law. For this purpose, we multiply both sides by the tracking vector *r:*

$$-\frac{G_0 M e^{\alpha}}{r^2}\frac{\boldsymbol{r}}{r}\times\boldsymbol{r} = \frac{d(\boldsymbol{v}_0 e^{\alpha})}{dt}\times\boldsymbol{r}\,. \tag{17a}$$

The lhs of the latter equation is null, and so must be the rhs:

$$\left[d\left(\boldsymbol{v}_0 e^{\alpha}\right)\big/dt\right]\times\boldsymbol{r} = 0\,. \tag{17b}$$

Let us add to this the vector quantity $(d\boldsymbol{r}/dt)\times\boldsymbol{v}_0 e^{\alpha}$ which – given that $\boldsymbol{v}_{hf}$ and $\boldsymbol{v}_0$ lie in the same direction – amounts to zero. Hence, we obtain:

$$\left(d\boldsymbol{r}/dt\right)\times\boldsymbol{v}_0 e^{\alpha} + \left[d\left(\boldsymbol{v}_0 e^{\alpha}\right)\big/dt\right]\times\boldsymbol{r} = \left[d\left(\boldsymbol{r}\times\boldsymbol{v}_0 e^{\alpha}\right)\big/dt\right] = 0. \tag{18}$$

The integral of this vector equation must then be a *constant vector* which we designate by $\boldsymbol{p}$*:*

$$\boldsymbol{r}\times\boldsymbol{v}_0 e^{\alpha} = \boldsymbol{p}\,. \tag{19}$$

The vector $\boldsymbol{p}$ obviously represents the same constant quantity both in view of the local observer and in view of the distant observer. All the same, recalling that we have to revert to $\boldsymbol{r}_0$ when $\boldsymbol{p}$ is assessed by the distant observer [see Eq. (2b)], and using Eq. (12), we get for the distant observer

$$\boldsymbol{r}_0 e^{\alpha}\times\boldsymbol{v} e^{2\alpha} e^{\alpha} = \boldsymbol{r}_0\times\boldsymbol{v} e^{4\alpha} = \boldsymbol{p}\,. \tag{20}$$

In order to compose the *total angular momentum,* we have to multiply Eq. (19) by the overall constant relativistic mass $m_{Overall}=E_{Overall}/c^2$ the way coined by YA [see Eq. (7)]; thus, the angular momentum vector $\boldsymbol{P}$ becomes a new constant to be written as:

$$m_{Overall} e^{4\alpha}\,\boldsymbol{r}_0\times\boldsymbol{v} = \boldsymbol{P}\,. \tag{21}$$

This quantity constitutes a *constant* of the motion of the test particle as long as we conceive it to be a single-entity at this stage. The *other constant* is provided by the total energy expressed by Eq. (7) which, in effect, is what we wrote straightforwardly as a special advantage provided by the current approach.

Based on the stated properties, we can present the acceleration vector $d\boldsymbol{v}/dt$ of the test object in terms of its tangential component and radial component to retrieve further information about the motion that the test object will delineate. The calculations are furnished in Appendix C, which yield

$$\frac{d\boldsymbol{v}}{dt} = -\frac{G_0 M}{r^2}\left[e^{-4a}\left(1+\frac{v_0^2}{c^2}\right)\frac{\boldsymbol{r}_0}{r_0} - \frac{4\left(\boldsymbol{r}_0\cdot\boldsymbol{v}\right)\boldsymbol{v}}{r_0 c^2}\right]. \tag{22a}$$

The force term over here is still written as assessed by the local observer attached to the test object [see Eq. (2d)]. The above equation, when viewed by the remote observer, must then be written via Eqs. (2c) and (11c) as

$$\frac{d\boldsymbol{v}}{dt} = -\frac{G_0 M}{r_0^2}e^{-2a}\left[e^{-4a}\left(1+\frac{v_0^2}{c^2}\right)\frac{\boldsymbol{r}_0}{r_0} - \frac{4\left(\boldsymbol{r}_0\cdot\boldsymbol{v}\right)\boldsymbol{v}}{r_0 c^2}\right]. \tag{22b}$$

Note that the term $e^{-2\sim}$ does not essentially change the final solution, for, it can be considered merely to serve in reducing the host mass M, which would simply push the orbits of the test object just a little bit outward were we to operate under a weak gravitational field.

Eqs. (22a) and (22b) thence furnish all the measured results that are indistinguishable from those led to by GTR (see, *e.g.*, [16]).

Recall that, although we did so far work in a spherically symmetric geometry in just the way set up through the rest mass decrease conceptualization [see Eq. (4) and Theorems 1 and 2], the possible extension of our approach to a non-spherical symmetry is somewhat trivial. For example, the gravitational redshift of a test object located near a spherically non-symmetric host mass constitutes just a matter of the familiar type of Newtonian integration of the effect created by an infinitely small mass of the given geometry over the entire host body.

5. Application of QTG to select problems

Next to the gravitational redshift – which, as we have shown, represents a quantum mechanical phenomenon – we will consider two other known astronomical phenomena as classical examples with regards to the applicability of Eq. (22a): *i.e.*, i) the gravitational deflection of visible light, and ii) the precession of the perihelion of a planet.

The solution to the first one is trivial. Indeed, consider a light beam propagating from r_0=-∞ to r_0=+∞ tangentially to the source of gravity (*e.g.*, a star). The second term within the brackets on the rhs of Eq. (22a) can be written as $-4cos\theta$, with θ being the angle drawn by the tracking vector $\boldsymbol{r}_0$, and the velocity vector $\boldsymbol{c}$ being associated with light. We can assume for simplicity that the light beam travels rectilinearly towards the impact point of the source of gravity. In such a case, θ varies from ® to 0, and the integral of cosθ is therefore vanishing throughout the path of light. On the other hand, we can consider the exponential term $e^{-4\alpha}$ as unity, given that, say, for the Solar surface, α turns out to be about 10^{-6}. The first term within the brackets on the rhs of Eq. (22a), in the case we have v_0=c, then becomes 2.

What all this really amounts to is just stating that the Newtonian force term per unit mass – *i.e.*, G_0M/r_0^2 – gets merely multiplied by virtually speaking, factor 2. So, just like in GTR, the radial component of the gravitational force exerted by the source of gravity on the client object amounts to twice the classical Newtonian force – but *extraordinarily*, only in the *wave-like case* of QTG, which led to Eq. (22a). Take notice that this was already conspicuous from Eq. (12).

Next, we consider, still in the wave-like case, the precession of the perihelion of a planet as scrutinized by a distant observer, and work out in Appendix D the solution that QTG furnishes; which amounts to an identical result as compared to what is led to by GTR [17] within the measurement precision. As seen till now, the calculations are straightforward and, unlike GTR, are based on the two constants we could write directly via our Eqs. (1) and (20) even without the determination of the metric of space-time.

We finalize this Section with the *most striking result* of QTG: *i.e.*, its power to describe *projectile-like entities* where Theorem 3 no longer holds – wherefore the equation of motion reduces to Eq. (13b) which, after re-arrangement, gets to be written as

$$\frac{d\boldsymbol{v}}{dt} = -\frac{G_0 M}{r^2}\left(1-\frac{v_0^{\,2}}{c^2}\right)\frac{\boldsymbol{r}}{r} \,. \tag{23}$$

One can see that this equation yields practically zero gravitational deflection when the velocity of the test object tends to c, and such a result becomes exciting when we realize that the said equation is well applicable to *high-energy γ-quanta* that always delineate projectile-like behavior. Thus, QTG predicts almost no gravitational bending for such projectile-like entities; our prediction regarding this can be found in [18]. All this remains entirely at odds with metric theories of gravity where the metric of space-time cannot depend on the energy of the particles. It is therefore no wonder that experimental results on the subject matter are definitely in conflict with GTR [19].

6. The space-time metric in QTG

It can be shown that the motional equation (22b), if we overlooked the term $e^{-2\alpha}$ multiplying the force term on the rhs, could be obtained independently via the minimization of action as defined in the standard way through the *ad hoc* metric expression

$$c^2 d\tau^2 = e^{-2\alpha} c^2 dt^2 - dr^2 e^{2\alpha} - r_0^2 e^{2\alpha} \left(d\theta^2 + sin^2\theta \, d\varphi^2 \right) \tag{24a}$$

in spherical coordinates – thus for the *quantal case only.* Astonishingly enough, this is, except for r_0, nothing else but the Yilmaz metric [20, 21]. In arriving at it, Hüseyin Yilmaz basically aimed to repair the singularity pitfall of GTR after having noticed that no singularity occurs in Einstein’s accelerated elevator gedanken experiment.

In view of Eq. (11d), the above metric equation can be re-phrased as

$$c^2 d\tau^2 = e^{-2\alpha} c^2 dt^2 - dr^2 e^{2\alpha} - r^2 \left(d\theta^2 + sin^2\theta \, d\varphi^2 \right), \tag{24b}$$

which is, this time, reduced to the Schwartzschild metric in the case of weak gravity where $e^{2\alpha} \approx 1-2\alpha$ – thus establishing a *remarkable bridge that was unrealized until now* between the Yilmaz metric and the Schwartzschild metric in the given gravity constraints.

Eqs. (22a) and (22b), together with Eqs. (24a) and (24b), but separately from each other, lead to all of the verified relativistic effects – *viz.*, gravitational bending of light, advance of the perihelion of elliptical orbits, Shapiro delay (see, *e.g.*, [22]), *etc.* – all within the quotidian measurement precision. At the same time, QTG drastically modifies the concept of a “black hole” (BH). Essentially, just the way we had elaborated on earlier [14], Eqs. (22a) and (22b), and thereby Eqs. (24a) and (24b) as well, entail no singularities or event horizons; hence any particle gathering a sufficient amount of energy inside of a “BH of the QTG type” can break out. In such a way, we resolve the *information paradox* [14]. Note further that, as underlined in Section 2, the coordinate r entering into the parameter α in the above metric expressions should be reverted to r_0; in which case, at sufficiently high α, the present approach hints at *anti-gravity.*

Anyway, the final umpire in choosing between QTG and standard theory remains experimentation. More importantly, the metric expression given by Eq. (24a) that we have provided with regards to our solution Eq. (22a) is not even mandatory, but is rather constrained to the framework of our single-entity model and constitutes nothing but an artefact (just like, in fact, our single-entity model itself). This will become even clearer following the derivation we aim to present with regards to our two-entity approach to be tackled in a subsequent paper.

Note that, in the corpuscular case of QTG we reviewed above, a different metric can straightforwardly be written (using the standard notation) in view of Eqs. (11a) and (11b) [1]:

$$s_0^2 = c^2 t_0^2 - r_0^2, \; s^2 = s_0^2 e^{2\alpha}. \qquad (24c), (24d)$$

7. Quantum mechanical deployment of the present approach

T. Yarman, together with the late Rozanov, made an attempt to develop a quantum mechanical deployment of the current undertaking in [23]. They started with Eq. (7), and via squaring it, they wrote

$$p_0^2(r) c^2 + m_{0\infty}^2 c^4 \, e^{-2\alpha} = E_{Overall}^2 , \tag{25}$$

with p_0 being the magnitude of the overall relativistic momentum of the test object moving with the velocity v_0.

Next, they considered the quantum mechanical equivalent of the momentum of the test object to write (with the standard notation):

$$\boldsymbol{p}_0(r) = -i\hbar\nabla \ , \tag{26}$$

where $\hbar$ is the reduced Planck Constant.

For a stationary system, Eq. (26) leads to a novel quantum mechanical equation

$$-\hbar^2\nabla^2 \cdot \ (r)c^2 + m_{0\infty}^2 c^4 e^{-2\alpha} \cdot \ (r) = E_{Overall}^2 \cdot \ (r) \ , \tag{27}$$

with $\cdot \ (r)$ being the eigenfunction.

This equation can be reduced to a Schrödinger-type equation through the equality

$$E_{Schr} = E_{Overall} - m_{0\infty}c_0^2 \tag{28}$$

if α and $E_{Schr} \,/\, (\, m_{0\infty}c^2 \,)$ are very small as compared to unity. Thus:

$$-\frac{1}{2m_{0\infty}}\hbar^2\nabla^2 \cdot_{\ Schr} - G_0\frac{M\ m_{0\infty}}{r_0} \cdot_{\ Schr} = E_{Schr} \cdot_{\ Schr} \ , \tag{29}$$

where $\cdot_{\ Schr}$ becomes the new eigenfunction within the adopted approximation.

This approach interestingly leads to the *Planck mass* if we wished to pin down the mass of the two identical objects bound to each other at the ground state with twice the *Planck length* as their separation distance. It can equally be applied to the atomic world as T. Yarman originally suggested in [3]. But all this lies outside of the scope of the current article.

The exciting quest, on the other hand, is that our root equation (7) engenders both a quantal description – but in a corpuscular manner [Eq. (22a)] – and a full quantum mechanical description of the system at hand [Eq. (29)]; and both appear to be correct. The former description would mean that electrons in the atomic world, still behaving quantally (given how we obtained the description through a “de Broglie setting” – *i.e.*, Eqs. (10a) and (10c)), move virtually in *geodesics;* and the latter would mean that we can profit from our quantum mechanical insights to describe complex gravitational systems behaving quantally, too. This yet falls outside of the scope of the present contribution as well.

We would like to add that our root equation (7) allowed us both i) to develop a cosmological model in [24] based on the current approach, where we could naturally deduce dark energy as a *fossil acceleration* of the early accelerated expansion the universe might have delineated at the beginning (which appears to be at the order of $10^{-9} \times$ Earth’s acceleration g), and ii) to explain the outcome of the measurements achieved by the LIGO and VIRGO installations [15].

8. Conclusion

Eq. (1), as the basis of Yarman’s Approach (YA), expresses the law of energy conservation for gravity and, in general, for any non-radiating field. An attractive advantage of the undertaking at hand is that it is written straightforwardly in an integral form and remains in full symbiosis with QM. It also reflects the known fact that any non-radiating field can be given as a function of the state (see, *e.g.*, [5]).

Next, we considered Eqs. (10c) and (10d), and furthermore Eq. (12), which constitute, in the quantal case of QTG, a plausible approximation to represent the structure made of the mass of the wavepacket hf/c^2 and that of the core $m_{0\infty}e^{-\alpha}$. These normally move conjointly but with different velocities, for they are conjectured to tear themselves apart over the long haul when engaging gravity, given that the core $m_{0\infty}e^{-\alpha}$ recoils whence fueling the speeding up of *hf* in a gravitational fall.

Such an event is conjectured for the first time herein, and our solution well predicts the emission of positrons and electrons from nearby a supermassive body. It is owing to the fact that an EM radiation falling in will eventually get torn apart and break down into its constituents. We remind that the quantal mass $m_{hf}=hf/c^2$ is based on the original core's mass $m_{0\infty}$ which, in turn, remains locally unaltered; recall that what is going to determine whether an object will interact quantally with gravity is evidently the mass value $m_{hf}=hf/c^2$ it assumes locally.

We could thus treat a quantally behaving object made separately of the wavepacket *hf* and the core $m_{0\infty}e^{-\alpha}$ as a single-entity. Thereby, we fixed the motion of the object made of *hf* and $m_{0\infty}e^{-\alpha}$ to the core's motion, which a fortiori lags behind hf/c^2; this brought up Eq. (12) as a plausible basis which can, in effect, be interpreted as the slowing down of the velocity of the test object in our single-entity case. Yet, in our approach, this is merely a useful artefact. It is further worth recalling that we tapped the rhs of Eq. (12), and more essentially, the philosophy we introduced to describe it, in our Eq. (15c), which constitutes an extension of Eq. (14a) – which, in turn, promptly led to the Pound & Rebka results [11].

In gravitation, said behaviour, as implied by Eq. (12), remarkably turned out to be the same as that framed by GTR *(barring yet metric singularities and the notion of an "event horizon");* this is so much so that we ended up with a metric expression (24a) which is compatible with our motional equation (22a).

A crucial question thence arises: How could Einstein succeed to put together GTR that well copes with the centennial observations based on the equivalence between the effect of acceleration and the effect of gravitation in conjunction with his Clock Hypothesis (CH)?

As is known, this hypothesis supposes that Einstein's gedanken rotating clock is affected by just its tangential motion, and acceleration has no effect on it, as such [2].

However, our recent experiments [25-27] severely put at stake the CH.

How then could the CH work so well under the framework of GTR?

The answer now becomes clear; as the spatial and temporal transformations experienced by the clock in a tangentially uniform translational motion the way assumed by Einstein – *i.e., mass increase, size contraction,* and *period of time stretching* – precisely occur solely in the quantal case of a free-falling object, which constitute the germane analogues of our Theorems 1, 2 and 3 – with the exception that *the relativistic mass increase of the object at hand, following Eqs. (8c) and (8d), is fueled by a tiny rest mass deficit and the recoil of the photon kernel in QTG's wave-like regime.*

Without this intricate piece of the puzzle, GTR was and could never be harmonized with QM.

Whenever the test object does not behave quantally, QTG predicts practically no light bending [18]; whereas GTR fails entirely (just like all other metric theories) in this regard – seeing as high-energy γ-quanta are observed not to bend when traversing over Earth's surface [19].

More importantly, the metric expression we have provided under Eq. (24a) with regards to our solution Eq. (22a) is not even mandatory, but is rather constrained to the framework of our single-entity model insofar as designating nothing other than an artefact (just like, in fact, our single-entity formulation itself). This is so much so that the metric to be written for a corpuscle-like behaving object [see Eq. (24d)] is not even the same as the one we wrote for a wave-like behaving object [see Eq. (24a)].

Anyway, the final umpire in choosing between the current approach and GTR remains to be experimentation.

Additionally, the present approach can be applied to all bound systems no matter what the force field coming into play may be. A good example is our prediction about the decay-rate retardation of a muon bound to an atomic nucleus [6].

Otherwise, GTR (known to be incompatible with QM) and the *wave-like case* of the current undertaking come to remarkably coincide, whereby QTG predicts almost the same results as GTR; be it through totally different means and save for the abandonment of singularities, event horizons, and black hole information paradoxes in our gravitation theory [14] that always upholds the law of energy conservation and remains in full symbiosis with QM.

Appendix A: The insight de Broglie had behind writing $m_ic^2=hf_i$ and his derivation of the wavelength relationship [8]

It is unfortunately not widely recalled nowadays that the classical de Broglie relationship about the wavelength λ_B for a test object of momentum *p, i.e.*

$$\lambda_B=h/p, \qquad (A1)$$

which constitutes the foundation of Quantum Mechanics (QM), is derived based on Eq. (10a) of the text [8]. Note that de Broglie considered here the test object as being initially at rest. Thus, de Broglie assigned an *intrinsic periodical phenomenon* of frequency $f_{0\infty}$ to the test object of rest mass $m_{0\infty}$ at hand and wrote

$$m_{0\infty}c^2=hf_{0\infty}. \qquad (A2)$$

We can grasp this via thinking of $f_{0\infty}$ as the frequency of the subsequent EM radiation had the original mass $m_{0\infty}$ been entirely annihilated. Such an argumentation was not known to de Broglie at all by the time he published his doctorate thesis in 1924, though [8]. Given that there may be several oscillatory motions inside $m_{0\infty}c^2$, one can perhaps more realistically denominate $hf_{0\infty}$ as the "overall wavepacket energy" encompassing altogether the energy $m_{0\infty}c^2$ and whose characteristic frequency is after all $f_{0\infty}$.

Further on, as a generalization, one can apprehend Eq. (10a) with $m_{0\infty}$ already being in motion. In de Broglie's understanding, the *overall oscillatory energy* $hf_{0\infty}$ is not absolutely confined within the boundaries of $m_{0\infty}c^2$ but rather has tails extending to the entire space, though with magnitudes quickly dying away along both directions of the itinerary the motion takes place in. An example de Broglie provides for this is the Bohr quantization rule he derived based on Eq. (A2). It brings about a standing wave that materializes athwart the fixed orbit of the electron as it supposedly moves around the proton in an Hydrogen atom – and we will review this shortly in the next Appendix.

The oscillatory energy $hf_{0\infty}$ is indeed not confined within the bounds of $m_{0\infty}c^2$ at all. It contrariwise extends to the entire orbit in the case of a charge revolving around a proton, insofar as culminating in the known picture of atomic orbitals.

No matter how "archaic" Eq. (A2) may look nowadays, it still constitutes, as we will summarize momentarily, the basis of QM. In effect, de Broglie thought that, when $m_{0\infty}$ is brought to a translational motion with the velocity *v*, it gets increased *special relativistically* by the Lorentz coefficient coming into play. According to his Eq. (ii), so must $f_{0\infty}$ too; which then becomes $f = f_{0\infty} / \sqrt{1-v^2/c^2}$. Conjointly, $f_{0\infty}$ must, as implied by the Special Theory of Relativity (STR), become fainter to assume the value $f^* = f_{0\infty}\sqrt{1-v^2/c^2}$. He could finally remove such an annoying dichotomy via adopting the idea that the *phases of the two oscillatory motions of frequencies f and f* are in constant harmony with each other* if the wave representing the increased frequency *f* moves with the velocity c^2/v. (Note anyway that a superluminal displacement of the phase not carrying any energy whatsoever is not prohibited by STR).

Thence, de Broglie came to write, with regards to his wavelength λ_B of Eq. (i),

$$\frac{c^2}{v} = \lambda_B f = \lambda_B f_{0\infty} / \sqrt{1 - v^2 / c^2} \, . \tag{A3}$$

This equation, via using Eq. (ii), leads, in turn, to the usual de Broglie relationship – *i.e.*, Eq. (A1):

$$\lambda_B = \frac{h}{m_{0\infty} v \big/ \sqrt{1 - v^2 / c^2}} = \frac{h}{p} \, . \tag{A4}$$

As one can see [*cf*. Eq. (A3)], had the rest mass $m_{0\infty}$ been brought to a motion of velocity v, the de Broglie wavelength λ_B would be made of the increased de Broglie frequency f. The passage from Eq. (iv) to QM is straightforward if one writes, as usual,

$$\boldsymbol{p}(r) = -i\hbar\nabla \ \ . \tag{A5}$$

Clearly, Eqs. (A4) and (A5) are in full conformity with each other. All this is tantamount to saying that Eq. (A2) we brought up in the text in reference to Louis de Broglie is not “archaic” after all.

Appendix B: Louis de Broglie’s derivation of the Bohr quantization rule as it pertains to the Hydrogen (H) atom, which hints at his insight behind writing $m_i c^2 = h f_i$: How in the present undertaking does the H atom get torn apart in a gravitational fall?

It will be useful to recall that Louis de Broglie, via his relationship Eq. (A3), was able to derive the Bohr quantization rule as it pertains to the Hydrogen (H) atom [8] featuring an electron revolving around the proton. This is transcribed, along with the familiar notation, as

$$2\pi r_i m_i v_i = nh, \ n = 1, 2, \ ... \tag{B1}$$

or the same, as

$$\lambda_{Bi} p_i = nh, \ n = 1, 2, \ ... \tag{B2}$$

It is written at an *initial state (i)* in empty space where the proton is assumed to be at rest. All quantities of interest are defined accordingly; $m_i=\gamma_i m_{0\infty}$ is the relativistic mass of the electron in its orbit, $m_{0\infty}$ is the rest mass of the electron, γ_i is the Lorentz coefficient in relation to the velocity v_i of the electron in its orbit, r_i is the radius of the orbit of the electron, λ_{Bi} is the de Broglie wavelength associated with the electron in its orbit (thereby exhibiting a stationary wave in the orbit), and p_i is the momentum of the said electron in its orbit.

Let us designate the initial stationary characteristic wave frequency f_i of the electron in its orbit in vacuum as

$$f_i=m_i c^2/h=\gamma_i m_{0\infty} c^2/h. \tag{B3}$$

This is the de Broglie frequency entering into Eq. (A3), where we will have to write

$$c^2/v_i=\lambda_{Bi} f_i \tag{B4}$$

for the electron in the H atom in vacuum.

Thus, we see that the de Broglie wavelength λ_{Bi}, together with the de Broglie frequency f_i coming into play with Eq. (B3), both go beyond the boundaries of the electron mass $m_{0\infty}$ inasmuch as covering up the entire orbit.

Suppose now that the H atom undergoes a gravitational fall; this is certainly a more complicated occurrence than the fall of a single-entity, which was undertaken in the text. Still, one interesting case is where the electron’s orbit remains perpendicular to the radial direction throughout the fall. The de Broglie characteristic wave frequency f of the electron in the said H atom at the given altitude would, as implied by Eq. (B3), have increased to the extent that it reads as

$$hf=mc^2=\gamma_{Fall}\gamma_i m_{0\infty}c^2, \tag{B5}$$

where $\gamma_{Fall}=\gamma$ is the Lorentz coefficient associated with the fall velocity $v=v_{Fall}$.

Note that Eq. (B5) embodies no other quantity than γ in relation with the gravitational fall in question. The increase of hf_i is, on the other hand, ensured by the decrease of the electron's rest mass $m_{0\infty}$ (in accordance with Theorems 1 and 2) which, as explained in the text, must a fortiori recoil. Accordingly, the orbit of the electron of rest mass $m_{0\infty}e^{-\alpha}$ will not any more be centered at the proton's center, nor shall the orbit of the instantaneous quantal mass hf/c^2. According to the present model, they altogether constitute a torus around the proton's center. The lower part of the torus encompasses the orbit of the quantal mass hf/c^2, and the upper part encompasses the orbit of the corpuscular electron of rest mass $m_{0\infty}e^{-\alpha}$.

It would further be interesting to consider what happens in the general case where the H atom gravitates without having the electronic orbital plane perpendicular to the radial direction. This however remains outside of the scope of the current contribution.

In brief, there arises two distinct masses in a fall in the example we considered: i) the quantal mass $m=hf/c^2$ which, in the present case, reads as $m=\gamma_{Fall}\gamma_i m_{0\infty}$, and ii) the corpuscular mass $me^{-\alpha}$ which, again in the present case, reads as $me^{-\alpha}=\gamma_{Fall}\gamma_i m_{0\infty}e^{-\alpha}$. They are locally indistinguishable, for, the proper observer has no means to determine that the core's mass $m_{0\infty}$ gets decreased. So, locally, both masses in question are the same. But, as viewed by the distant observer, $m=hf/c^2$ speeds up during the fall. The necessary energy for this is supplied by the core's mass $me^{-\alpha}$, which a fortiori recoils. The two masses are then no more concentric as assessed by the distant observer.

It is surely interesting that Richard Feynman's intuition from many decades ago, which we have come across when we studied the famous Feynman Lectures, "Optics: the Principle of Least Time", Chapter 26 – and more precisely what he wrote under Sub-Section 26-5, *i.e.*, *"A more precise statement of Fermat's principle"* – aligns with the topic at hand. He verbatim says this [28]:

- *The following is another difficulty with the principle of least time, and one which people who do not like this kind of a theory could never stomach. With Snell's theory we can "understand" light. Light goes along, it sees a surface, it bends because it does something at the surface. The idea of causality, that it goes from one point to another, and another, and so on, is easy to understand. But the principle of least time is a completely different philosophical principle about the way nature works. Instead of saying it is a causal thing, that when we do one thing, something else happens, and so on, it says this: we set up the situation, and light decides which is the shortest time, or the extreme one, and chooses that path. But what does it do, how does it find out? Does it smell the nearby paths, and check them against each other? The answer is, yes, it does, in a way. That is the feature which is, of course, not known in geometrical optics, and which is involved in the idea of wavelength; the wavelength tells us approximately how far away the light must "smell" the path in order to check it. It is hard to demonstrate this fact on a large scale with light, because the wavelengths are so terribly short. But with radiowaves, say 3-cm waves, the distances over which the radiowaves are checking are larger.*

Such an analogy serves to describe exactly what happens with regards to that which we tried to convey in the text around Eqs. (10a) – (10e). There, we elaborated on how the de Broglie frequency f of a wave-like behaving test object in a gravitational fall gets increased, and the necessary impetus is supplied by a minimal depletion of the core's rest mass $m_{0\infty}e^{-\alpha}$, which necessarily makes the latter recoil.

We consequently tackled the same idea in this Appendix with regards to a gravitating H atom, where we discovered that, according to the present approach, the H atom's said orbit becomes de-centered (see above).

As to the fall of a single-entity, we can now re-phrase what we said above in the light of Feynman's insight in reference to de Broglie's original writing of $hf_i=m_ic^2$ [*cf.* Eq. (10a) of the text and also Appendix A]:

- *When a wave-like behaving test object undergoes a gravitational fall, the entire wavepacket slides downward toward gravity to the extent that it is no more mostly localized inside the test object's mass boundaries, which thence allows the wavepacket's forward tail to "smell" further ahead, and the particle thus chooses the path it finally does. For this, the wavepacket and the factual mass de Broglie equated with the energy of the test mass at hand must effectively get de-centered.*

Appendix C: Equation of motion for the single-entity setup

First of all, we address Eq. (20) and obtain $\boldsymbol{r}_0\times\boldsymbol{v}=\boldsymbol{p}e^{-4\alpha}$. We then take its time derivative

$$\frac{d\left(\boldsymbol{r}_0\times\boldsymbol{v}\right)}{dt}=\boldsymbol{r}_0\times\frac{d\boldsymbol{v}}{dt}=-4\boldsymbol{p}\left(\frac{d\sim}{dt}\right)e^{-4\sim}\ , \tag{C1}$$

where the term $d\boldsymbol{r}_0/dt\times\boldsymbol{v}$ is, owing to Eq. (11d) of the text, practically equal to $d\boldsymbol{r}/dt\times\boldsymbol{v}$; the latter amounts to $\boldsymbol{v}\times\boldsymbol{v}$, thus vanishes. Accordingly, via Eq. (20), we arrive at

$$\boldsymbol{r}_0\times\frac{d\boldsymbol{v}}{dt}=\boldsymbol{r}_0\times\boldsymbol{v}\left(-4\frac{d\sim}{dt}\right). \tag{C2}$$

We notice that the vector $d\boldsymbol{v}/dt$ is not exactly oriented along the radial direction due to the fact that what is angular momentum-wise conserved is $\boldsymbol{r}_0\times\boldsymbol{v}e^{4\alpha}=\boldsymbol{p}$, and not the quantity $\boldsymbol{r}_0\times\boldsymbol{v}$ [see Eq. (20)]; so $\boldsymbol{r}_0\times d\boldsymbol{v}/dt$ does not vanish.

Next, we write for the total acceleration vector:

$$\frac{d\boldsymbol{v}}{dt}=\left(\frac{d\boldsymbol{v}}{dt}\right)_{tangential}+\left(\frac{d\boldsymbol{v}}{dt}\right)_{radial}. \tag{C3}$$

Therefore,

$$\boldsymbol{r}_0\times\frac{d\boldsymbol{v}}{dt}=\boldsymbol{r}_0\times\left(\frac{d\boldsymbol{v}}{dt}\right)_{tangential}+\boldsymbol{r}_0\times\left(\frac{d\boldsymbol{v}}{dt}\right)_{radial}. \tag{C4}$$

The last term of Eq. (C4) vanishes, and hence,

$$\boldsymbol{r}_0\times\frac{d\boldsymbol{v}}{dt}=\boldsymbol{r}_0\times\left(\frac{d\boldsymbol{v}}{dt}\right)_{tangential}. \tag{C5}$$

Thus, in view of Eq. (C2) and the definition of α we posed along with Eq. (5),

$$\left(\frac{d\boldsymbol{v}}{dt}\right)_{tangential}=\boldsymbol{v}\left(\frac{4\sim}{r}\frac{dr}{dt}\right). \tag{C6}$$

Now, we recall that $\boldsymbol{r}$ should eventually be converted to $\boldsymbol{r}_0$ as implied by Eq. (2c). We have, on the other hand, the equality

$$\frac{dr}{dt} = \frac{\boldsymbol{r}_0}{r_0} \cdot \frac{d\,\boldsymbol{r}}{dt} = \frac{\boldsymbol{r}_0}{r_0} \cdot \boldsymbol{v} \quad . \tag{C7}$$

Thus:

$$\left(\frac{d\,\boldsymbol{v}}{dt} \right)_{tangential} = \boldsymbol{v} \left(\frac{4\sim}{r} \frac{\boldsymbol{r}_0\,.\boldsymbol{v}}{r_0} \right) = 4\boldsymbol{v} \left(\frac{G_0 M}{r^2 c^2} \frac{\boldsymbol{r}_0\,.\boldsymbol{v}}{r_0} \right). \tag{C8}$$

Therefore, in the orbital plane we obtain

$$\left(\frac{d\,\boldsymbol{v}}{dt} \right) = 4\boldsymbol{v} \left(\frac{G_0 M}{r^2 c^2} \frac{\boldsymbol{r}_0\,.\boldsymbol{v}}{r_0} \right) + \left(\frac{d\,\boldsymbol{v}}{dt} \right)_{radial} \quad . \tag{C9}$$

The last term on the rhs is yet to be determined; this means we have to calculate the difference between the lhs and the first term of the rhs. Thus, we return to Eq. (7) and re-phrase the overall energy expression for the unit mass in the form

$$E_{unit\ mass} = c^2 e^{-\sim} \left(1 - v^2 e^{4\sim} / c^2 \right)^{-1/2}, \tag{C10}$$

which yields

$$\boldsymbol{v} \cdot \boldsymbol{v} = v^2 = c^2 e^{-4\sim} - c^6 e^{-6\sim} \Big/ E^2_{unit\ mass} \ . \tag{C11}$$

The derivative of this equation with respect time is

$$2\boldsymbol{v} \cdot \frac{d\boldsymbol{v}}{dt} = \frac{dr}{dt} \left(\frac{4\sim}{r} c^2 e^{-4\sim} - \frac{6\sim}{r E^2_{unit\ mass}} c^6 e^{-6\sim} \right). \tag{C12}$$

We then plug Eq. (C9) into Eq. (C12):

$$2\boldsymbol{v} \cdot 4\boldsymbol{v} \left(\frac{G_0 M}{r^2 c^2} \frac{\boldsymbol{r}_0\,.\boldsymbol{v}}{r_0} \right) + 2\boldsymbol{v} \cdot \left(\frac{d\,\boldsymbol{v}}{dt} \right)_{radial} = \frac{dr}{dt} \left(\frac{4\sim}{r} c^2 e^{-4\sim} - \frac{6\sim}{r E^2_{unit\ mass}} c^6 e^{-6\sim} \right). \tag{C13}$$

Recall that *dr/dt* can be written as $\boldsymbol{v} \cdot \boldsymbol{r}_0 / r_0$ [see Eq. (C7)]. Further, we re-arrange and use Eq. (C10) :

$$\boldsymbol{v} \cdot \left(\frac{d\,\boldsymbol{v}}{dt} \right)_{radial} = \boldsymbol{v}\,. \frac{\boldsymbol{r}_0}{r_0} \left[\frac{2\sim}{r} c^2 e^{-4\sim} - \frac{3\sim \left(1 - v^2 e^{4\sim} / c^2 \right)}{r c^4 e^{-2\sim}} c^6 e^{-6\sim} \right] - 4v^2 \left(\frac{G_0 M}{r^2 c^2} \frac{\boldsymbol{r}_0\,.\boldsymbol{v}}{r_0} \right). \tag{C14}$$

This leads to

$$\left(\frac{d\,\boldsymbol{v}}{dt} \right)_{radial} = \frac{\boldsymbol{r}_0}{r_0} \left[\frac{2\sim}{r} c^2 e^{-4\sim} - \frac{3\sim \left(1 - v^2 e^{4\sim} / c^2 \right)}{r c^4 e^{-2\sim}} c^6 e^{-6\sim} \right] - 4v^2 \left(\frac{G_0 M}{r^2 c^2} \frac{\boldsymbol{r}_0}{r_0} \right), \tag{C15}$$

or to

$$\left(\frac{d\,\boldsymbol{v}}{dt} \right)_{radial} = \frac{\boldsymbol{r}_0}{r_0} \left(2 \frac{G_0 M}{r^2} e^{-4\sim} - \frac{G_0 M}{r^2} 3 \left(1 - v^2 e^{4\sim} / c^2 \right) e^{-4\sim} \right) - 4v^2 \left(\frac{G_0 M}{r^2 c^2} \frac{\boldsymbol{r}_0}{r_0} \right), \tag{C16}$$

or to

$$\left(\frac{d\boldsymbol{v}}{dt}\right)_{radial}=\frac{\boldsymbol{r}_0}{r_0}\left[2\frac{G_0M}{r^2}e^{-4\alpha}-3\frac{G_0M}{r^2}e^{-4\alpha}+3\frac{G_0M}{r^2}\frac{v^2}{c^2}\right]-4v^2\left(\frac{G_0M}{r^2c^2}\frac{\boldsymbol{r}_0}{r_0}\right), \qquad \text{(C17)}$$

or to

$$\left(\frac{d\boldsymbol{v}}{dt}\right)_{radial}=-\frac{\boldsymbol{r}_0}{r_0}\left[\frac{G_0M}{r^2}e^{-4\alpha}+\frac{G_0M}{r^2}\frac{v_0^{\ 2}}{c^2}e^{-4\alpha}\right]. \qquad \text{(C18)}$$

Therefore,

$$\left(\frac{d\boldsymbol{v}}{dt}\right)_{radial}=-\frac{G_0M}{r^2}\left[e^{-4a}\left(1+\frac{v_0^2}{c^2}\right)\frac{\boldsymbol{r}_0}{r_0}\right]. \qquad \text{(C19)}$$

We can thus compose the total acceleration as follows:

$$\frac{d\boldsymbol{v}}{dt}=-\frac{G_0M}{r^2}\left[e^{-4a}\left(1+\frac{v_0^2}{c^2}\right)\frac{\boldsymbol{r}_0}{r_0}-\frac{4\left(\boldsymbol{r}_0\cdot\boldsymbol{v}\right)\boldsymbol{v}}{r_0c^2}\right]. \qquad \text{(C20)}$$

Appendix D: Orbital precession calculation for the single-entity case

To calculate the precession of the perihelion of a planet, we start with Eq. (20):

$$r_0v\sin\theta = pe^{-4\alpha}, \qquad \text{(D1)}$$

where θ is the angle between the vectors $\boldsymbol{r_0}$ and $\boldsymbol{v}$. Note that $v_\perp=v\sin\theta$ constitutes the magnitude of the velocity component lying along the direction perpendicular to $\boldsymbol{r_0}$.

Hence, the angular velocity $\omega=d\varphi/dt=v_\perp/r_0$ becomes

$$\omega=\frac{d\varphi}{dt}=\frac{v_\perp}{r_0}=\frac{pe^{-4\alpha}}{r_0^2}. \qquad \text{(D2)}$$

Introducing the radial component of the velocity $v_{//}=dr/dt$, and using the relationship $v^2=v^2_{//}+v^2_\perp$, we obtain

$$v^2=\left(dr/dt\right)^2+r_0^2\left(d\varphi/dt\right)^2. \qquad \text{(D3)}$$

Next, combining Eqs. (7) and (12) of the text, we get

$$v^2=c^2\left(e^{-4\alpha}-\frac{c^6e^{-6\alpha}}{E^2_{unit\ mass}}\right). \qquad \text{(D4)}$$

$E_{unit\ mass}$ is, by construct, *a constant scalar* written for a unit mass; $E_{unit\ mass}/c^2$ amounts to unity within the span of about 10^{-8}, *e.g.*, in the case of Mercury. Therefore, we can write

$$\left(dr/dt\right)^2+r_0^2\left(d\varphi/dt\right)^2=c^2\left(e^{-4\alpha}-e^{-6\alpha}\right). \qquad \text{(D5)}$$

Using Eq. (D2), we get

$$\left(dr/dt\right)^2+r_0^2\left(\frac{pe^{-4\alpha}}{r_0^2}\right)^2=c^2\left(e^{-4\alpha}-e^{-6\alpha}\right), \qquad \text{(D6)}$$

or

$$\left(dr/dt\right)^2 = c^2\left(e^{-4\sim} - e^{-6\sim}\right) - r_0^2\left(\frac{pe^{-4\sim}}{r_0^2}\right)^2, \qquad \text{(D7)}$$

or

$$dt = dr\left[c^2\left(e^{-4\sim} - e^{-6\sim}\right) - r_0^2\left(\frac{pe^{-4\sim}}{r_0^2}\right)^2\right]^{-1/2}. \qquad \text{(D8)}$$

Next, we use Eq. (D2) to replace *dt:*

$$d\ddot{\ } = \frac{dr}{r_0^2}\left[\frac{c^2\left(e^{4\sim} - e^{6\sim}\right)}{p^2} - \frac{1}{r_0^2}\right]^{-1/2}. \qquad \text{(D9)}$$

Let us pose $\rho = GM/c^2$, $u = 1/r_0$ and square both sides:

$$\left(\frac{du}{d\ddot{\ }}\right)^2 = \frac{c^2\left(e^{4\circ u} - e^{6\circ u}\right)}{p^2} - u^2. \qquad \text{(D10)}$$

We now consider the derivative of this equation with respect to *φ:*

$$2\left(\frac{d^2u}{d\ddot{\ }^2}\right)\frac{du}{d\ddot{\ }} = 4\circ\frac{du}{d\ddot{\ }}\frac{c^2e^{4\circ u}}{p^2} - 2\circ\frac{c^2e^{2\circ u}}{p^2}\frac{du}{d\ddot{\ }} - 2u\frac{du}{d\ddot{\ }}, \qquad \text{(D11)}$$

which we re-arrange as

$$\left(\frac{d^2u}{d\ddot{\ }^2}\right) = 2\circ\frac{c^2e^{4\circ u}}{p^2} - \circ\frac{c^2e^{2\circ u}}{p^2} - u. \qquad \text{(D12)}$$

In a weak gravitational field, one can write $e^{\circ u} \approx 1 + \circ u$. Hence, Eq. (D12) yields:

$$\left(\frac{d^2u}{d\ddot{\ }^2}\right) + u\left(1 - 6\circ^2\frac{c^2}{p^2}\right) = \circ\frac{c^2}{p^2} \qquad \text{(D13)}$$

versus the corresponding Newtonian equation

$$\left(\frac{d^2u_{Newtonian}}{d\ddot{\ }^2}\right) + u_{Newtonian} = \circ\frac{c^2}{p^2}. \qquad \text{(D14)}$$

The function *u(φ)* of Eq. (D13) thus becomes

$$u(\ddot{\ }) = \left(\circ c^2/p^2\right)/\left(1 - 6\circ^2c^2/p^2\right) + C_1\cos\left(\sqrt{1 - 6\circ^2\frac{c^2}{p^2}}\,\ddot{\ }\right), \qquad \text{(D15)}$$

where C_1 is a suitable constant. Therefore,

$$u(\ddot{\ }) \cong \left(\circ c^2/p^2\right) + C_1\cos\left(\sqrt{1 - 6\circ^2\frac{c^2}{p^2}}\,\ddot{\ }\right) \qquad \text{(D16)}$$

versus the corresponding Newtonian equation

$$u_{Newtonian}(\ddot{\ }) = \circ c^2/p^2 + C_1\cos(\ddot{\ }). \qquad \text{(D17)}$$

The precession angle during one revolution is therefore equal to

$$\text{¤}_{precession} = 2\text{®}\left(1 - \sqrt{1 - 6\frac{G^2M^2}{c^2p^2}}\right), \tag{D18}$$

where we have used the definition $\rho = GM/c^2$. In a weak gravity, this amounts to

$$\varphi_{precession} = 6\pi\frac{G^2M^2}{c^2p^2}, \tag{D19}$$

which is identical to the prediction of GTR within the measurement precision.

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